CAIM 2018

20^{th}–23^{th} September, 2018
Technical University of Moldova, Chişinău, Moldova

Book of Abstracts

Organized by

Technical University of Moldova	ROMAI	Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy
Mathematical Society of Moldova	AOŞR	The Tiraspol State University

Volume Editor: Stelian Ion

Printed in Moldova: Casa Editorial-poligrafica "Bons Offices"
Adress: Chisinau, str. Feredeului 4/6, MD-2005, Republica Moldova
`e-mail adress: bons@bons.md`,

Scientific Committee

Organizing Committee:

President: Costică Moroşanu
Mitrofan Choban
Călin Ioan Gheorghiu
Stelian Ion

Vice President: Viorel Bostan
Anca Veronica Ion
Carmen Rocşoreanu
Şerban Vlad

Local Organizing Commitee

Viorel Bostan
Mitrofan Choban
Valuţă Ion
Mircea Bernic
Libomir Chiriac
Victor Orlov

Dumitru Cozma
Ursu Vasile
Leonid Dohotaru
Galina Rusu
Vladimir Izbaş
Elena Cojuhari

Contents

Plenary talks

Overview of Nonlinear Partial Differential Equation-based Structural Inpainting Techniques

Tudor Barbu

Institute of Computer Science of the Romanian Academy, Iaşi
e-mail: tudor.barbu@iit.academiaromana-is.ro

Image interpolation, known also as inpainting or completion, represents the process of recovering the missing or highly deteriorated regions of the image, as plausibly as possible, by using the information achieved from the known surrounding areas. The inpainting techniques are divided into three main categories: structure-based, texture-based and combined reconstruction approaches.
We consider only the structural image interpolation domain that comprises variational and partial differential equation (PDE) - based inpainting models. A comprehensive overview of the state of the art nonlinear diffusion-based inpainting techniques is provided here. Our own contributions to this image processing field are also highlighted.
Nonlinear PDE-based interpolation models of various orders are described in this survey. Many of them follow the variational principles, while other PDE inpainting models do not derive from variational schemes, being directly given as evolutionary equations. The state of the art second-order diffusion-based interpolation schemes, given in variational or PDE-based form, are presented first. Thus, we describe the Harmonic Inpainting, the variational inpainting models based on the Mumford-Shah functional and the total variation-based reconstruction algorithms, such as the influential TV Inpainting and other variational schemes derived from it.
Higher-order PDE-based interpolation approaches following variational principles, such as TV^2 Inpainting, Total Generalized Variation (TGV) Inpainting or Euler's Elastica Inpainting, are disscused and compared next. Then, we focus on the non-variational high-order PDE completion models. The third-order PDE-based structural inpainting solutions include the pioneering interpolation model introduced by Bertamio et. al, Navier-Stokes equation-based inpainting and Curvature-driven Diffusion (CDD) Inpainting scheme. The state of the art fourth-order differential models for image interpolation that are described here include Cahn-Hillard Inpainting, $TV\text{-}H^{-1}$ Inpainting and LCIS Inpainting. Finite difference-based numerical approximation algorithms and interpolation experiments are provided for the surveyed techniques.
We have also conducted a high amount of research in the diffusion-based image restoration and interpolation domains in the last decade. Some of the most important structure-based inpainting techniques developed by us are also included in this overview. So, we briefly describe some nonlinear parabolic second-order PDE-based inpainting models that can be achieved from variational problems and are derived from our past anisotropic diffusion-based restoration methods by introducing image masks corresponding to the inpainting regions. Variational hybrid interpolation techniques that combine second- and fourth-order nonlinear diffusions are also presented here. Some structural inpainting approaches based on PDE models not following variational principles, which have been proposed by us, are disscused next. They include a nonlinear hyperbolic PDE-based image reconstruction technique and a second-order anisotropic diffusion-based interpolation framwework. Several numerical experiments and method comparison that illustrate the effectiveness of our inpainting methods are also described in this survey.
Keywords. Structural inpainting, Variational scheme, Nonlinear diffusion, Second-order PDE model, High-order PDE, Total variation, Finite difference method, Numerical approximation.

Scattered and Digital Topologies in Information Sciences

Mitrofan M. Cioban, Ivan A. Budanaev

Tiraspol State University, Institute of Mathematics and Computer Science
e-mail: mmchoban@gmail.com, ivan.budanaev@gmail.com

Some of the central problems in computer science and, in particular, in programming are the correctness and the similarity problems, which contain:

- the question of whether a program computes a given function;
- the problem to decide whether an element of the space is equal to a fixed element;
- whether two elements of a given space are equal and whether one approximates the other in the specialized order;
- geometric inequalities in information spaces with special distances;
- variational problems in information spaces with special distances;
- geometric control theory in information spaces with special distances;
- the calculation of the weighted means of two strings;
- describe the proper similarity of two strings;
- the problem to calculate the distance between strings;
- the problem to solve the problem of text editing and correction;
- the problem to appreciate changeability of information over time.

A topological space X is called a pseudo-discrete space if the intersection of any family of open sets is open. By definition, the space X is a pseudo-discrete space if and only if the sets $O(x)$, $x \in X$, are open in X. A topological space X is called an Alexandroff space if it is a pseudo-discrete T_0-space. A connected Alexandroff space is called a topological digital space.

We mention the following universal theorem

Theorem 1. *Let $\mathcal{P}$, Γ and $\mathcal{Q}$ be the properties of spaces with the following conditions: any space with property $\mathcal{P}$ has the properties $\mathcal{Q}$ and Γ; a closed subspace of the space with the property Γ is a space with the property Γ; if $Y = Z \cup S$ is a space with the property Γ, where S is a closed subspace with property $\mathcal{P}$ and Z is a subspace with locally property $\mathcal{Q}$ in Y, then the space Y has the property $\mathcal{Q}$; if S and Z are open subspaces of the space Y with the property Γ, F is a subspace of Y with the property $\mathcal{P}$ and $x \in S \setminus Z \subset F$, then there exist an open subset U of Y and a subspace Φ with the property Γ such that $x \in U$, $U \subset \Phi \subset Z \cup (F \setminus Z)$ and $\Phi \setminus Z$*

Then any $\mathcal{P}$-decomposable space X with the property Γ has the property $\mathcal{Q}$.

Theorem 1 opens the possibility of studying $\mathcal{P}$-decomposable spaces using induction and algorithms.

The image classification problem is to find a fragmentation of the image under research into certain regions such that each region represents a class of elementary partitions with the same label. Assume that the domain X of the plane $\mathbb{R}^2$ represented the image of the original Φ and that image is represented by an observed data function $I : X \to \mathbb{R}$ of the level intensity. We have $I(X) = \{c_i : 1 \leq i \leq n\}$. The function I is constructed in the following way: we determine for the image

X the levels $\{c_i : 1 \le i \le n\} \subset \omega$; find a family $\{O_i : 1 \le i \le n\}$ of open subsets of X, where the $O = \cup\{O_i : 1 \le i \le n\}$ is dense in X, $O_i \cap O_j = \emptyset$ for $1 \le i < j \le n$ and O_i is the set of points of the intensity c_i; for any $i \in \{1, 2, ..., n\}$ and any $x \in O_i$ we put $I(x) = c_i$; if $x \in X \setminus \cup\{O_i : 1 \le i \le n\}$, then $I(x) = sup\{i : x \in cl_X O_i\}$; by the method of digitalization we construct a finite subset K of X which represent the image of th original.

On $\mathbb{Z} = \{0, 1, -1, 2, -2, ..., n, -n, ...\}$ one can consider the topology of Khalimsky $\mathcal{T}_{Kh}$ with the open base $\mathcal{B}_{Kh} = \{\{2n-1\} : n \in \mathbb{Z}\} \cup \{\{2n-1, 2n, 2n+1\} : n \in \mathbb{Z}\}$. The space $(\mathbb{Z}, \mathcal{T}_{Kh})$ is called the Khalimsky line, $(\mathbb{Z}^2, \mathcal{T}^2_{Kh})$ is called the Khalimsky plane, $(\mathbb{Z}^3, \mathcal{T}^3_{Kh})$ is called the Khalimsky space.

Let D be a topological space and $g : D \to \mathbb{Z}$ be a function. For each $n \in \mathbb{Z}$ we put $O(g, n) = \cup\{U \subset X : g(U) = \{n\} : U \textit{ is open in } X\}$. A continuous function f of D in $(\mathbb{Z}, \mathcal{T}_l)$ is a level intensity function on D provided $f(X) = \{0, 1, 2, ..., n\}$ for some $n \in \mathbb{N}$ and $O(f, i) \subset f^{-1}(i) \subset cl_D O(f, i)$ for any $i \in \{0, 1, 2, ..., n\}$. Any intensity function $f : D \to \mathbb{Z}$ determine on D the property $\mathcal{P}(f)$: a subset U of the subspace Y of the space D has the property $\mathcal{P}(f)$ if the set U is open in Y and $f(U)$ is an open singleton subset of $f(Y)$ as the subspace of the space $(\mathbb{Z}, \mathcal{T}_l)$. Relatively to this property D is a $\mathcal{P}(f)$-scattered space.

The space $\mathbb{Z} = \{0, 1, -1, 2, -2, ..., n, -n, ...\}$ is called the discrete line. For the process of digitalization are important the digital topologies on $\mathbb{Z}$. We say that the topology $\mathcal{T}$ on $\mathbb{Z}$ is symmetric if $(\mathbb{Z}, \mathcal{T})$ is a scattered Alexandroff space, the set $\{0\}$ is not open in $(\mathbb{Z}, \mathcal{T})$ and for any $n \in \mathbb{Z}$ the mapping $S_n : \mathbb{Z} \to \mathbb{Z}$, where $S_n(x) = 2n - x$ for each $x \in \mathbb{Z}$, is a homeomorphism.

Theorem 2. *For a topology $\mathcal{T}$ on $\mathbb{Z}$ the following assertions are equivalent:*

1. The topology $\mathcal{T}$ is symmetric.

2. There exists a non-empty subset $L \subset \{2n-1 : n \in \mathbb{N}\}$ such that: $U_0 = \{0\} \cup L \cup \{-n : n \in L\}$ is the minimal open neighbourhood of the point 0 in the space $(\mathbb{Z}, \mathcal{T})$; the family $\mathcal{B}(L) = \{T_{2n}(U_0) : n \in \mathbb{Z}\} \cup \{\{2n-1\} : n \in \mathbb{Z}\}$ is an open base of the topology $\mathcal{T}$ on $\mathbb{Z}$.

The topology of Khalimsky $\mathcal{T}_{Kh}$ with the open base $\mathcal{B}_{Kh} = \{\{2n-1\} : n \in \mathbb{Z}\} \cup \{\{2n-1, 2n, 2n+1\} : n \in \mathbb{Z}\}$ is of the form $\mathcal{T}(L)$ for $L = \{1\} = L_0$. Therefore the topology of Khalimsky is the unique minimal digital symmetric topology on the discrete line $\mathbb{Z}$.

Recent developments on numerical solutions for hyperbolic systems of conservation laws

Rolf Jeltsch

Seminar for Applied Mathematics, ETH Zurich
e-mail: rolf.jeltsch@sam.math.ethz.ch

In 1757 Euler developed the famous Euler equations describing the flow of a compressible gas. This is a system of hyperbolic conservation laws in three space dimensions. However until recently one could not show convergence of numerical schemes to the 'classical' weak entropy solutions. By adapting the concept of measure-valued and statistical solutions to multidimensional systems Siddhatha Mishra and his coauthors could recently show convergence of numerical schemes. Mishra has presented these results at the ICM 2018 in Rio de Janeiro. After a brief introduction to the field these developments will be described.

The national e-Infrastructure RENAM for the development of high-performance computing and information technologies

Ion Tighineanu[1], Petru Bogatencov[2], Grigore Secrieru[2]

[1]*Academy of Sciences of Moldova,*
[2]*RENAM Association and Institute of Mathematics and Computer Science*
e-mail: tiginyanu@asm.md, bogatencov@renam.md, secrieru@renam.md

Over the past several years the dependence of research and education on access to high-speed networking infrastructures, to large-scale computing and other e-Infrastructures' services is rapidly increasing [1]. The use of modern high-performance computing and information technologies allows solving complex problems of mathematics, physics, climatology, life science and other scientific fields [2]. The report is devoted to the analysis of approaches and solutions for the development of the national infrastructure of the RENAM network (Research and Educational Networking Association of Moldova, www.renam.md), which will ensure the deployment of modern information technologies and services for research and education. The prospects of creating new regional cross-border optical channels and other components of the electronic RENAM-GEANT (Gigabit European Advanced Network Technologies, www.geant.net) platform on the basis of the EU EaPConnect project are described [3, 4].

Bibliography

[1] Bogatencov P., Secrieru G., *Regional e-infrastructure and services for research and education in E&P Countries.* Eastern European Journal for Regional Studies (EEJRS). Vol.3, Issue 1, June 2017, pp. 89-101.

[2] Bogatencov P., Secrieru G., Tighineanu I., *E-infrastructura RENAM - platforma interoperabila de colaborare, resurse si servicii informationale in cercetare si educatie.* Revista ACADEMOS nr. 2(45), 2017, pp. 19-26.

[3] ***, *Project*, https://www.eapconnect.eu

[4] ***, GEANT *Pan-European Network*, https://www.geant.org

1. Partial Differential Equations

Estimates for Solutions to Partial Quasilinear Differentila Equations

Iurie Baltag

Technical University of Moldova
e-mail: iubaltag@mail.ru

Let's consider

$$L(u) = a(x,t)u_{tt} + b(x,t)u_{tx} + c(x,t)u_{xx} + d(x,t)u_t + h(x,t)u_x, \quad x \in R^1.$$

The second order quasilinear equations are studied in the following form:

$$L(u) + r(u)\left[a(x,t)u_t^2 + b(x,t)u_t u_x + c(x,t)u_x^2\right] + f(x,t,u) = 0,\ x \in R^1. \tag{1}$$

The objective is to reduce this equation to a linear equation and to study the solutions of the equation (1) depending on the solutions of the linear equation obtained and the functions $r(u)$ and $f(x,t,u)$. For this purpose we make the substitution $u = z(v)$, $v = v(x,y)$. (2)
If $z(v)$ will be a nontrivial solution to the ordinary differential equation $z'' + r(z)(z')^2 = 0$ (3), then equation (1) will be reduced to the following linear equation for determining the function $v(x,y)$: $L(v) + g(x,t)v = 0$, $x \in R^1$ (4), where $f(x,t,z(v)) \cdot (z')^{-1} = g(x,t)v$.
This will specifically be if we take $f \equiv 0$ and the condition $z'(v) \neq 0$ is met.
Here are some particular cases of the substitution $u = z(v)$ of (2) and the function $f(x,t,u)$ of equation (1), for which it is reduced to equation (4)

$$1)\ r(u) = \alpha,\ \alpha \neq 0;\ u = \frac{1}{\alpha}\ln v;\ f = g(x,t);$$

$$2)\ r(u) = -\frac{1}{u};\ u = e^v;\ f = g(x,t)u\ln u;$$

$$3)\ r(u) = \frac{\alpha}{u},\ \alpha \neq -1;\ u = v^{\frac{1}{1+\alpha}};\ f = g(x,t)u^{2\alpha+1};$$

$$4)\ r(u) = \frac{\alpha}{u} + (\alpha+1)u^\alpha,\ \alpha \neq -1;\ u = [\ln v(\alpha+1)]^{\frac{1}{1+\alpha}};\ f = g(x,t)u^\alpha;$$

$$5)\ r(u) = \frac{u}{1-u^2};\ u = \sin v;\ f = g(x,t)\arcsin u \cdot \sqrt{1-u^2}.$$

Thus, if we are able to determine the general (or particular) solution of equation (4), then the general solution (or the corresponding particular one) of equation (1) will be expressed by relation (2). Here are some examples of how to apply this method to the study of the solution to Cauchy's problem for some quasilinear equations.
Example 1. Cauchy's problem for the hyperbolic equation with variable coefficients:

$$L(u) + r(u)\left[u_t^2 - au_x^2\right] + f(u) = 0, \quad L(u) = u_{tt} - au_{xx} + bu_t + cu_x;$$

$$u(x,0) = \varphi_1(x),\ u_t(x,0) = \varphi_2(x). \tag{5}$$

Here the coefficients a, b, c, f are functions that depend on x and t with $a(x,t) \geq a_0 > 0$. By performing the substitution (2), for the function $v(x,t)$ we will get the next problem of Cauchy:

$$L(v) + g(x,t)v = 0, \quad v(x,0) = \psi_1(x), \quad v_t(x,0) = \psi_2(x). \tag{6}$$

I refer to paper [3] where the following theorem is proved: if in the area of $S(x,\tau)=\left\{x\in R^1,\,0\le\tau\le t\right\}$ function a admits the derivatives bounded up to including order 5 and the functions b,c,g derivatives up to order 4, then for the solution of the problem (6) the following estimation takes place:

$$From\,|\psi_1(x)|\le M_1,\ |\psi_2(x)|\le M_2\Rightarrow|v(x,t)|\le C_1(t)M_1+C_2(t)M_2. \tag{7}$$

Thus, by determining the corresponding substitution, we can obtain estimates of the solutions of problem (5) based on the estimates of type (7) for the solution of problem (6). From these estimates results the uniqueness of the solution of problem (5) and the continuous dependence of the solution on the initial conditions of the problem. So, to emphasize: in case 1) the estimates for the solution of the problem (5) are of the type (7), and in other cases these estimates have another type.
This will be made obvious in the case of the equations with constant coefficients, shown in the examples below.
Example 2. Cauchy's problem for the hyperbolic equation:

$$L(u)+r(u)\left[u_t^2-a^2u_x^2\right]=0,\quad L(u)=u_{tt}-a^2u_{xx};$$

$$u(x,0)=\varphi_1(x),\ u_t(x,0)=\varphi_2(x). \tag{8}$$

Applying the substitution (2) with $f=0$ for the function $v(x,t)$, we will get Cauchy's problem for the vibrating string equation: $L(v)=0;\ v(x,0)=\psi_1(x),\ v_t(x,0)=\psi_2(x)$. If the function $\psi_1(x)$ is double derivable and $\psi_2(x)$ derivable, then the solution to this problem is given by the formula of D'Alembert:

$$v(x,t)=\frac{1}{2}\left[\psi_1(x-at)+\psi_1(x+at)\right]+\frac{1}{2a}\int\limits_{x-at}^{x+at}\psi_2(y)dy.$$

$$From\,|\psi_1(x)|\le M_1,\ |\psi_2(x)|\le M_2\Rightarrow|v(x,t)|\le M_1+tM_2.$$

Let's examine a couple of particular cases from 1) to 5) examined above and the corresponding estimates that are obtained for the solution to problem (8), emphasizing the suplimentary conditions to the innitial conditions:

$$1)\,r(u)=\alpha;\ u=\frac{1}{\alpha}\ln v;\ \ \psi_1(x)=e^{\alpha\varphi_1(x)};\ \ \psi_2(x)=\alpha\varphi_2(x)e^{\alpha\varphi_1(x)},a\varphi_2(x)>0;$$

$$|u(x,t)|\le\left|\frac{1}{\alpha}\ln\left(e^{\alpha M_1}+t|\alpha|M_2e^{\alpha M_1}\right)\right|\le\left|\frac{1}{\alpha}\left(\alpha M_1+\ln(1+t|\alpha|M_2)\right)\right|\le M_1+tM_2.$$

$$2)\,r(u)=-\frac{1}{u};\ u=e^v;\ \psi_1(x)=\ln\varphi_1(x),\ \varphi_1(x)\ge c>0;\ \psi_2(x)=\frac{\varphi_2(x)}{\varphi_1(x)};$$

$$|u|\le e^{\ln M_1+\frac{1}{c}tM_2}\le M_1e^{c_1tM_2}.$$

$$3)\,r(u)=\frac{n}{u};\ u=\sqrt[n+1]{v};\ \psi_1(x)=\varphi_1^{n+1}(x);\ \psi_2(x)=(n+1)\varphi_1^n(x)\varphi_2(x);$$

$$|u|\le\sqrt[n+1]{M_1+t(n+1)M_1^nM_2},with\ \varphi_1(x)\varphi_2(x)\ge0\ for\ n\ unequal.$$

Example 3. Cauchy's problem for the parabolic equation:

$$L(u)-r(u)a^2u_x^2=0,\quad L(u)=u_t-a^2u_{xx};\ u(x,0)=\varphi(x). \tag{9}$$

Applying substitution (2), for function v Cauchy's problem for the heat equation in a homogeneous bar is obtained: $L(v) = 0;\ v(x,0) = \psi(x)$. If the function $\psi(x)$ is continuous, then the solution to this problem is given by the following formula:

$$v(x,t) = \frac{1}{2a\sqrt{\pi t}} \int\limits_{-\infty}^{+\infty} \psi(y) e^{-\frac{(x-y)^2}{4a^2 t}} dy = \frac{1}{\sqrt{\pi}} \int\limits_{-\infty}^{+\infty} \psi\left(x + 2az\sqrt{t}\right) e^{-z^2} dz,$$

because $\int\limits_{-\infty}^{+\infty} e^{-z^2} dz = \sqrt{\pi}$, from $|\psi(x)| \le M \Rightarrow |v(x,t)| \le M$.

Note that different specific cases can be examined and the corresponding estimates can be deducted as in the solutions to problem (9).

Bibliography

[1] Arsenin V. *Metodi matematicheskoi fiziki i specialnie functii.* Moskva, Nauka, 1984.

[2] Bitsadze A. *Nekotorie klassi uravnenii v chastnih proizvodnih.* Moskva, Nauka, 1981.

[3] Baltag Iu.; Perjan A. *Ob ogranichenosti razreshaiuschego operatora zadachi Cauchy dlia odnomernogo ghiperbolicheskogo uravnenia vtorogo poriadka v prostranstvah L_p.* Kishinev, 1990, Mat. issled., N. 112.

[4] Courant, R. *Kurs differencialnogo i integralnogo ischislenia.* Moskva, Nauka, 1970.

[5] Courant, R.; Hilbert, D. *Metodi matematicheskoi fiziki.* Moskva, Nauka, 1951.

Weak solutions for a class of higher-order PDEs

Maria-Magdalena Boureanu

Department of Applied Mathematics, University of Craiova, Romania
e-mail: mmboureanu@yahoo.com

We are concerned with the weak solvability of a class of fourth-order elliptic problems involving variable exponents. In the qualitative study of PDEs, the impact of an exponent that varies has been intensively investigated over the last decades, but the interest in variable exponent problems of fourth-order is much more recent. We continue this fresh line of research and we rely on the critical point theory to obtain existence and multiplicity results for our problem.

Explicit solutions of the differential systems and mathematical modelling in electrodynamics

Dmitrieva Irina

Odessa National Academy of Telecommunications
e-mail: dmitrievairina2017@gmail.com

Several new analytic methods are proposed here basing on the diagonalization procedure for arbitrary square system of PDEs (partial differential equations) with the piecewise constant coefficients. Engineering applications are done in terms of the boundary value problems regarding the general wave PDE included all scalar components of the electromagnetic field vector intensities.

Those mentioned results are inspired and supported by [1] - [8]. The tentative constructive generalizations and relevant computer simulations are based on [9].
The outline of research is the following:

1 Introduction: The classical and modern trends in the electromagnetic field theory.

2 Main constructive results: Two new diagonalizing procedures for the systems of PDEs and their applications to the mathematical modelling in electrodynamics.

3 Discretization scheme: Numerical implementation and computer simulation of the relevant electromagnetic engineering problems.

4 Conclusions: Specific features of the proposed analytic techniques.

Bibliography

[1] J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 1st ed., vol. 1. Oxford: Clarendon Press, 1873. - 455 p.

[2] A. Sommerfeld, Electrodynamics. New York: Academic Press, 1959. - 371 p.

[3] A. Angot, Complements de Mathematiques: A l'usage des ingenieurs de l'electrotechnique et des telecommunications. Paris: Edition de la Revue D'optique, 1961. - 836 p.

[4] A. N. Tikhonov, and A. A. Samarskii, Equations of Mathematical Physics. New York: Pergamon Press Ltd., 1963. - 777 p.

[5] R. Mitra, Computer Techniques for Electromagnetics, 1st ed. Intl. Series for Monographs in Electrical Engineering. Oxford - New York - Toronto - Sydney - Braunschweig: Pergamon Press, 1973. - 416 p.

[6] A. Le Mehaute, Fractal Geometries: Theory and Applications, Engl. transl. by J. Howlett. New York: CRC Press, 1991. - 220 p.

[7] C. Caloz, and T. Itoh, Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications. The Engineering Approach. New Jersey: John Wiley and Sons Inc., 2006. - 364 p.

[8] R. Marques, F. Martin, and M. Sorolla, Metamaterials with Negative Parameters: Theory, Design and Microwave Applications. New Jersey: John Wiley and Sons Inc., 2008. - 315 p.

[9] I. Yu. Dmitrieva, "Analytical research methods of electromagnetic wave propagation in guided systems", Abstract of the Dr. - Engineering habilitation thesis, specialty "antennas and microwave structures', Odessa National Academy of Telecommunications, Odessa, March 2017. - 44 p. (in Ukrainian).

Inverse problem for a two-dimensional strongly degenerate heat equation

Mykola Ivanchov, Vitaliy Vlasov

Ivan Franko National University of Lviv, Lviv, Ukraine
e-mail: mykola.ivanchov@lnu.edu.ua

We consider an inverse problem for a two-dimensional degenerate heat equation in a rectangular domain. Direct problems of this type are mathematical models of various processes such as seawater desalination, movement of liquid in porous medium, financial market behavior.
Inverse problems arise when certain parameters of these processes are unknown. Var- ious types of inverse problems for non-degenerate equations are well investigated and some results may be found in many monograph. In the domain $Q_T := (x, y, t) : 0 < x < h, 0 < y < l, 0 < t < T$ we consider a two- dimensional heat equation with unknown leading coefficient depending on the time variable. We suppose that the equation degenerates at the initial moment as a power with a given exponent $\beta \geq 1$. We choose the case of mixed Dirichlet-Neumann boundary conditions. The additional condition (so-called overdetermination condition) is taken accordingly to the physical sense and it presents the value of the heat flux on the part of the boundary of domain. So, the problem consists of finding a pair of functions $(a(t), u(x, y, t)), a(t) > 0, t \in [0, T]$ that satisfy the degenerate heat equation

$$u_t = t^\beta a(t)\triangle u + f(x, y, t), (x, y, t) \in Q_T,$$

initial condition

$$u(x, y, 0) = \phi(x, y, 0), (x, y) \in \widetilde{D} := [0, h] \times [0, l],$$

boundary conditions

$$u(0, y, t) = \mu_1(y, t), u(h, y, t) = \mu_2(y, t), (y, t) \in [0, l] \times [0, T],$$

$$u_y(x, 0, t) = \nu_1(x, t), u_y(x, l, t) = \nu_2(x, t), (x, t) \in [0, h] \times [0, T]$$

and overdetermination condition

$$a(t)u_x(0, y_0, t) = \chi(t), t \in (0, T],$$

where $y_0 \in (0, l)$ is some arbitrary fixed point. Our goal is to determine the conditions of existence and uniqueness of classical solution of the problem.

Viscosity solutions for functional parabolic PDEs with Neumann nonlinear boundary conditions. A stochastic approach via BSDEs with time–delayed generators

Lucian Maticiuc

"Alexandru Ioan Cuza" University of Iaşi, Romania
e-mail: `lucian.maticiuc@uaic.ro`

We provide a probabilistic representation of a viscosity solution for the path dependent nonlinear Kolmogorov equation with Neumann nonlinear boundary conditions:

$$\begin{cases} -\dfrac{\partial u}{\partial t}(t,\phi) - \mathcal{L}u(t,\phi) - f(t,\phi,u(t,\phi),\partial_x u(t,\phi)\sigma(t,\phi),(u(\cdot,\phi))_t) = 0, \\ \qquad\qquad\qquad\qquad t\in[0,T),\ \phi\in\mathcal{C}([0,T];\mathrm{Int}\,(G)), \\ u(T,\phi) = h(\phi), \quad \phi\in\mathcal{C}([0,T];\bar{G}), \\ \dfrac{\partial u}{\partial n}(t,\phi) = g(t,\phi,u(t,\phi),(u(\cdot,\phi))_t) = 0, \\ \qquad\qquad\qquad\qquad t\in[0,T],\ \phi\in\mathcal{C}([0,T];\mathrm{Fr}\,(G)), \end{cases} \tag{1}$$

where G is a C^2 convex, open and bounded subset of $\mathbb{R}^d$, $\mathcal{L}$ is a second order differential operator and

$$(u(\cdot,\phi))_t := (u(t+\theta,\phi))_{\theta\in[-\delta,0]}\,.$$

We show that the functional $u:[0,T]\times\mathcal{C}([0,T];\bar{G})\to\mathbb{R}$, $u(t,\phi):=Y^{t,\phi}(t)$, is a viscosity solution for (1), where $(Y^{t,\phi},Z^{t,\phi})$ is the unique solution of a certain backward stochastic differential equation (BSDE for short) with time delayed-generators.

Errors analysis for two methods approximating the classical Caginalp's model

Costică Moroşanu

Faculty of Mathematics, "Al. I. Cuza" University,
Bd. Carol I, No. 11, 700506, Iaşi, ROMANIA
e-mail: `costica.morosanu@uaic.ro`

The paper concerns with the error analysis of two time-stepping schemes used in the discretization of the phase-field transition system with a classical regular potential (Caginalp's model) and Neumann boundary conditions. Using energy methods, we establish L^∞ error estimates for the implicit Euler and a fractional steps method. A numerical experiment validates the theoretical results (see [1]), comparing the accuracy of the methods (see [2], [3]).

MSC: 35K55, 35K57, 65M06, 65M12, 65Y20, 80Axx.

Keywords. nonlinear PDE of parabolic type, reaction-diffusion equations, finite difference methods, fractional steps method, stability and convergence of numerical methods, performance of numerical algorithms, thermodynamics, phase-changes.

Bibliography

[1] C. Moroşanu, Analysis and optimal control of phase-field transition system: Fractional steps methods, Bentham Science Publishers, 2012,

http://dx.doi.org/10.2174/97816080535061120101.

[2] C. Moroşanu, *Qualitative and quantitative analysis for a nonlinear reaction-diffusion equation*, ROMAI J., Vol. 12, No. 2 (2016), pp. 85-113, https://rj.romai.ro/arhiva/2016/2/Morosanu.pdf

[3] C. Moroşanu, S. Pavăl and C. Trenchea, *Analysis of stability and errors of three methods associated to the nonlinear reaction-diffusion equation supplied with homogeneous Neumann boundary conditions*, Journal of Applied Analysis and Computation, Vol. 7, No. 1 (2017), pp. 1-19, DOI:10.11948/2017001

Construction and analysis of approximate schemes for the evolution equation of fractional order

Ivan Secrieru

State University of Moldova, Chişinău, Republic of Moldova
e-mail: secryah@yahoo.com

The evolution equations modeling many process that appear in physique, ecologies, hydrogeology, finance etc. For example, the mathematical model of the problem to transport any substance in atmosphere, because of the diffusion and advection factors, presents an evolution equation.The classical model of this problem use the derivatives of entire order for unknown function [1]. In recent years many authors use the partial derivatives of fractional order by the space variables to modeling such process.
In this article is considered the same problem with two space variables of the form

$$\begin{gathered}\frac{\partial\varphi}{\partial t}-d_+(x)\frac{\partial^\alpha\varphi}{\partial_+x^\alpha}-d_-(x)\frac{\partial^\alpha\varphi}{\partial_-x^\alpha}-d_+(y)\frac{\partial^\alpha\varphi}{\partial_+y^\alpha}-d_-(y)\frac{\partial^\alpha\varphi}{\partial_-y^\alpha}=f(x,y,t),\\ \varphi(x,y,0)=s(x,y), \qquad (1)\\ \varphi(x,y,t)=0 \qquad on \qquad the \qquad \partial D,\end{gathered}$$

in the domain $D=[0,a]\times[0,b]$ with the boundary ∂D and the time interval $[0,T]$, where $1<\alpha\leq 2, 0<x<a, 0<y<b, 0\leq t\leq T, d_+(x)\geq 0, d_-(x)\geq 0$. The left-hand (+) and the right-hand (-) fractional derivatives of order α in (1) are defined by Riemann-Liouville formulas and will be approximated using the Grunwald formulas [2].The first order time derivative is discretized by the central finite differences. The obtained approximate scheme, with some adequate suppositions, verify the conditions of stability and convergence.

Bibliography

[1] G.I. Marchuk, *Mathematical modeling in problem to protect the environment*, Nauka, Moscow, 1982 (in Russian).

[2] I.Secrieru, V.Ticau, *Weighted approximate scheme for fractional order diffusion equation*, Buletinul Institutul Politehnic din Iasi, tomul LVII(LXI), fasc.1, sectia MATEMATICA, MECANICA TEORETICA,FIZICA, Ed. POLITEHNIUM, Iasi, Romania, 2011.

The Border Problem of the Ring Domain Deformation

N.V. Snizhko

Zaporizhzhia National Technical University, Ukraine
e-mail: snizhko.nataliia@gmail.com

In this work, the method of Muskhelishvili's complex potentials is used to solve the boundary value problem of elasticity theory for a domain in the form of a ring with piecewise constant boundary conditions on the contour. The solution is obtained in an analytical form and it is put to a form suitable for numerical simulation. It is established that in the neighborhood of the contour there is deformation of the region close to the shift (on the sections of the boundary with a nonzero boundary condition) or to radial compression (on the parts of the boundary with the zero boundary condition).

Bibliography

[1] , Anpilohov D., Snizhko N., *The angular deformation of the ring with reference to the centrifugal forces*, Lobachevskii J. of Math. – 2017. – **38**, N3. – P. 395-399.

[2] Anpilohov D., Snizhko N., *On one border problem of ring domain deformation*, Bukovinian Mathematical Journal. – 2018. – **6**, N1-2.

2. ODEs; Dynamical Systems

Topological configurations of singularities for quadratic differential systems

Joan C. Artes, Jaume Llibre, Dana Schlomiuk, Nicolae Vulpe

Departament de Matemàtiques, Universitat Autònoma de Barcelona, Spain;
Departament de Matemàtiques, Universitat Autònoma de Barcelona, Spain;
Département de Mathématiques et de Statistiques, Université de Montréal, Canada;
Institute of Mathematics and Computer Science, Moldova

e-mail: artes@mat.uab.cat, jllibre@mat.uab.cat, dasch@dms.umontreal.ca, nvulpe@gmail.com

In [1-7] the authors proved that there are at least 1879 (and at most 1880) different geometric configurations of singularities of quadratic differential systems in the plane. This classification is completely algebraic and done in terms of invariant polynomials and it is finer than the classification of quadratic systems according to the topological classification of singularities.
The long term project is the classification of phase portraits of all quadratic systems under topological equivalence. A first step in this direction is to obtain the classification of quadratic systems under topological equivalence of local phase portraits around singularities.
In this paper we extract the local topological information around all singularities from the 1879 geometric equivalence classes. We prove that there are exactly 208 topologically distinct global topological configurations of singularities for the whole quadratic class. From here the next goal would be to obtain a bound for the number of possible different phase portraits, modulo limit cycles.

Bibliography

[1] J. C. Artés, J. Llibre, D. Schlomiuk and N. Vulpe, *Configurations of singularities for quadratic differential systems with total finite multiplicity lower than 2,* Bul. Acad. Ştiinţe Repub. Mold. Mat., 2013, no. 1, 72–124.

[2] J. C. Artés, J. Llibre, D. Schlomiuk and N. Vulpe, *Geometric configurations of singularities for quadratic differential systems with three distinct real simple finite singularities*, J. Fixed Point Theory Appl. **14** (2013), no. 2, 555–618.

[3] J. C. Artés, J. Llibre, D. Schlomiuk and N. Vulpe, *Geometric configurations of singularities for quadratic differential systems with total finite multiplicity* $m_f = 2$, Electron. J. Differential Equations, **2014** (2014), No. 159, 79 pp.

[4] J. C. Artés, J. Llibre, D. Schlomiuk and N. Vulpe, *Global configurations of singularities for quadratic differential systems with total finite multiplicity three and at most two real singularities*, Qual. Theory Dyn. Syst., **13** (2014), no. 2, 305–351.

[5] J. C. Artés, J. Llibre, A.C. Rezende, D. Schlomiuk and N. Vulpe, *Global configurations of singularities for quadratic differential systems with exactly two finite singularities of total multiplicity four*, Electron. J. Qual. Theory Differ. Equ., 2014, No. 60, 43 pp.

[6] J. C. Artés, J. Llibre, D. Schlomiuk and N. Vulpe, *Global configurations of singularities for quadratic differential systems with exactly three finite singularities of total multiplicity four*, Electron. J. Qual. Theory Differ. Equ., 2015, No. 49, 60 pp.

[7] J. C. Artés, J. Llibre, D. Schlomiuk and N. Vulpe, *From topological to geometric equivalence in the classification of singularities at infinity for quadratic vector fields.* Rocky Mountain J. of Math., **45** (2015), no. 1, 29–113.

Acknowledgement. *The second author is partially supported by FP7-PEOPLE-2012-IRSES-316338 and by the grant 12.839.08.05F from SCSTD of ASM.*

Averaging Method in Multifrequency Systems with Delay and Nonlocal Conditions

Yaroslav Bihun, Roman Petryshyn, Inessa Krasnokutska

Yurii Fedkovych Chernivtsi National University, Ukraine
e-mail: yaroslav.bihun@gmail.com, r.petryshyn@chnu.edu.ua, i.krasnokutska@chnu.edu.ua

We consider a system of differential equations of the form [1, 2]

$$\frac{dx}{d\tau} = X(\tau, x_\Lambda, \varphi_\Theta), \quad \frac{d\varphi}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y(\tau, x_\Lambda, \varphi_\Theta)$$

with initial conditions, multipoint and boundary integral conditions, for example [3],

$$a(\tau_0) = a_0, \quad \int\limits_{\tau_1}^{\tau_2} \Big[\sum_{j=1}^{s} b_j(\tau, a_\Lambda(\tau))\varphi_{\theta_j}(\tau) + g(\tau, a_\Lambda(\tau), \varphi_\Theta(\tau)) \Big] d\tau = d.$$

Here $0 \le \tau \le L$, $x \in D \subset \mathbb{R}^n$, $\varphi \in \mathbb{T}^m$, $\Lambda = (\lambda_1, \dots, \lambda_p)$, $\Theta = (\theta_1, \dots, \theta_q)$, $\lambda_i, \theta_j \in (0,1)$, $x_{\lambda_i}(\tau) = x(\lambda_i \tau)$, $\varphi_{\theta_j}(\tau) = \varphi(\theta_j \tau)$, $\varepsilon \in (0, \varepsilon_0]$, $\varepsilon_0 \ll 1$, $0 \leqslant \tau_0 \leqslant L$, $0 \leqslant \tau_1 < \tau_2 \leqslant L$.
The complexity of the research of the problem is the existence of resonances. Resonance condition in point $\tau \in [0, L]$ is

$$\sum_{\nu=1}^{q} \theta_\nu (k_\nu, \omega(\theta_\nu \tau)) = 0, \quad k_\nu \in \mathbb{R}^m, \quad \|k\| \neq 0.$$

Averaging in system (1) is carried out on fast variables φ_Θ on the torus T^m. The averaged problem takes the form

$$\frac{d\overline{x}}{d\tau} = X_0(\tau, \overline{x}_\Lambda,), \quad \frac{d\overline{\varphi}}{d\tau} = \frac{\omega(\tau)}{\varepsilon} + Y_0(\tau, \overline{x}_\Lambda),$$

$$\overline{a}(\tau_0) = a_0, \quad \int\limits_{\tau_1}^{\tau_2} \Big[\sum_{j=1}^{s} b_j(\tau, \overline{a}_\Lambda(\tau))\overline{\varphi}_{\theta_j}(\tau) + g_0(\tau, \overline{a}_\Lambda(\tau)) \Big] d\tau = d.$$

The existence and uniqueness of solution of the problem and the estimation error $\|x(\tau,\varepsilon) - \overline{x}(\tau)\| \le c_1 \varepsilon^\alpha$, where $\alpha = (mq)^{-1}$, $c_1 = const > 0$ of averaging method is obtained.

Bibliography

[1] A. M. Samoilenko, R. I. Petryshyn, *Multifrequency Oscillations of Nonlinear Systems*, Kluwer, Dordrecht V-Boston V-London, Netherlands, 2004, 317 p.

[2] Ya. Bihun, R. Petryshyn, I. Krasnokutska, *Averaging in Multifrequency Systems with Linearly Transformed Arguments and with Point and Integral Conditions*, Bukovinian Math. Journal, **4**, No. 3–4 , 30–35 (2016).

[3] M. Benchohra, J. Henderson, R. Luka, A. Ouahab, *Boundary Value Problems for Systems of Diferential*, Diference and Fractional Equation, Kluwer, Dordrecht-Boston-London, Netherlands, 2016, 307 p.

Configurations of invariant lines of total multiplicity 7 of cubic systems with four real distinct infinite singularities

Cristina Bujac , Dana Schlomiuk, Nicolae Vulpe

Institute of Mathematics and Computer Science, Moldova;
Département de Mathématiques et de Statistiques, Université de Montréal, Canada;
Institute of Mathematics and Computer Science, Moldova
e-mail: crisulicica@yahoo.com, dasch@dms.umontreal.ca, nvulpe@gmail.com

Consider the family of planar cubic polynomial differential systems. Following [1] we call *configuration of invariant lines* of a cubic system the set of (complex) invariant straight lines (which may have real coefficients), including the line at infinity, of the system, each endowed with its own multiplicity and together with all the real singular points of this system located on these invariant straight lines, each one endowed with its own multiplicity.

Our main goal is to classify the family of cubic systems according to their geometric properties encoded in the configurations of invariant straight lines of total multiplicity seven (including the line at infinity with its own multiplicity), which these systems possess.

Here we consider only the subfamily of cubic systems with four real distinct infinite singularities which we denote by $\mathbf{CSL}_7^{4s\infty}$. We prove that there are exactly 94 distinct configurations of invariant straight lines for this class and present corresponding examples for the realization of each one of the detected configurations.

We remark that cubic systems with nine (the maximum number) of invariant lines for cubic systems are considered in [2], whereas cubic systems with eight invariant lines (considered with their multiplicities) are investigated in [3-7].

Bibliography

[1] D. Schlomiuk, N. Vulpe, *Global topological classification of Lotka-Volterra quadratic differential systems*, Electronic Journal of Differential Equations, Vol. 2012 (2012), No. 64, 1–69.

[2] J. Llibre, N. Vulpe , *Planar cubic polynomial differential systems with the maximum number of invariant straight lines*, Rocky Mountain J. Math. **38** (2006), 1301–1373.

[3] C. Bujac, N. Vulpe, *Cubic differential systems with invariant lines of total multiplicity eight and with four distinct infinite singularities.* J. Math. Anal. Appl. **423**(2015), 1025–1080.

[4] C. Bujac, N. Vulpe, *Cubic systems with invariant straight lines of total multiplicity eight and with three distinct infinite singularities.* Qual. Theory Dyn. Syst. **14**(2015), No. 1, 109–137.

[5] C. Bujac, N. Vulpe, *Classification of cubic differential systems with invariant straight lines of total multiplicity eight and two distinct infinite singularities*, Electron. J. Qual. Theory Differ. Equ. **2015**, No. 60, 1–38. MR3418574

[6] C. Bujac, *One subfamily of cubic systems with invariant lines of total multiplicity eight and with two distinct real infinite singularities*, Bul. Acad. Ştiinţe Repub. Mold., Mat. **77**(2015), No. 1, 48–86.

[7] C. Bujac, N. Vulpe, *Cubic Differential Systems with Invariant Straight Lines of Total Multiplicity Eight possessing One Infinite Singularity*, Qual. Theory Dyn. Syst. **16**(2017), No. 1, 1–30.

Acknowledgement. *The second author is partially supported by FP7-PEOPLE-2012-IRSES-316338 and by the grant 12.839.08.05F from SCSTD of ASM.*

A rational basis of $GL(2,\mathbb{R})$-comitants for the bidimensional polynomial system of differential equations of the fifth degree

Iurie Calin[1,2], Victor Orlov[1,3]

[1] *Institute of Mathematics and Computer Science,*
[2] *Moldova State University,*
[3] *Technical University of Moldova, Chişinău, Republic of Moldova*

e-mail: `iucalin@yahoo.com, orlovictor@gmail.com`

Let us consider the system of differential equations of the fifth degree

$$\frac{dx}{dt} = P_0 + \sum_{i=1}^{5} P_i(x,y), \qquad \frac{dy}{dt} = Q_0 + \sum_{i=1}^{5} Q_i(x,y), \tag{1}$$

where $P_i(x,y),\ Q_i(x,y)$ are homogeneous polynomials of degree i in x and y with real coefficients. The following $GL(2,\mathbb{R})$-comitants [1] have the first degree with respect to the coefficients of the system (1):

$$\begin{gathered} R_i = P_i(x,y)y - Q_i(x,y)x, \quad i=\overline{0,5} \\ S_i = \frac{1}{i}\left(\frac{\partial P_i(x,y)}{\partial x} + \frac{\partial Q_i(x,y)}{\partial y}\right), \quad i=\overline{1,5}. \end{gathered} \tag{2}$$

Using the comitants (2) as elementary "bricks" and the notion of transvectant [2] the following

$GL(2,\mathbb{R})$-comitants of the system (1) were constructed:

$$K_{001} = R_0, \quad K_{002} = (R_0, R_1)^{(1)},$$
$$K_{101} = R_1, \quad K_{102} = S_1,$$
$$K_{103} = (R_1, R_1)^{(2)}, \quad K_{201} = R_2,$$
$$K_{202} = (R_2, R_1)^{(1)}, \quad K_{203} = (R_2, R_1)^{(2)},$$
$$K_{204} = ((R_2, R_1)^{(2)}, R_1)^{(1)}, \quad K_{205} = S_2,$$
$$K_{206} = (S_2, R_1)^{(1)}, \quad K_{301} = R_3,$$
$$K_{302} = (R_3, R_1)^{(1)}, \quad K_{303} = (R_3, R_1)^{(2)},$$
$$K_{304} = ((R_3, R_1)^{(2)}, R_1)^{(1)}, \quad K_{305} = ((R_3, R_1)^{(2)}, R_1)^{(2)},$$
$$K_{306} = S_3, \quad K_{307} = (S_3, R_1)^{(1)},$$
$$K_{308} = (S_3, R_1)^{(2)}, \quad K_{401} = R_4,$$
$$K_{402} = (R_4, R_1)^{(1)}, \quad K_{403} = (R_4, R_1)^{(2)},$$
$$K_{404} = ((R_4, R_1)^{(2)}, R_1)^{(1)}, \quad K_{405} = ((R_4, R_1)^{(2)}, R_1)^{(2)},$$
$$K_{406} = (((R_4, R_1)^{(2)}, R_1)^{(2)}, R_1)^{(1)}, \quad K_{407} = S_4,$$
$$K_{408} = (S_4, R_1)^{(1)}, \quad K_{409} = (S_4, R_1)^{(2)},$$
$$K_{410} = ((S_4, R_1)^{(2)}, R_1)^{(1)}, \quad K_{501} = R_5,$$

$$K_{502} = (R_5, R_1)^{(1)}, \quad K_{503} = (R_5, R_1)^{(2)},$$
$$K_{504} = ((R_5, R_1)^{(2)}, R_1)^{(1)}, \quad K_{505} = ((R_5, R_1)^{(2)}, R_1)^{(2)},$$
$$K_{506} = (((R_5, R_1)^{(2)}, R_1)^{(2)}, R_1)^{(1)}, \quad K_{507} = (((R_5, R_1)^{(2)}, R_1)^{(2)}, R_1)^{(2)},$$
$$K_{508} = S_5, \quad K_{509} = (S_5, R_1)^{(1)},$$
$$K_{510} = (S_5, R_1)^{(2)}, \quad K_{511} = ((S_5, R_1)^{(2)}, R_1)^{(1)},$$
$$K_{512} = ((S_5, R_1)^{(2)}, R_1)^{(2)}.$$

We denote by A the coefficient space of the system (1).

Definition 1. *The set S of the comitants is called a rational basis on $M \subseteq A$ of the comitants for the system (1) with respect to the group $GL(2,\mathbb{R})$ if any comitant of the system (1) with respect to the group $GL(2,\mathbb{R})$ can be expressed as a rational function of elements of the set S.*

Definition 2. *A rational basis on $M \subseteq A$ of the comitants for the system (1) with respect to the group $GL(2,\mathbb{R})$ is called minimal if by the removal from it of any comitant it ceases to be a rational basis.* In [3] was established a method for construction the rational bases of $GL(2,\mathbb{R})$-comitants for the bidimensional polynomial systems of differential equations by using different comitants of the system. In this paper we will present a rational basis of $GL(2,\mathbb{R})$-comitants for the bidimensional polynomial system of differential equations of the fifth degree in the case, when the comitant of the linear part $R_1 \not\equiv 0$. **Theorem.** *The set of $GL(2,\mathbb{R})$-comitants*

$$\{K_{001},\ K_{002},\ K_{101},\ K_{102},\ K_{103},\ K_{201},\ K_{202},\ K_{203},\ K_{204},\ K_{205},\ K_{206},$$
$$K_{301},\ K_{302},\ K_{303},\ K_{304},\ K_{305},\ K_{306},\ K_{307},\ K_{308},\ K_{401},\ K_{402},\ K_{403},$$
$$K_{404},\ K_{405},\ K_{406},\ K_{407},\ K_{408},\ K_{409},\ K_{410},\ K_{501},\ K_{502},\ K_{503},\ K_{504},$$
$$K_{505},\ K_{506},\ K_{507},\ K_{508},\ K_{509},\ K_{510},\ K_{511},\ K_{512}\}$$

is a minimal rational basis of the $GL(2,\mathbb{R})$*-comitants for the system* (1) *of differential equations of the fifth degree on* $M = \{a \in A \mid R_1 \not\equiv 0 \ (K_{101} \not\equiv 0)\}$.

Acknowledgement. *This research is partially supported by the grant 15.817.02.03F from SCSTD of ASM*

Bibliography

[1] K. S. Sibirsky, *Introduction to the Algebraic Theory of Invariants of Differential Equations.* Manchester University Press, 1988.

[2] G. B. Gurevich, *Foundations of the Theory of Algebraic Invariants.* Noordhoff, Groningen, 1964.

[3] Iu. Calin, *On rational bases of $GL(2,\mathbb{R})$-comitants of planar polynomial systems of differential equations.* Buletinul Academiei de Stiinte a Republicii Moldova. Matematica, 2003, nr. 2 (42), pp. 69-86.

The sufficient center conditions for a class of bidimensional polynomial systems of differential equations with nonlinearities of the fourth degree

Iurie Calin[1,2] Stanislav Ciubotaru[1]

[1]*Institute of Mathematics and Computer Science,*
[2]*Moldova State University, Chişinău, Republic of Moldova*
e-mail: iucalin@yahoo.com, stanislav.ciubotaru@yahoo.com

Let us consider the system of differential equations with nonlinearities of the fourth degree

$$\frac{dx}{dt} = P_1(x,y) + P_4(x,y), \quad \frac{dy}{dt} = Q_1(x,y) + Q_4(x,y), \tag{1}$$

where $P_i(x,y)$ and $Q_i(x,y)$ are homogeneous polynomials of degree i in x and y with real coefficients. We shall consider the following polynomials:

$$R_i = P_i(x,y)y - Q_i(x,y)x; \quad S_i = \frac{1}{i}\left(\frac{\partial P_i(x,y)}{\partial x} + \frac{\partial Q_i(x,y)}{\partial y}\right), \quad i = 1, 4,$$

which in fact are $GL(2,\mathbb{R})$-comitants [1, 2] of the first degree with respect to the coefficients of system (1). Let us consider the following $GL(2,\mathbb{R})$-comitants and $GL(2,\mathbb{R})$-invariants for the system (1), constructed by using the comitants R_i and S_i ($i = 1,4$) and the notion of the transvectant [3] (in the list below, the bracket "$[\![$" is used in order to avoid placing the otherwise necessary parenthesis "("):

$$I_1 = S_1, \quad I_2 = (R_1, R_1)^{(2)}, \quad I_3 = [\![S_4, R_1)^{(2)}, R_1)^{(1)}, (S_4, R_1)^{(2)})^{(1)},$$
$$I_4 = [\![R_4, R_1)^{(2)}, R_1)^{(2)}, R_1)^{(1)}, ((R_4, R_1)^{(2)}, R_1)^{(2)})^{(1)}.$$

The system (1) can be written in the following coefficient form:

$$\frac{dx}{dt} = cx + dy + gx^4 + 4hx^3y + 6kx^2y^2 + 4lxy^3 + my^4,$$
$$\frac{dy}{dt} = ex + fy + nx^4 + 4px^3y + 6qx^2y^2 + 4rxy^3 + sy^4. \tag{2}$$

In [4] were established the necessary and sufficient center conditions for the system (1) (or (2)) with $I_1 = 0,\ I_2 > 0,\ I_3 = I_4 = 0$.

In this paper we consider the class of systems (1) (or (2)) with the conditions $I_3 = 0,\ I_1 = 0, I_2 > 0$. The conditions $I_1 = 0, I_2 > 0$ mean that the eigenvalues of the Jacobian matrix at the singular point $(0, 0)$ are pure imaginary, i.e., the system has the center or a weak focus at $(0, 0)$. The system (2) with $I_1 = 0,\ I_2 > 0$ and $I_3 = 0$ can be reduced by a centeraffine transformation and time scaling to the form:

$$\begin{aligned} \frac{dx}{dt} &= y + gx^4 + 4hx^3y + 6kx^2y^2 + 4lxy^3 + my^4, \\ \frac{dy}{dt} &= -x + nx^4 + 4px^3y - 6hx^2y^2 - 4(g+k+p)xy^3 - ly^4. \end{aligned} \tag{3}$$

In this paper the sufficient center conditions for the origin of coordinates of the phase plane for the bidimensional polynomial systems of differential equations with nonlinearities of the fourth degree with $I_1 = 0,\ I_2 > 0,\ I_3 = 0$ were established.

Theorem. *The system* (3) *has singular point of the center type in the origin of the coordinates of the phase plane of the system if one of the series of the conditions:*

$$\begin{aligned} &1)\quad g + p = 0; \\ &2)\quad n = h = l = 0; \\ &3)\quad n = -2h - 3l, \quad \sqrt{3}g - 16h + 6\sqrt{3}k - 3\sqrt{3}m + 8\sqrt{3}p = 0, \\ &\qquad 5\sqrt{3}g + 14\sqrt{3}k + 16l + \sqrt{3}m + 8\sqrt{3}p = 0; \\ &4)\quad n = -2h - 3l, \quad \sqrt{3}g + 16h + 6\sqrt{3}k - 3\sqrt{3}m + 8\sqrt{3}p = 0, \\ &\qquad 5\sqrt{3}g + 14\sqrt{3}k - 16l + \sqrt{3}m + 8\sqrt{3}p = 0 \end{aligned}$$

is fulfilled.

Acknowledgement. *This research is partially supported by the grant 15.817.02.03F from SCSTD of ASM*

Bibliography

[1] K. S. Sibirsky, *Introduction to the Algebraic Theory of Invariants of Differential Equations.* Manchester University Press, 1988.

[2] N.I. Vulpe, *Polynomial bases of comitants of differential systems and their applications in qualitative theory.* Kishinev, Shtiintsa, 1986, (in Russian).

[3] G. B. Gurevich, *Foundations of the Theory of Algebraic Invariants.* Noordhoff, Groningen, 1964.

[4] Iu. Calin, St. Ciubotaru, *The Lyapunov quantities and the center conditions for a class of bidimensional polynomial systems of differential equations with nonlinearities of the fourth degree.* Buletinul Academiei de Ştiinţe a Republicii Moldova, Chişinău, 2017.

The sufficient center conditions for some classes of bidimensional cubic differential systems

Iurie Calin[1,2] Valeriu Baltag[1]

[1] *Institute of Mathematics and Computer Science,*
[2] *Moldova State University, Chişinău, Republic of Moldova*
e-mail: iucalin@yahoo.com, vbaltag99@gmail.com

,

Let us consider the cubic system of differential equations

$$\begin{aligned} \frac{dx}{dt} &= P_1(x,y) + P_2(x,y) + P_3(x,y) = P(x,y), \\ \frac{dy}{dt} &= Q_1(x,y) + Q_2(x,y) + Q_3(x,y) = Q(x,y), \end{aligned} \tag{1}$$

where $P_i(x,\ y),\ Q_i(x,\ y)$ are homogeneous polynomials of degree i in x and y with real coefficients. The following $GL(2,\mathbb{R})$-comitants [1] have the first degree with respect to the coefficients of the system (1):

$$R_i = P_i(x,y)y - Q_i(x,y)x,\ S_i = \frac{1}{i}\left(\frac{\partial P_i(x,y)}{\partial x} + \frac{\partial Q_i(x,y)}{\partial y}\right),\ i = 1,2,3. \tag{2}$$

The definition of the transvectant of two polynomials is well known in the classical invariant theory [2].

Definition. *Let $f(x,y)$ and $\varphi(x,y)$ be homogeneous polynomials in x and y with real coefficients of the degrees $\rho \in \mathbb{N}^*$ and $\theta \in \mathbb{N}^*$, respectively, and $k \in \mathbb{N}^*$. The polynomial*

$$(f,\varphi)^{(k)} = \frac{(\rho-k)!(\theta-k)!}{\rho!\theta!} \sum_{h=0}^{k} (-1)^h \binom{k}{h} \frac{\partial^k f}{\partial x^{k-h}\partial y^h} \frac{\partial^k \varphi}{\partial x^h \partial y^{k-h}}$$

is called the transvectant of the index k of the polynomials f and φ.

Remark. *If the polynomials f and φ are $GL(2,\mathbb{R})$-comitants of the degrees $\rho \in \mathbb{N}^*$ and $\theta \in \mathbb{N}^*$, respectively, for the system (1), then the transvectant of the index $k \le \min(\rho,\theta)$ is a $GL(2,\mathbb{R})$-comitant of the degree $\rho+\theta-2k$ for the system (1). If $k > \min(\rho,\theta)$, then $(f,\varphi)^{(k)} = 0$.*

By using the transvectants for the system (1) the following $GL(2,\mathbb{R})$-invariants were constructed:

$$I_1 = S_1, \qquad I_2 = (R_1,R_1)^{(2)}, \qquad I_4 = (R_1,S_3)^{(2)}.$$

The system (1) can be written in the following coefficient form:

$$\begin{aligned} \frac{dx}{dt} &= cx + dy + gx^2 + 2hxy + ky^2 + px^3 + 3qx^2y + 3rxy^2 + sy^3, \\ \frac{dy}{dt} &= ex + fy + lx^2 + 2mxy + ny^2 + tx^3 + 3ux^2y + 3vxy^2 + wy^3. \end{aligned} \tag{3}$$

In this paper only the cubic differential systems (1) (or (2)) with $S_2 \equiv 0$, $I_1 = 0$ and $I_2 > 0$ were considered.

By using a center-affine transformation and time scaling the system (3) with $I_1 = 0$, $I_2 > 0$ and $S_2 \equiv 0$ can be reduced to the form:

$$\frac{dx}{dt} = y + gx^2 + 2hxy + ky^2 + px^3 + 3qx^2y + 3rxy^2 + sy^3,$$
$$\frac{dy}{dt} = -x + lx^2 - 2gxy - hy^2 + tx^3 + 3ux^2y - 3qxy^2 + wy^3. \quad (4)$$

In this paper the sufficient center conditions for the origin of coordinates of the phase plane for the cubic differential system with $I_1 = 0$, $I_2 > 0$, $S_2 \equiv 0$ were established.

Theorem. *The system* (4) *has singular point of the center type in the origin of the coordinates, if* $w = -p - r - u$ $(I_4 = 0)$ *and one of the series of the conditions*

$$1)\quad p + u = 0;$$
$$2)\quad k - l = g + h = r + u = s + t = 0;$$
$$3)\quad k + l = g - h = r + u = s + t = 0$$

is fulfilled.

Acknowledgement. *This research is partially supported by the grant 15.817.02.03F from SCSTD of ASM*

Bibliography

[1] *K. S. Sibirsky, Introduction to the Algebraic Theory of Invariants of Differential Equations. Manchester University Press, 1988.*

[2] *G. B. Gurevich, Foundations of the Theory of Algebraic Invariants. Noordhoff, Groningen, 1964.*

Intervals of linear stability of geometrical parameters in the restricted eight bodies problem with incomplete symmetry

Elena Cebotaru

Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: elena.cebotaru@mate.utm.md

We consider the Newtonian restricted eight bodies problem with incomplete symmetry. We investigate the linear stability of this configuration by some numerical methods. For geometric parameter intervals of stability and instability are found, the corresponding theorem are formulated and proved. All relevant and numerical calculation are done with the computer algebra system Mathematica.
Keywords: Newtonian problem; differential equation of motion; configuration; particular solutions; equilibrium points; linear stability.

Bibliography

[1] E. A. Grebenikov, *On a mathematical problems of homographic dynamics*, Moscow: MAKS Press, 2010, 253 p. (in Russian)

[2] E. A. Grebenikov, D. Kozak-Skovorodkina, M. Yakubyak, *Computer algebra methods in the many-body problem*, Second edition, revised and supplemented, Moscow: The Peoples' Friendship University of Russia, 2002, 211 p. ISBN: 5-209-01451-7. (in Russian)

[3] A. Wintner, *The Analytical Foundations of Celestial Mechanics,* Princeton, NJ,USA: Princeton University Press, 1941, 448 p.
[4] S. Wolfram, *The Mathematica Book,* Cambridge: University Press, 1996.

Minimal polynomial basis of $GL(2,\mathbb{R})$-comitants and of $GL(2,\mathbb{R})$-invariants of the planar system of differential equations with nonlinearities of the fourth degree

Stanislav Ciubotaru[1], Iurie Calin[1,2]

[1]*Institute of Mathematics and Computer Science,*
[2]*Moldova State University, Chişinău, Republic of Moldova*
e-mail: iucalin@yahoo.com, stanislav.ciubotaru@yahoo.com

Let us consider the system of differential equations with nonlinearities of the fourth degree

$$\frac{dx}{dt} = P_1(x,y) + P_4(x,y), \quad \frac{dy}{dt} = Q_1(x,y) + Q_4(x,y), \tag{1}$$

where P_i and Q_i are homogeneous polynomials of degree i in x and y with real coefficients. We denote by A the 14-dimensional coefficient space of the system (1), by $\mathbf{a} \in A$ the vector of coefficients, by $\boldsymbol{q} \in \mathcal{Q} \subseteq Aff(2,\mathbb{R})$ a nondegenerate linear transformation of the phase plane of the system (1), by $\mathbf{q}$ the transformation matrix and by $r_{\boldsymbol{q}}(\mathbf{a})$ the linear representation of coefficients of the transformed system in the space A.

Definition 1. [1] *A polynomial* $\mathcal{K}(\mathbf{a},\mathbf{x})$ *in coefficients of system* (1) *and coordinates of the vector* $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$ *is called a comitant of system* (1) *with respect to the group* $\mathcal{Q}$ *if there exists a function* $\lambda : \mathcal{Q} \to \mathbb{R}$ *such that*

$$\mathcal{K}(r_{\boldsymbol{q}}(\mathbf{a}), \mathbf{q}\mathbf{x}) \equiv \lambda(\boldsymbol{q})\mathcal{K}(\mathbf{a},\mathbf{x})$$

for every $\boldsymbol{q} \in \mathcal{Q},\ \mathbf{a} \in A$ *and* $\mathbf{x} \in \mathbb{R}^2$.

If $\mathcal{Q}$ is the group $GL(2,\mathbb{R})$, then the comitant is called $GL(2,\mathbb{R})$-comitant or centroaffine comitant. In what follows only $GL(2,\mathbb{R})$-comitants are considered. The function $\lambda(\boldsymbol{q})$ is called a multiplicator. It is known [1] that the function $\lambda(\boldsymbol{q})$ has the form $\lambda(\boldsymbol{q}) = \Delta_{\boldsymbol{q}}^{-g}$, where $\Delta_{\boldsymbol{q}} = \det \boldsymbol{q}$ and g is an integer, which is called the weight of the comitant $\mathcal{K}(\mathbf{a},\mathbf{x})$. If $g = 0$, then the comitant is called absolute, otherwise it is relative.

If a comitant does not depend on the coordinates of the vectors $\mathbf{x}$, then it is called *invariant.*

We say that a comitant $\mathcal{K}(\mathbf{a},\mathbf{x})$ has the type $(\rho,\ g,\ l_1,\ l_4)$ if it has the degree ρ with respect to coordinates of the vector $\mathbf{x}$, the weight g and the degree l_i, $i = 1,4$ with respect to the coefficients of the homogenity of degree i for the system (1).

Definition 2. [4] *Let* f *and* φ *be polynomials in the coordinates of the vector* $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$ *of the degrees* r *and* ρ*, respectively. The polynomial*

$$(f,\varphi)^{(k)} = \frac{(r-k)!(\rho-k)!}{r!\rho!} \sum_{h=0}^{k} (-1)^h \binom{k}{h} \frac{\partial^k f}{\partial x^{k-h} \partial y^h} \frac{\partial^k \varphi}{\partial x^h \partial y^{k-h}}$$

is called the transvectant of index k *of polynomials* f *and* φ.

If polynomials f and φ are $GL(2,\mathbb{R})$-comitants of system (1), then the transvectant of the index $k \leq \min(r,\rho)$ is a $GL(2,\mathbb{R})$-comitant of the system (1). **Definition 3.** [1, 3] *The set*

$\mathcal{S}$ of comitants (respectively, invariants) is called a polynomial basis of comitants (respectively, invariants) for system (1) with respect to a group $\mathcal{Q}$ if any comitant (respectively, invariant) of system (1) with respect to the group $\mathcal{Q}$ can be expressed in the form of a polynomial of elements of the set $\mathcal{S}$.

Definition 4. [3] *The set $\mathcal{S}$ of comitants (respectively, invariants) of degrees less or equal to δ is called a polynomial basis of comitants for the system* (1) *up to the δ degree with respect to the group $\mathcal{Q}$ if any comitant (respectively, invariants) of the degree less or equal to δ of system* (1) *with respect to the group $\mathcal{Q}$ can be expressed as a polynomial of elements of this set $\mathcal{S}$.*

Definition 5. [1, 3] *A polynomial basis of comitants (respectively, invariants) for system (1) with respect to a group $\mathcal{Q}$ is called minimal if by the removal from it of any comitant (respectively, invariant) it ceases to be a polynomial basis.*

The theory of algebraic invariants and comitants for polynomial autonomous systems of differential equations has been developed by C. Sibirschi [1, 2] and his disciples. One of the important problems concerning this theory is the construction of minimal polynomial bases of the invariants and comitants of the mentioned systems, with respect to different subgroups of the affine group of the transformations of their phase planes, in particular with respect to the subgroup $GL(2,\mathbb{R})$. Some important results in this direction are obtained by academician C. Sibirschi [1, 2] and N.Vulpe [3]. We remark, that polynomial bases for different combinations of homogeneous polynomials $P^j_m(x^1,x^2)$ $(j = 1, 2,\ m = 0, 1, 2, 3)$ in system were considered by E. Gasinskaya-Kirnitskaya, Dang Dinh Bich, D. Boularas, M. Popa, V. Ciobanu, V. Danilyuk, E. Naidenova.

We shall consider the following polynomials:

$$R_i = P_i y - Q_i x; \quad S_i = \frac{1}{i}\left(\frac{\partial P_i}{\partial x} + \frac{\partial Q_i}{\partial y}\right), \qquad (i = 1, 4), \tag{2}$$

which in fact are $GL(2,\mathbb{R})$-comitants of the first degree with respect to the coefficients of system (1).

Using the comitants (2) as elementary "bricks" and the notion of transvectant we have constructed 419 irreducible $GL(2,\mathbb{R})$-comitants of system (1) and hence, the next results is proved:
Theorem. *A minimal polynomial basis of $GL(2,\mathbb{R})$-comitants (respectively, of $GL(2,\mathbb{R})$-invariants) of system (1) up to 18 degree consists from 419 elements (respectively, 182 elements) which must be of the following 111 (respectively, 42) types:*

$$\begin{array}{llll}
(2,\ -1,\ 1,\ 0) - 1; & (5,\ -1,\ 0,\ 1) - 1; & (3,\ 0,\ 0,\ 1) - 1; \\
(2,\ 2,\ 0,\ 2) - 3; & (4,\ 1,\ 0,\ 2) - 1; & (6,\ 0,\ 0,\ 2) - 2; \\
(1,\ 1,\ 1,\ 1) - 1; & (3,\ 0,\ 1,\ 1) - 2; & (5,\ -1,\ 1,\ 1) - 1; \\
(0,\ 0,\ 2,\ 0) - 1; & (1,\ 4,\ 0,\ 3) - 2; & (3,\ 3,\ 0,\ 3) - 6; \\
(5,\ 2,\ 0,\ 3) - 3; & (7,\ 1,\ 0,\ 3) - 1; & (9,\ 0,\ 0,\ 3) - 1; \\
(0,\ 3,\ 1,\ 2) - 3; & (2,\ 2,\ 1,\ 2) - 4; & (4,\ 1,\ 1,\ 2) - 3; \\
(6,\ 0,\ 1,\ 2) - 1; & (1,\ 1,\ 2,\ 1) - 2; & (3,\ 0,\ 2,\ 1) - 1; \\
(0,\ 6,\ 0,\ 4) - 6; & (2,\ 5,\ 0,\ 4) - 6; & (4,\ 4,\ 0,\ 4) - 6;
\end{array}$$

(6, 3, 0, 4) – 2;	(1, 4, 1, 3) – 8;	(3, 3, 1, 3) – 6;
(5, 2, 1, 3) – 1;	(7, 1, 1, 3) – 1;	(0, 3, 2, 2) – 1;
(2, 2, 2, 2) – 3;	(4, 1, 2, 2) – 1;	(1, 1, 3, 1) – 1;
(1, 7, 0, 5) – 11;	(3, 6, 0, 5) – 9;	(5, 5, 0, 5) – 1;
(7, 4, 0, 5) – 4;	(0, 6, 1, 4) – 6;	(2, 5, 1, 4) – 9;
(4, 4, 1, 4) – 3;	(1, 4, 2, 3) – 9;	(3, 3, 2, 3) – 1;
(5, 2, 2, 3) – 1;	(0, 3, 3, 2) – 3;	(2, 2, 3, 2) – 2;
(0, 9, 0, 6) – 7;	(2, 8, 0, 6) – 12;	(4, 7, 0, 6) – 2;
(1, 7, 1, 5) – 20;	(3, 6, 1, 5) – 1;	(5, 5, 1, 5) – 1;
(0, 6, 2, 4) – 8;	(2, 5, 2, 4) – 5;	(1, 4, 3, 3) – 4;
(3, 3, 3, 3) – 1;	(0, 3, 4, 2) – 1;	(1, 10, 0, 7) – 20;
(3, 9, 0, 7) – 1;	(5, 8, 0, 7) – 1;	(0, 9, 1, 6) – 15;
(2, 8, 1, 6) – 5;	(1, 7, 2, 5) – 7;	(3, 6, 2, 5) – 1;
(0, 6, 3, 4) – 10;	(1, 4, 4, 3) – 2;	(0, 3, 5, 2) – 1;
(0, 12, 0, 8) – 15;	(2, 11, 0, 8) – 4;	(1, 10, 1, 7) – 7;
(3, 9, 1, 7) – 1;	(0, 9, 2, 6) – 16;	(1, 7, 3, 5) – 2;
(0, 6, 4, 4) – 5;	(1, 4, 5, 3) – 1;	(1, 13, 0, 9) – 5;
(3, 12, 0, 9) – 1;	(0, 12, 1, 8) – 19;	(1, 10, 2, 7) – 2;
(0, 9, 3, 6) – 5;	(1, 7, 4, 5) – 1;	(0, 6, 5, 4) – 3;
(0, 15, 0, 10) – 14;	(1, 13, 1, 9) – 2;	(0, 12, 2, 8) – 5;
(1, 10, 3, 7) – 1;	(0, 9, 4, 6) – 3;	(0, 6, 6, 4) – 1;
(1, 16, 0, 11) – 2;	(0, 15, 1, 10) – 5;	(1, 13, 2, 9) – 1;
(0, 12, 3, 8) – 3;	(0, 9, 5, 6) – 1;	(0, 6, 7, 4) – 1;
(0, 18, 0, 12) – 4;	(1, 16, 1, 11) – 1;	(0, 15, 2, 10) – 3;
(0, 12, 4, 8) – 1;	(0, 9, 6, 6) – 1;	(1, 19, 0, 3) – 1;
(0, 18, 1, 12) – 3;	(0, 15, 3, 10) – 1;	(0, 12, 5, 8) – 1;
(0, 21, 0, 14) – 2;	(0, 18, 2, 12) – 1;	(0, 15, 4, 10) – 1;
(0, 21, 1, 14) – 1;	(0, 18, 3, 12) – 1;	(0, 24, 0, 16) – 1;
(0, 21, 2, 14) – 1;	(0, 24, 1, 16) – 1;	(0, 27, 0, 18) – 1.

Remark. *Besides each of the types given in the theorem above, we have also indicated the number of the irreducible comitants (or invariants) having this type.*

We establishe a conjecture that the minimal polynomial basis of $GL(2,\mathbb{R})$-comitants (respectively, of $GL(2,\mathbb{R})$-invariants) of system (1) consists from 419 elements (respectively, 182 elements) which must be of the above 111 (respectively, 42) types.

Acknowledgement. *This research is partially supported by the grant 15.817.02.03F from SCSTD of ASM*

Bibliography

[1] K. S. Sibirsky, *Introduction to the Algebraic Theory of Invariants of Differential Equations.* Manchester University Press, 1988.

[2] K. S. Sibirsky, *Algebraic invariants of differential equations and matrices.* Kishinev, Shtiintsa, 1976, (in Russian).

[3] N.I. Vulpe, *Polynomial bases of comitants of differential systems and their applications in qualitative theory.* Kishinev, Shtiintsa, 1986, (in Russian).

[4] G. B. Gurevich, *Foundations of the Theory of Algebraic Invariants.* Noordhoff, Groningen, 1964.

Applications of fast-slow differentiable dynamical systems in neuroscience and engineering

Dana Constantinescu

Univesity of Craiova, Romania
e-mail: constantinescu.dana@ucv.ro

This paper reviews some results from the theory of fast-slow dynamical systems and presents the detailed analysis of some systems of interest in neuroscience, electrical engineering and fusion plasma physics. The singular perturbation methods are used order to explain the different types of oscillations occurring in systems that are obtained by weak perturbations of integrable systems. There are also analyzed systems for which these methods cannot be used and specific explanations for their oscillatory behavior are found. The results are interpreted from practical point of view.

Center conditions for a cubic system with two invariant straight lines and one invariant cubic

Dumitru Cozma, Anatoli Dascalescu

Tiraspol State University, Chişinau, Republic of Moldova
e-mail: dcozma@gmail.com, anatol.dascalescu@gmail.com

We consider the cubic differential system of the form

$$\begin{aligned}\dot{x} &= y + ax^2 + cxy - y^2 + [(a-1)(p+c-b)+g]x^3+ \\ &\quad + [(b-p)(p+c-b) - a - n - 1]x^2y + pxy^2, \\ \dot{y} &= -x - gx^2 - dxy - by^2 + (a-1)(d+n-1)x^3+ \\ &\quad +[(b-p)(d+n+1) - g]x^2y + nxy^2 + by^3,\end{aligned} \tag{1}$$

where the variables $x = x(t), y = y(t)$ and coefficients a, b, c, d, g, p, n are assumed to be real. The origin $O(0,0)$ is a singular point of a center or a focus type for (1), i.e. a fine focus.

It is easy to verify that the cubic system (1) has two invariant straight lines of the form $l_1 \equiv 1 + A_1x - y = 0,\ \ l_2 \equiv 1 + A_2x - y = 0$, where A_1, A_2 are distinct solutions of the equations $A^2 + (b - c - p)A - d - n - 1 = 0$, and we determine the conditions under which the cubic system (1) has also one irreducible invariant cubic curve of the form

$$\Phi(x, y) \equiv x^2 + y^2 + a_{30}x^3 + a_{21}x^2y + a_{12}xy^2 + a_{03}y^3 = 0$$

with $(a_{30}, a_{21}, a_{12}, a_{03}) \neq 0$ and $a_{30}, a_{21}, a_{12}, a_{03} \in \mathbb{R}$. The problem of the center for cubic system (1) with: two parallel invariant straight lines and one invariant cubic $\Phi = 0$ was solved in [1]; a bundle of three algebraic curves $l_1 = 0$, $l_2 = 0$ and $\Phi = 0$ was solved in [2].

In this paper we study the problem of the center for cubic system (1) having three algebraic solutions $l_1 = 0,\ l_2 = 0,\ \Phi = 0$ in generic position and prove the following theorem:

Theorem 1. *Let the cubic system (1) have two invariant straight lines $l_1 = 0,\ l_2 = 0$ and one irreducible invariant cubic $\Phi = 0$. Then a fine focus $O(0,0)$ is a center if and only if the first three Lyapunov quantities vanish.*

Bibliography

[1] D. Cozma, *The problem of the center for cubic systems with two parallel invariant straight lines and one invariant cubic*, ROMAI Journal, 2015, no. 2, 63–75.

[2] D. Cozma, A. Dascalescu, *Integrability conditions for a class of cubic differential systems with a bundle of two invariant straight lines and one invariant cubic*, Bull. Acad. Sci. of Moldova, Mathematics, 2018, no.1, 120–138.

Global stability results for models of commensalism

Paul Georgescu[1], Daniel Maxin[2], Hong Zhang[3,4]

[1] *Technical University of Iaşi, Romania*
[2] *Valparaiso University, U.S.A.*
[3] *Jiangsu University, P.R. China*
[4] *Changzhou Institute of Technology, P.R. China*
e-mail: v.p.georgescu@gmail.com

We analyze the global stability of the coexisting equilibria for several models of commensalism, first by devising a procedure to modify several Lyapunov functionals which were introduced earlier for corresponding models of mutualism, further confirming their usefulness. It is seen that commensalism promotes global stability, in connection with higher order self-limiting terms which prevent unboundedness. We then use the theory of asymptotically autonomous systems to prove global stability results for models of commensalism which are subject to Allee effects, finding that commensalisms of appropriate strength can overcome the influence of strong Allee effects.

Invariant conditions of stability of unperturbed motion described by cubic differential system with quadratic part of Darboux type

Natalia Neagu[1,2], Victor Orlov[3,4], Mihail Popa[3]

[1] *Tiraspol State University,*
[2] *Ion Creangă State Pedagogical University,*
[3] *Institute of Mathematics and ComputerScience,*
[4] *Technical University of Moldova, Chisinau, Republic of Moldova*
e-mail: neagu_natusik@mail.ru, orlovictor@gmail.com, mihailpomd@gmail.com

In [1] the center-affine invariant conditions of stability of unperturbed motion, described by critical two-dimensional differential systems with quadratic nonlinearities $s(1,2)$, cubic nonlinearities $s(1,3)$ and fourth-order nonlinearities $s(1,4)$, were obtained.
We consider the two-dimensional cubic differential system $s(1,2,3)$ of perturbed motion of the form

$$\dot{x}^j = a^j_\alpha x^\alpha + a^j_{\alpha\beta} x^\alpha x^\beta + a^j_{\alpha\beta\gamma} x^\alpha x^\beta x^\gamma \equiv \sum_{i=1}^{3} P^j_i,\ (j,\alpha,\beta,\gamma = 1,2), \tag{1}$$

where coefficients $a^j_{\alpha\beta}$ and $a^j_{\alpha\beta\gamma}$ are symmetric tensors in lower indices in which the total convolution is done. Coefficients and variables in (1) are given over the field of real numbers.
Let φ and ψ be homogeneous comitants of degree ρ_1 and ρ_2 respectively of the phase variables

$x = x^1$ and $y = x^2$ of a two-dimensional polynomial differential system. Then by [2] the transvectant

$$(\varphi, \psi)^{(j)} = \frac{(\rho_1 - j)(\rho_2 - j)}{\rho_1!\rho_1!} \sum_{i=0}^{j} (-1)^j \begin{pmatrix} j \\ i \end{pmatrix} \frac{\partial^j \varphi}{\partial x^{j-i} \partial y^i} \frac{\partial^j \psi}{\partial x^i \partial y^{j-i}} \tag{2}$$

is also a comitant for this system.
In the works of Iurie Calin, see for example [3], it is shown that by means of the transvectant (2) all generators of the Sibirsky algebras of comitants and invariants for any system of type (1) can be constructed.
According to [3], we write the following comitants of the system (1):

$$R_i = P_i^1 x^2 - P_i^2 x^1, \quad S_i = \frac{1}{i}\left(\frac{\partial P_i^1}{\partial x^1} + \frac{\partial P_i^2}{\partial x^2}\right), \; i = 1, 2, 3. \tag{3}$$

Later on, we will need the following comitants and invariants of system (1), built by operations (2) and (3), presented by Iurie Calin:

$$I_1 = S_1, \quad I_2 = (R_1, R_1)^{(2)}, \quad I_3 = \left((R_3, R_1)^{(2)}, R_1\right)^{(2)}, \; I_4 = (S_3, R_1)^{(2)},$$

$$K_2 = R_1, \quad K_5 = S_2, \quad K_8 = R_3, \quad K_9 = (R_3, R_1)^{(1)}, \quad K_{10} = (R_3, R_1)^{(2)},$$

$$K_{11} = \left((R_3, R_1)^{(2)}, R_1\right)^{(1)}, \; K_{14} = (S_2, R_1)^{(1)}, \; K_{15} = S_3, \; K_{16} = (S_3, R_1)^{(1)}.$$

Let for system (1) the invariant conditions $I_1^2 - I_2 = 0$, $I_1 < 0$ are hold. Then the system (1) becomes critical system of Lyapunov type [1].
Let for system (1) the invariant condition $R_2 \equiv 0$ is hold. Then the quadratic part of this system takes the Darboux form: $P_2^1 = x^1(a_{11}^1 x^1 + 2a_{12}^1 x^2)$, $P_2^2 = x^2(a_{11}^1 x^1 + 2a_{12}^1 x^2)$.

Theorem. *Let for system of perturbed motion* (1) *the invariant conditions* $I_1^2 - I_2 = 0$, $I_1 < 0$ *and* $R_2 \equiv 0$ *be satisfied. Then the stability of the unperturbed motion is described by one of the following twelve possible cases:*

I. $\mathcal{N}_1 \not\equiv 0$, *then the unperturbed motion is unstable;*
II. $\mathcal{N}_1 \equiv 0, \quad \mathcal{N}_2 > 0$, *then the unperturbed motion is stable;*
III. $\mathcal{N}_1 \equiv 0, \quad \mathcal{N}_2 < 0$, *then the unperturbed motion is unstable;*
IV. $\mathcal{N}_1 \equiv \mathcal{N}_2 \equiv 0, \quad K_5\mathcal{N}_3 \not\equiv 0$, *then the unperturbed motion is unstable;*
V. $\mathcal{N}_1 \equiv \mathcal{N}_2 \equiv K_5 \equiv 0, \quad \mathcal{N}_3\mathcal{N}_4 > 0$, *then the unperturbed motion is unstable;*
VI. $\mathcal{N}_1 \equiv \mathcal{N}_2 \equiv K_5 \equiv 0, \quad \mathcal{N}_3\mathcal{N}_4 < 0$, *then the unperturbed motion is stable;*
VII. $\mathcal{N}_1 \equiv \mathcal{N}_2 \equiv \mathcal{N}_4 \equiv K_5 \equiv 0$, $\mathcal{N}_3 \not\equiv 0$, $\mathcal{N}_5 > 0$, *then the unperturbed motion is stable;*
VIII. $\mathcal{N}_1 \equiv \mathcal{N}_2 \equiv \mathcal{N}_4 \equiv K_5 \equiv 0$, $\mathcal{N}_3 \not\equiv 0$, $\mathcal{N}_5 < 0$, *then the unperturbed motion is unstable;*
IX. $\mathcal{N}_1 \equiv \mathcal{N}_2 \equiv \mathcal{N}_4 \equiv K_5 \equiv 0$, $S\mathcal{N}_3 > 0$, *then the unperturbed motion is unstable;*
X. $\mathcal{N}_1 \equiv \mathcal{N}_2 \equiv \mathcal{N}_4 \equiv K_5 \equiv 0$, $S\mathcal{N}_3 < 0$, *then the unperturbed motion is stable;*
XI. $\mathcal{N}_1 \equiv \mathcal{N}_2 \equiv 0$, $\mathcal{N}_3 \equiv 0$, *then the unperturbed motion is stable;*
XII. $\mathcal{N}_1 \equiv \mathcal{N}_2 \equiv \mathcal{N}_4 \equiv \mathcal{N}_5 \equiv K_5 \equiv S \equiv 0$, *then the unperturbed motion is stable;*

where $\mathcal{N}_1 = 2K_{14} - I_1K_5$, $\mathcal{N}_2 = 2I_1^2K_{10} - 4I_1K_{11} - 3I_1I_2K_{15} - 3I_1^2K_{16} + 4I_3K_2 + 3I_1I_4K_2$, $\mathcal{N}_3 = -12I_1K_{10}K_2 + 8K_{11}K_2 + 3I_1^2K_{15}K_2 - 6I_1K_{16}K_2 + 6I_4K_2^2 - 4I_1^3K_8 + 8I_1^2K_9$, $\mathcal{N}_4 = 2I_3 + I_1I_4$, $\mathcal{N}_5 = 2K_{10} + I_1K_{15} - K_{16}$, $S = 3K_{15}K_2 - 2I_1K_8 - 4K_9$.

In the last two cases the unperturbed motion belongs to some continuous series of stabilized motions, and all motions, which are sufficiently close to unperturbed motion, will be stable. In this case for sufficiently small perturbations any perturbed motion will asymptotically approach to one of the stabilized motions of the mentioned series.

Bibliography

[1] Neagu, N.; Orlov, V.; Popa, M. *Invariant conditions of stability of unperturbed motion governed by some differential systems in the plane*, Bul. Acad. Ştiinţe Repub. Mold. Mat., 2017, no. 3(85), 88-106.

[2] Vulpe, N.I., *Polynomial bases of comitants of differential systems and their applications in qualitative theory*, Kishinev, Ştiinţa, 1986 (in Russian).

[3] Calin, Iu., *On rational bases of $GL(2,R)$-comitants of planar polynomial systems of differential equations*, Bul. Acad. Ştiinţe Repub. Mold. Mat., 2003, no. 2(42), 69-86.

Canonical forms for cubic differential systems with affine real invariant straight lines of total parallel multiplicity six and configurations $\big(2(m), 2(n), 1, 1\big)$

Vitalie Puţuntică

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: vitputuntica@mail.ru

We consider the real cubic differential system

$$\begin{cases} \dot{x} = P_0 + P_1(x,y) + P_2(x,y) + P_3(x,y) \equiv P(x,y), \\ \dot{y} = Q_0 + Q_1(x,y) + Q_2(x,y) + Q_3(x,y) \equiv Q(x,y), \ \gcd(P,Q) = 1, \end{cases} \tag{1}$$

and the vector field $\mathbb{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}$ associated to system (1).
A straight line $l \equiv \alpha x + \beta y + \gamma = 0$, $\alpha, \beta, \gamma \in \mathbb{C}$ is invariant for (1) if there exists a polynomial $K_l \in \mathbb{C}[x,y]$, $\deg(K_l) \leq 2$ such that the identity

$$\alpha P(x,y) + \beta Q(x,y) \equiv (\alpha x + \beta y + \gamma) K_l(x,y), \ (x,y) \in \mathbb{R}^2 \tag{2}$$

holds. The polynomial $K_l(x,y)$ is called cofactor of the invariant straight line l. If m is the greatest natural number such that l^m divides $\mathbb{X}(l)$ then we say that l has parallel multiplicity m. By present a great number of works have been dedicated to the investigation of polynomial differential systems with invariant straight lines.
The classification of all cubic systems with the maximum number of invariant straight lines, including the line at infinity, and taking into account their geometric multiplicities, is given in [1], [4], [5].
The cubic systems with exactly eight and exactly seven distinct affine invariant straight lines have been studied in [4], [5]; with invariant straight lines of total geometric (parallel) multiplicity eight (seven) - in [2], [3], [8], and with six real invariant straight lines along two (three) directions - in [6], [7]. In [9] it was shown that in the class of cubic differential systems the maximal multiplicity of an affine real straight line is seven.
In this paper are obtained canonical forms for cubic differential systems with affine real invariant straight lines of total parallel multiplicity six and configurations $\big(2(m), 2(n), 1, 1\big)$. It was proved
Theorem. *Assume that a cubic system* (1) *possesses affine real invariant straight lines of total parallel multiplicity six and of four distinct directions. At least two of these lines are multiplicity one. Then via an affine transformation and time rescaling this system can be brought to one of the three systems:*

$$1) \begin{cases} \dot{x} = x(x+1)(a+bx+y), \\ \dot{y} = y(y+1)\big(a+(a+b)x+(1-a)y\big), \\ ab(a+b)(2a-b-1) \neq 0, (a-1)(b-a+1) < 0, a, b \in R, \end{cases}$$

$\big(l_1 = x, l_2 = x+1, l_3 = y, l_4 = y+1, l_5 = y-x, l_6 = y - \frac{b}{a-1}x\big)$;

2) $\begin{cases} \dot{x} = x(x+1)\big(a + (2a-1)x + y\big), \\ \dot{y} = y(y+1)\big(a + x + (2a-1)y\big), \\ a(a-1)(3a-1)(3a-2) \neq 0, a \in R, \end{cases}$

$\big(l_1 = x, l_2 = x+1, l_3 = y, l_4 = y+1, l_5 = y-x, l_6 = y+x+1\big)$;

3) $\begin{cases} \dot{x} = x^2(bx + y), \\ \dot{y} = y^2\big((a+b)x + (1-a)y\big), \\ ab(b+1)(a+b)(2a-b-1)(a-2b-2) \neq 0, \\ (a-1)(b-a+1) < 0, a, b \in R, \end{cases}$

$\big(l_1 = l_2 = x, l_3 = l_4 = y, l_5 = y-x, l_6 = y - \frac{b}{a-1}x\big)$.

Bibliography

[1] Bujac C., O*ne new class of cubic systems with maximum number of invariant lines omitted in the classification of J.Llibre and N.Vulpe.* In Bul. Acad. Ştiinţe Repub. Mold. Mat., 2014, No. **2(75)**, p. 102-105.

[2] Bujac C. and Vulpe N., *Cubic systems with invariant straight lines of total multiplicity eight and with three distinct infinite singularities*, Qual. Theory Dyn. Syst. **14** (2015), No. 1, 109–137.

[3] Bujac C. and Vulpe N., *Cubic systems with invariant lines of total multiplicity eight and with four distinct infinite singularities*, Journal of Mathematical Analysis and Applications, **423** (2015), No. 2, 1025–1080.

[4] Llibre J. and Vulpe N., *Planar cubic polynomial differential systems with the maximum number of invariant straight lines.* Rocky Mountain J. Math. **36** (2006), No 4, 1301–1373.

[5] Lyubimova R.A., *About one differential equation with invariant straight lines.* Diff. and int. eq., Gorky Universitet. **8** (1984), 66–69; **1** (1997), 19-22.

[6] Puţuntică V. and Şubă A. *The cubic differential system with six real invariant straight lines along two directions.* Studia Universitatis. 2008, No **8(13)**, 5–16.

[7] Puţuntică V. and Şubă A. *The cubic differential system with six real invariant straight lines along three directions.* Bulletin of ASRM. Mathematics. 2009, No **2(60)**, 111–130.

[8] Şubă A., Repeşco V. and Puţuntică V., *Cubic systems with invariant affine straight lines of total parallel multiplicity seven.* Electron. J. Diff. Equ., Vol. 2013 (2013), No. **274**, 1–22. http://ejde.math.txstate.edu/

[9] Şubă A. and Vacaraş O., *Cubic differential systems with an invariant straight line of maximal multiplicity.* Annals of the University of Craiova. Mathematics and Computer Science Series, **42** (2015), No. 2, 427–449.

The comitants of Lyapunov system with respect to the rotation group and applications

Pricop Victor

"Ion Creangă" State Pedagogical University,
Institute of Mathematics and Computer Science,
Chişinău, Republic of Moldova
e-mail: pricopv@mail.ru

Let us consider the Lyapunov system $s\mathcal{L}(1, m_1, ..., m_\ell)$

$$\dot{x} = y + \sum_{i=1}^{\ell} P_{m_i}(x, y),\ \dot{y} = -x + \sum_{i=1}^{\ell} Q_{m_i}(x, y), \tag{1}$$

where P_{m_i} and Q_{m_i} are homogeneous polynomials of degree m_i with respect to phase variables x and y. The set $\{1, m_1, ..., m_\ell\}$ consists of a finite number of distinct natural numbers. With A is denoted the set of coefficients of P_{m_i} and Q_{m_i}.
We investigate the action of the rotation group $SO(2, \mathbb{R})$ on the system (1).
Following [1] analogically were defined the comitants of differential systems with respect to the rotation group.
The Lie operator of the representation of the group $SO(2, \mathbb{R})$ in the space $E^N(x, y, A)$ of the system (1) was defined [2].
Using this Lie operator was determined the criterion when a polynomial is a comitant of Lyapunov system with respect to the rotation group.

Theorem 1. *The number of functionally independent focus quantities θ in the center and focus problem for the Lyapunov system $s\mathcal{L}(1, m_1, ..., m_\ell)$ does not exceed the number*

$$2\left(\sum_{i=1}^{\ell} m_i + \ell\right) + 1.$$

Bibliography

[1] Sibirsky K. S., *Algebraic invariants of differential equations and matrices*, Shtiintsa, Kishinev, 1976 (in Russian).

[2] Popa M. N., Pricop V. V., *The center-focus problem: algebraic solutions and hypotheses*, Kishinev, ASM, Institute of Mathematics and Computer Science, 2018 (in Russian).

Qualitative study of cubic differential systems with invariant straight lines of total multiplicity seven along one direction

Vadim Repeşco

Tiraspol State University, Chişinău, Republic of Moldova
e-mail:

Consider the general cubic differential system $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$, where $P, Q \in \mathbb{R}[x, y]$, $\max\{\deg P, \deg Q\} = 3$ and $GCD\,(P, Q) = 1$.
According to [1], we can construct a Darboux first integral for a cubic differential system, if this system has sufficiently many invariant straight lines considered with their multiplicities. In [2] we showed that there are exactly 26 canonical forms of cubic differential systems which possess invariant straight lines of total multiplicity at least seven (including the invariant straight line at the infinity) along one direction.
In this paper, using qualitative methods for dynamical systems, we investigate the systems obtained in [2] and show trajectories behavior on the Poincaré disk.
Acknowledgement. *Paper supported by project for young researchers 16.80012.02.01F*

Bibliography

[1] Llibre J., Xiang Zhang, *On the Darboux Integrability of Polynomial Differential Systems*, Qual. Theory Dyn. Syst., 2012, 129–144.

[2] Repeşco V., *Canonical forms of cubic differential systems with invariant straight lines of total multiplicity seven along one direction*, The 25th Conference on Applied and Industrial Mathematics, 14-17 september, 2017, Iaşi.

Positive Solutions for a System of Riemann-Liouville Fractional Differential Equattionswith Multi-Point Fractional Boundary Conditions

Rodica Luca Tudorache

Gh. Asachi Technical University, Department of Mathematics, 11 Blvd. Carol I, Iasi 700506, Romania.
e-mail: rluca@math.tuiasi.ro; rlucatudor@yahoo.com

We study the system of nonlinear ordinary fractional differential equations

$$(S) \qquad \begin{cases} D_{0+}^{\alpha}u(t) + \lambda f(t, u(t), v(t), w(t)) = 0, & t \in (0,1), \\ D_{0+}^{\beta}v(t) + \mu g(t, u(t), v(t), w(t)) = 0, & t \in (0,1), \\ D_{0+}^{\gamma}w(t) + \nu h(t, u(t), v(t), w(t)) = 0, & t \in (0,1), \end{cases}$$

with the multi-point boundary conditions which contain fractional derivatives

$$(BC) \qquad \begin{cases} u^{(j)}(0) = 0,\ j = 0, \dots, n-2;\ D_{0+}^{p_1}u(1) = \sum_{i=1}^{N} a_i D_{0+}^{q_1}u(\xi_i), \\ v^{(j)}(0) = 0,\ j = 0, \dots, m-2;\ D_{0+}^{p_2}v(1) = \sum_{i=1}^{M} b_i D_{0+}^{q_2}v(\eta_i), \\ w^{(j)}(0) = 0,\ j = 0, \dots, l-2;\ D_{0+}^{p_3}w(1) = \sum_{i=1}^{L} c_i D_{0+}^{q_3}w(\zeta_i), \end{cases}$$

where λ, μ, $\nu > 0$, α, β, $\gamma \in \mathbb{R}$, $\alpha \in (n-1, n]$, $\beta \in (m-1, m]$, $\gamma \in (l-1, l]$, n, m, $l \in \mathbb{N}$, n, m, $l \geq 3$, p_1, p_2, p_3, q_1, q_2, $q_3 \in \mathbb{R}$, $p_1 \in [1, n-2]$, $p_2 \in [1, m-2]$, $p_3 \in [1, l-2]$, $q_1 \in [0, p_1]$, $q_2 \in [0, p_2]$, $q_3 \in [0, p_3]$, ξ_i, $a_i \in \mathbb{R}$ for all $i = 1, \ldots, N$ ($N \in \mathbb{N}$), $0 < \xi_1 < \cdots < \xi_N \leq 1$, η_i, $b_i \in \mathbb{R}$ for all $i = 1, \ldots, M$ ($M \in \mathbb{N}$), $0 < \eta_1 < \cdots < \eta_M \leq 1$, ζ_i, $c_i \in \mathbb{R}$ for all $i = 1, \ldots, L$ ($L \in \mathbb{N}$), $0 < \zeta_1 < \cdots < \zeta_L \leq 1$, and D_{0+}^k denotes the Riemann-Liouville derivative of order k (for $k = \alpha, \beta, \gamma, p_1, q_1, p_2, q_2, p_3, q_3$).
Under some assumptions on the functions f, g and h, we give intervals for the parameters λ, μ and ν such that positive solutions of $(S) - (BC)$ exist (see [1]). The nonexistence of positive solutions for the above problem is also investigated. In the proof of our existence results we use the Guo-Krasnosel'skii fixed point theorem.

Bibliography

[1] R. Luca, *Positive solutions for a system of Riemann-Liouville fractional differential equations with multi-point fractional boundary conditions*, Bound. Value Probl., 2017:102 (2017), 1-35.

Boolean asynchronous systems: the concept of attractor

Serban E. Vlad

Oradea City Hall
e-mail: `serban_e_vlad@yahoo.com`

The Boolean asynchronous systems are systems generated by the functions $\Phi : \{0,1\}^n \longrightarrow \{0,1\}^n$ which iterate their coordinates independently on each other. Our purpose is to introduce their attractors by analogy with the dynamical systems literature.
The attractors are defined by Andrew Ilachinski in a real space, real time context (David Ruelle, Floris Takens, Jean-Pierre Eckmann and Robert Devaney are also cited) as sets that fulfill invariance, attractivity, minimality and topological transitivity is mentioned also.
John Milnor refers to real space, discrete time dynamical systems. He defines the trapped attractors, the trapping neighborhoods (or trapped attracting sets) and finally the attractors, in a manner that proves to be equivalent to that of D. V. Anosov, V. I. Arnold, Michael Brin, Garrett Stuck, Boris Hasselblatt, Anatole Katok and Jurgen Jost.
Such suggestions that we have grouped around the ideas of Ilachinski and Milnor bring in the Boolean asynchronous context a unique concept of attractor.

3. Mathematical Modeling

Modeling the error ratio in digital optical communications

Alexandra Băluţă, Diana Rotaru, Mihaela Ilie, D. Fălie, Eugen Vasile

Politehnica University of Bucharest, Bucharest, Romania
e-mail: evasevas@yahoo.com

In classical or quantum digital optical communications, the useful transmitted information may decreases due to the random noises and perturbations that cannot be eliminated and which matter in the case of very low level optical signals. The paper examines the errors of the transmitted bits or qubits which are conditioned by different random optoelectronic noises which are mathematically modeled to be appropriate to different physical phenomenon. In the classical communications case it was pointed out the possibility to obtain an error ratio error ratio of 10-9 as well as of the usual value of 10-6. In the quantum communications case, the error rates are significantly higher, usually around a few percent. This case is distinct from the bit error ratio used in standard communications and is analyzed within the formalism specific to quantum physics. Quantum error ratio need to be corrected down to 10-9 with different algorithms than those used in classical communications. The main purpose of these algorithms is to keep the secrecy of the transmitted information. Based on normalized standard models (with terminology and definitions revisited) numerical simulations have performed in the MathCAD software environment.
random noise, bit error ratio / rate, quantum communications, MathCAD simulation

Elaboration of calculation model for precessional gearing

I. Bostan, V. Dulgheru, M. Vaculenco

Technical University of Moldova, Department "Fundamental of Machine Design", Chişinău, Republic of Moldova
e-mail: valeriu.dulgheru@bpm.utm.md

The paper presents the CAD (Autodesk Inventor) modelling and CAE simulation of precessional drives capable of high transmission ratio and torque for one stage compact construction. The simulations of the drives provide information concerning forces, torques and energies, as well as contact forces between gear teeth and satellite teeth. Also, there is presented the analysis of the results concerning the design optimization of the planetary precessional transmission.

Elaboration of calculation model for dynamic simulation of a vertical axis wind turbine rotor

V. Bostan, M. Guţu and I. Rabei

Technical University of Moldova, Department of Basics of Machinery Projecting, Chişinău, Republic of Moldova
e-mail: marin.gutu@pmai.utm.md

This paper describes the steps of elaboration of calculation model for dynamic simulation of a small vertical axis wind turbine rotor. The calculation model is based on the finite element analysis ANSYS CFX software. The CFD model is used to determine the performance of the wind turbine rotor for deferent settings. The verification of the proposed model is done by calculation of the rotor performance using QBlade software.

Computational issues regarding the parameter sensitivity in biological models

Gabriel Dimitriu

University of Medicine and Pharmacy "Grigore T. Popa", Department of Medical Informatics and Biostatistics, Iaşi, Romania
e-mail: dimitriu.gabriel@gmail.com

The aim of this talk is to present computational issues contained in parameter sensitivity studies associated to several biological models. The local sensitivity is carried out by using the QR decomposition with column pivoting to the relative sensitivity matrix. We evaluate the relative identifiability of the parameters and establish orderings with respect to their identifiability. Also, a global sensitivity analysis is performed with sensitivity heat maps of the model variables and parameter sensitivity spectra.

Bibliography

[1] G. Dimitriu, *Determination of the optimal inputs for an identification problem using sensitivity analysis*, Proceedings of the Second International Symposium on Sensitivity Analysis of Model Output (SAMO), Venice, Ca' Dolfin, April 19-22 (1998), 99–102.

[2] G. Dimitriu, V.L. Boiculese, *Sensitivity Study for a SEIT Epidemic Model*, Proceedings of the 5th IEEE International Conference on e-Health and Bioengineering, Iași, November 19-21 (2015).

[3] G. Dimitriu, M. Moscalu, V.L. Boiculese, *A local sensitivity study for an activated T-cell model*, Proceedings of the International Conference on e-Health and Bioengineering, Sinaia, June 22-24 (2017).

[4] G. Dimitriu, *Identifiability and Sensitivity Analyses for a Chronic Myelogenous Meukemia Model with T Cell Interaction*, Proceedings of the IEEE International Conference ICSTCC, Sinaia, October 19-21 (2017), 704–710.

[5] M. Domijan, P. Brown, B. Shulgin, D. Rand, *Using PeTTSy, Perturbation Theory Toolbox for Systems*, Warwick Systems Biology Centre.

[6] H. Moore, N.K. Li, *A mathematical model for chronic myelogenous leukemia (CML) and T cell interaction*, J. Theor. Biol., vol. 227 (2004), 513–523.

Applying stochastic analysis of experimental data to optimize the bioactive compounds extraction from agro-food industry waste

A. A. Ghendov-Mosanu[1], R. A. Sturza[2] and T.D. Chereches[3]

[1] *Department of Food Technology, Technical University of Moldova;*
[2] *Department of Chemistry, Technical University of Moldova;*
[3] *UPS PILOT ARM LTD, Bucharest, Romania*
e-mail: aliona.mosanu@tpa.utm.md, rodica.sturza@chim.utm.md, tudor.chereches@yahoo.com

Classical statistics are based on the law of large numbers, which calls for many experimental values. In the case of costly experiments with fewer practical values, the results can be questionable as classical statistics offer only one prediction horizon, which is a disadvantage in terms of the credibility of the conclusions. To reduce the uncertainty of complex experimental modeling, it is necessary to apply the stochastic analysis of the experimental data, the most used methods in this regard, using the bootstrap algorithms [1]. Bootstrap techniques consist of reshaping the experimental discrete series (re-sampling is another name for bootstrap) based on stochastic distribution laws, so not the usual classical ones (normal, Weibull, etc.); the most used are Markov processes, the Monte Carlo method and others [2]. Bootstrap algorithms consider each sample to be irreptable, which is true, since it is known that two absolutely identical results can never be obtained.

Experimental research focused on 6 products (bioactive compounds extracts from forest fruits and agro-food industry waste) and a maximum of 9 experimental parameters determined. The purpose of applying the stochastic analysis was to establish predictive elements, which allow good data interpolation, ensure the greatest credibility of the experimental results, including the values of factors of influence on the extraction process (concentration of ethyl alcohol, hydromodule, duration and temperature extraction, pulse number, field intensity, voltage) not found experimentally, as well as the determination of the most pronounced and the weakest interdependences of the measured parameters.

The principle of the bootstrap technique is the following, with an example for a certain parameter P in the $P1$-$P9$ range. We know from experimentation the finite discrete series, which is a vector of n values $P = \{P_1, P_2, ..., P_n\}$, with the unknown distribution function F and it is desired to estimate $Q^* = S(P)$ of the set of characteristic parameters $Q = f(F)$ related to the set of values of some parameter P. Applying this principle requires two calculation steps: 1. The determination of B samples with the same number of values and with the same specification (mean, standard deviation, median, dispersion, etc.) as those of the experimental series, subject to the unknown distribution law F: $P_i^* = \{P_{1i}^*, P_{2i}^*, ..., P_{ni}^*\}; i = 1, 2, ..., B$. 2. The estimation of the set of characteristic parameters for each sample obtained by sampling: $Q_i^* = S(P_i^*)$; $i = 1, 2, ..., B$.

Stochastic analysis of experimental data allowed: performing prediction with a higher prediction horizon than that provided by classic statistics, which gives a high veracity of results; obtaining a lot of reliable data, complementary to the experimental ones, which are usually of low volume; obtaining credible results in situations commonly encountered in practice, namely when experimental data is not subject to the distribution laws known in classical statistics.

Following the stochastic analysis of the experimental data, the following conclusions were drawn:

– the data obtained by applying bootstrap algorithms allow prediction with a high prediction horizon;

– the application of the booster algorithms provides acceptable results and in the cases of few experimental data, frequent situations in practice;

– the boostrap algorithm offers a flexible stochastic analysis method, since it allows the imposition of any specification on the experimental data.

Bibliography

[1] M. R. Chernick, *Bootstrap methods: A predictioner's guide*, John Wiley and Sons, New York (1999).

[2] C. Z. Mooney and R. D. Duval, *Bootstrapping: A nonparametric approach to statistical inference*, Newbury, Sage Publications (1993).

Numerical analysis of the dynamic loading of elastic-plastic buried structures

Elena Gutuleac, Grigore Secrieru

Institute of Mathematics and Computer Science, Chisinau, Republic of Moldova
e-mail: elena.gutuleac@math.md, secrieru@renam.md

Use of computer technology allows solving of the most complex applied problems, such as dynamic loading of a solid deformable body under the influence of a wide range of external loads (seismic, explosive and others). The study of seismic and explosive impact is based on a series of approximations and models, related to the environment's structure and distribution of the seismic impulse and shock waves in this environment.
For Moldova as priority challenges, we consider environment monitoring and forecasting of changes of environmental parameters, seismic data collection and processing, modelling of seismic waves influence on dangerous constructions.
The problem of computer estimation of operational condition of potentially dangerous objects is very actual for various regions. The potentially dangerous objects are objects where used, stored, transported or destroyed flammable, explosive and toxic substances (oil depots, gas stations, storages of fertilizers, ammunition depots).
Their damage or destruction in the event of seismic impact (or other force majeure) may lead to environmental disasters. Full-scale physical tests in the industry are difficult or expensive; therefore the significance of mathematical modelling increases. The modern computational capabilities allow solving of the above-menti-
oned problems with using numerical algorithms based on finding solutions of complex mathematical physics equations, to take for model creation a lot of information about objects, which interact with each other and with the environment in the model framework.
The possibilities of analytical methods and application of solutions based on physical experiments are quite limited. Researchers are trying to create to precise mathematical models, numerical algorithms and data analysis systems to obtain reliable numerical solutions for more efficient design of constructions. The implementation of these solutions is a complex task because of their large and number of parameters.
For a correct description of the elastic-plastic behaviour of constructions realistic equations of state for construction filling materials and explosives is necessary to use. The behaviour of various materials is described within the equation of state in the form of Mie-Gruneisen [1], taking into account a complex stress-strain behavior of substance.
The system of governing equations describes the motion of elasto-plastic medium under a shock and blast loading. Equations are written in Lagrange coordinates in a two-dimensional setting. In

this case the coordinates are "freezed" -in the medium so that they move and deform along with it [2, 3, 4].
Lagrangian coordinates allow to simulate free and contact boundaries as well as to implement boundary conditions accurately and correctly. The difference scheme proposed by Wilkins [1] but with certain modifications has taken as a basic scheme for solving the governing equations. This scheme is second order and is satisfactory for the problem that under our consideration. The corresponding state equations for the ground, fluid, explosive and detonation products are chosen. The developed methods and algorithms have tested and realized. Planning and execution of numerical experiments with analysis and visualization in graphs and 2D-images have implemented. In the framework of the described model the stress-strain state of buried structures has analyzed. The properties of structural materials, ground and filler have taken into account. The behavior of buried structures under explosive or seismic wave was studied. The influence of the surrounding environment on the deformation structure processes has investigated. The detailed picture of the process of ground, fluid and structure interaction under dynamic loading has obtained.

Bibliography

[1] Wilkins M.L. *Modelling the behavior of materials.* Struct. Impact and Grashworth. Proceeding of International Conference. V.2, London, New York, 1984, pp. 243–277.

[2] Rybakin B. *Computer Modeling of Dynamic Processes.* CSJM, v.8, N 2(23), 2000, pp. 150–180.

[3] Rybakin B., Russeva E., Secrieru G. *Numerical investigation of the process of detonation waves interaction with an elastoplastic target.* Buletinul ACM, Matematica, N.2(42), 2003, pp. 113–122.

[4] Gutuleac E. *Mathematical modeling of high-speed loads effects on storage tanks.* Computer Science Journal of Moldova, 2012, vol. 20, 1 (58), pp. 33–41.

Modeling the Plant Effect on Soil Erosion by Water

Stelian Ion, Ştefan-Gicu Cruceanu and Dorin Marinescu

"Gheorghe Mihoc - Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania

e-mail: ro_diff@yahoo.com, stefan.cruceanu@ima.ro, dorin.marinescu@ima.ro

Soil erosion, by water agent, is understood as a moving process of a certain quantity of soil particles from a soil surface point to another point. The presence of the plants strongly affect the erosion process, the plant stem interact with the water motion and the roots modify the physico-chemical properties of the soil. To take into account the plant effect we build a model by coupling the Saint-Venant type equations for water dynamics with a Hairsine-Rose type model for soil erosion. In this talk we analyse, analytically and numerically, the sensibility of the model with respect to the plant parameters.

Acknowledgement. *Partially Supported by a grant of Romanian Ministery of Reserach and Innovation, CCCDI-UEFISCDI, project number PN-III-P1-1.2-PCCDI-2017-0721/34PCCDI/2018, within PNCDI III*

New theoretical and applicative mathematical methods in the study of the fluids with free surfaces movement

Mircea Lupu[1,3], Cristian-George Constantinescu[2] and Gheorghe Radu[2]

[1] *Transilvania University of Brasov, Brasov, Romania;*
[2] *Henri Coanda Air Force Academy, Brasov, Romania;*
[3] *Academy of the Romanian Scientists, Bucharest, Romania*

e-mail: gh.radu@gmail.com

In the paper the authors presents new mathematical models and methods in the optimization of these phenomena with technical applications: the optimization of the hydraulic, a eolian turbine's blades or for the eliminating air pollutants and residual water purification; the actions hydro-pneumatics (robotics) to balance the ship in roll stability, optimizing the sails (wind powered) for extreme durability or propelling force, optimizing aircraft profiles for the drag or the lift forces, directing navigation, parachute brake, the wall, etc. The scientific results are accompanied by numerical calculation, integrating in the specialized literature from our country and foreign.
The inverse methods which lead to the Riemann-Hilbert boundary problems, and singular equation for the analytical functions. Here we solve the problems regarding of the fluids flow in the curvilinear obstacles presence, regarding of the profiles optimization for the minimal or maximal drag. The drag forces are expressed by the nonlinear integral operators and the extremum of the functionals is made by using the parametrical or the Jensen inequalities. The applications are for the aerodynamics profiles, brake deflectors, bow problems, wind turbines, ship sails, jets theory, etc.

Estimation of the mathematical model of the DC engine coupled with a reaction wheel

Vladimir Melnic[1], Irina Cojuhari[2]

[1] *Technical University of Moldova, The Centre of Excellence for Space Sciences and Technologies, Chişinău, Republic of Moldova;*
[2] *Technical University of Moldova, Faculty of Computers, Informatics and Microelectronics, Department of Software Engineering and Automatics, Chişinău, Republic of Moldova*

e-mail: vladimir.melnic@mate.utm.md, irina.cojuhari@ati.utm.md

Stabilization of the Microsatellite within the Centre of Space Technology is realized based on the control of the reaction wheels. In order to stabilize the microsatellites, it is necessary to choose the reaction wheels and also the mathematical modeling of the actuator system. In this paper, it was proposed to do the experimental identification of the mathematical model of the DC motor, coupled with the reaction wheel.
The experimental identification involves the acquisition of data, so that the experimental variation of the DC motor speeds at the reference speed of 8000 rpm was obtained as presented in the Figure 1.

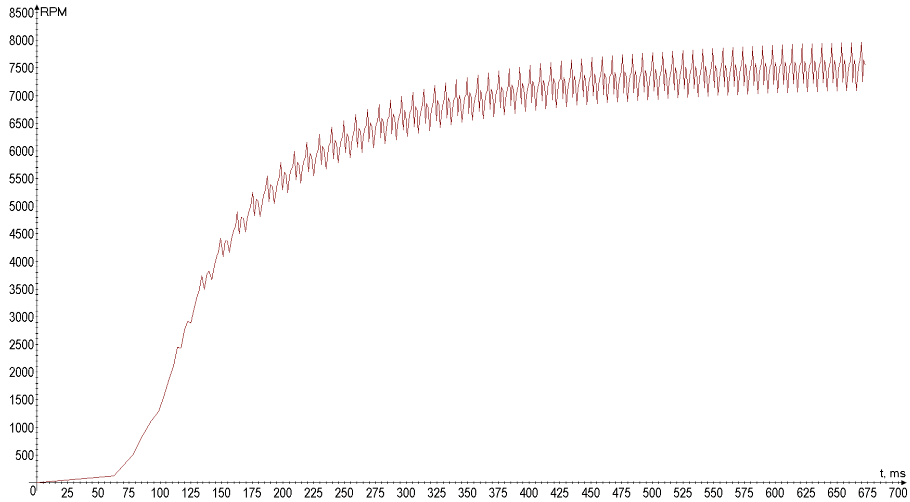


Figure 1. Experimental curve

1. Model of object with first order inertia:

$$H_1(s) = \frac{k}{Ts+1}.$$

2. Model of object with second order inertia:

$$H_1(s) = \frac{k}{(T_1 s+1)(T_2 s+1)}.$$

3. Model of object with third order inertia:

$$H_1(s) = \frac{k}{(T_1 s+1)(T_2 s+1)(T_3 s+1)}.$$

In the transfer functions are used the following notations: k is the transfer coefficient; T_1, T_2, T_3 - time constants.
To estimate the mathematical model of the control object it was proposed to use the Küpfmüller and Strejc methods, the obtained results were compared with results obtained using the module Process Models from System Identification Toolbox from MATLAB.

Bibliography

[1] ŞTEFĂNOIU D., CULIŢĂ J., STOICA P. *Fundamentele modelării şi identificării sistemelor.* Editura Printech, Bucureşti, 2005.

[2] BOBASU E. *Identificarea sistemelor.* Editura Didactică şi Pedagogică, Bucureşti, 2010.

[3] COJUHARI I., IZVOREANU B. *Ghid pentru proiectare de curs "Modelarea şi identificarea".* Editura UTM, Chişinău, 2015.

Numerical method for determining potential coefficients matrix for multiconductor transmission line

Vladimir Paţiuc

Moldova State University, Chişinău, Republic of Moldova
e-mail: patsiuk@mail.ru

Consider the numerical technique for computing the matrix of potential coefficients for multiconductor electrical lines, i.e. for a system consisting of an arbitrary number of electrical conductors. The transmission line equations [1] contain the matrix of the coefficients of electrostatic induction which allows to express the charge vector through the vector of potentials in each conductor. The method for calculating the exact values of its coefficients is proposed in [2]. For this purpose it is necessary to solve n (number of conductors) Dirichlet problems with known boundary conditions. At the same time it is proposed numerical method for calculating the potential coefficients matrix, which allows to express the vector of potentials through the charge vector in each conductor. In order to obtain the elements of this matrix we obtain the problem that differs from the classical Dirichlet problem for the Laplace equation. The difference consists in replacing the Dirichlet condition by some special boundary condition, which contains integrals over the boundary of the domain from the values of the unknown function. Such problems are called problems with nonlocal boundary conditions. The existence and uniqueness of the solution of such a problem are proved in this paper.

Bibliography

[1] Paul Clayton R., *Analysis of multiconductor transmission lines*, 2nd Ed. Wiley-IEEE Press, 2007, 780 p.

[2] Demirchean C.S., Korovkin N.B., Chechiurin V.L. *Teoreticheskie osnovi electrotehniki*, Vol. 3, St. Petersburg, 2004. 364 p. (in Russian).

Mathematical modelling of the screening of musical abilities and skills

Mihail Perebinos, Natalia Pasat

Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: mihaip52@yahoo.com

This article is a continuation of the bachelor's thesis "*ICT Services for the Screening of Musical Abilities and Skills*". Based on the results obtained, a mathematical model is proposed. It presents a multicriteria optimization. An integrated additive criteria is defined as a superposition of local criterias derived from the special conditions determined by the decision-maker. The solutions are designed to facilitate decision-maker in the "primary" quantitative assessment of the screening of musical skils and abilities that have a great impact on the overall development of person.

Key words: musical abilities, musical skills, screening, mathematical model, multicriteria optimization, integrated additive criteria, local criteria, decision-maker, "primary" quantitative assessment, weight coefficients, method Churchman - Ackoff.

Bibliography

[1] Pasat Natalia, Servicii TIC pentru screening-ul capacităţilor şi abilităţilor muzicale, teza de licenţă, Chişinău, 2018, UTM, FCIM, DIIS, Informatica Aplicată

[2] Kemeny J., Snell L. (1962), Mathematical Models in the Social Sciences. Ginn, Boston.

[3] Churchman and Ackoff(1954), *The method of successive comparisons*

Entropy and F-divergence construction using the Einstein sum

Vasile Patrascu

Research Center for Electronics and Information Technology,
Valahia University of Targoviste, Romania.
e-mail: patrascu.v@gmail.com

The Einstein sum was defined by the formula [1]:

$$\forall x, y \in (-1,1), x \oplus y = \frac{x+y}{1+xy} \tag{1}$$

Starting from the Einstein sum and considering the real parameter $a \in (1, \infty)$, we define the function $f : [0, \infty) \to (-\infty, \infty)$ with the formula:

$$f(x) = \ln\left(\frac{x+a}{1+ax}\right) \tag{2}$$

Using the function f, the following formulas for entropy H_f and f-divergence D_f are constructed [**??**,**??**] .

$$H_f(P) = \sum_{i=1}^{n} p_i \ln\left(\frac{p_i + a}{1 + ap_i}\right) \tag{3}$$

$$D_f(P||Q) = \sum_{i=1}^{n} p_i \ln\left(\frac{q_i + ap_i}{p_i + aq_i}\right) \tag{4}$$

where $P = (p_1, p_2, \ldots, p_n)$ and $Q = (q_1, p_2, \ldots, q_n)$ are two discrete probabilities distributions.

Bibliography

[1] A. Einstein, *On the Electrodynamics of moving bodies*, Annalen der Physik, 10 (322), pp. 891-921, 1905.

[2] I. Csiszar, *Eine informationstheoretische Ungleichung und ihre Anwendung auf den Beweis der Ergodizitat von Markoffschen Ketten*, Magyar. Tud. Akad. Mat. Kutato Int. Kozl. 8: pp. 85-108, 1963.

[3] T. Morimoto, *Markov processes and the H-theorem*, J. Phys. Soc. Jap. 18 (3): pp. 328-331, 1963.

Several solution for assessing Particulate Matter concentrations

Alin Pohoata[1], Daniel Dunea[2], Emil Lungu[1], Corneliu Salisteanu[3], Otilia Nedelcu[3]

[1] *Valahia University Targoviste, Faculty of Science and Arts;*
[2] *Valahia University Targoviste, Faculty of Environmental Engineering and Food Science;*
[3] *Valahia University Targoviste, Faculty of Electrical Engineering*
e-mail: alinpohoata@yahoo.com

Forecasting and analysis of the Particulate Matter (PM) concentrations is a subject of high interest for the public health. PM contains the inhalable particles that penetrate the thoracic region of the respiratory system determining numerous negative health effects particularly for younger children (0-10 years). We present in this article several methods of assessing the trends of PM concentrations, based on feedforward neural networks (FANN) combined with a wavelet decomposition of the time series values using smoothing filters to adjust the PM model outputs.

Modeling of the kinetics of fluorine sorption onto modified trepel

Veaceslav Zelentsov and Tatiana Datsko

The Institute of Applied Physics, Chişinău, Republic of Moldova
e-mail: zelen@yandex.ru

The application of kinetic models of pseudo first and second order for the description of experimental kinetic data related to the adsorption of fluorine ions on trepel modified by aluminum compounds (TMA) is discussed in this work. The pseudo-first-order kinetic model is expressed as [1]:

$$\frac{\mathrm{d}a_t}{\mathrm{d}t} = k_1(a_m - a_t) \tag{1}$$

Integrating within the boundary conditions $t = 0$ to $t = t$ and $a_t t = 0$ to $a_t = a_m$ gives the linearized form as

$$\ln (a_m - a_t) = \ln a_m - k_1 t \tag{2}$$

The pseudo-second order Mackay and Ho model [2] is expressed as

$$\frac{\mathrm{d}a_t}{\mathrm{d}t} = k_1(a_m - a_t)^2 \tag{3}$$

Rearranging and integrating Eq. (3) with respect to the boundary conditions $t = 0$ to $t = t$ and $a_t = 0$ to $a_t = a_m$ gives the linearized form as

$$\frac{t}{a_t} = \frac{1}{k_2} a_m^2 + \frac{t}{a_m} \tag{4}$$

where a_m and a_t[mmol/g] are the the amounts of fluoride ions adsorbed at equilibrium at time t, respectively; $k_1[\mathrm{min}^{-1}]$ is the pseudo-first-order rate constant and k_2[mmol/(g min] is the rate constant of the pseudo-second order kinetics.
Employing the linearized form of pseudo-first-order model ($\ln (am - at) = f(t)$, Eq. (2)) and linearized form of pseudo-second-order model (t/a_t, versus contact time (t), Eq. (4)) we determined the constants k_1, k_2, and a_m values using the slopes and intercept points of the linear plots. The results are illustrated in Table.1.

Table 1. The kinetic parameters of adsorption of fluorine ions onto TMA and correlation coefficients R2 of two kinetic models

Kinetic model	Parameters			
Pseudo 1	k_1 min^{-1} 0.050	a_{calc} mmol/g 0.041	a_{exp} mmol/g 0.241	R^2 0.6303
Pseudo 2	k_2 mmolg^{-1}min^{-1} 12.827	a_{calc} mmol/g 0.240	a_{exp} mmol/g 0.241	R^2 0.9999

Experimental conditions: $m = 0.2\text{g}, v = 50\text{ml}, T = 20\,°\text{C}, t = 120\text{min.}, Co = 1.03, \text{mmolF/L}$ The found rate constants of fluorine adsorption were used for calculation of the adsorption values of fluorine on TMA and for construction of kinetic curves. The correlation coefficients R^2 for the pseudo-first-order kinetic model is not high ($R^2 = 0.6303$), the determined values of a_m calculated from the equation (2) differ from the experimental values (Table 2). This indicates that adsorption of fluorine onto TMA does not follow the pseudo-first-order reaction model.

Table 2. Comparison of the experimental adsorption kinetics of fluorine ions on the trepel TMA sample with those calculated from the kinetic models

t, min	a_{exp} mmol/g	a_{calc} mmol/g by Eq. (4)	a_{calc} mmol/g by Eq. (2)	Relative error $\triangle$, %(Eq. (4)
1	0.194	0.182	0,001	6.0
2	0.216	0.208	0.023	2.3
3	0.223	0.218	0.034	4.0
5	0.229	0.227	0.054	5.7
10	0.234	0.234	0.095	6.0
20	0.236	0.238	0.015	4.7
30	0.238	0.239	0.187	3.0
60	0.239	0.240	0.0229	3.0
90	0.240	0.241	0.240	2.0
120	0.241	0.241	-	2.1

It can be seen from the Table 1 the R^2 for the pseudo-second-order kinetic model has the maximum value ($R^2 = 0.9999$) and a small relative error ($\triangle$,%), Table 2; this indicates that this model most accurately describes the experimental kinetic data on the fluorine adsorption on aluminum-modified trepel. The analytical expression is obtained for the pseudo-second-order equation, which has the following form:

$$a_t = \frac{0.744}{1 + 3.078t}$$

The equation makes it possible to calculate theoretically the values of fluorine adsorption to TMA for any time of the sorption process.

Bibliography

[1] G. Sujana, R. S. Thakur, and S. B. Rao, *Removal of fluoride from aqueous solution by using alum sludge*, Journal of Colloid and Interface Science, **206**(1998) 94–101

[2] G. McKay, Y.S. Ho, *Pseudo-second-order model for sorption processes*, Process Biochem. **34**(1999) 451–465.

4. Real, Complex, Functional and Numerical Analysis

Boundary Value Problem Solution Existence For Linear Integro-Differential Equations With Delays

Igor Chervko, Andrew Dorosh, Larissa Piddubna

Yuriy Fedkovych Chernivtsy National University, Ukraine
e-mail: i.cherevko@chnu.edu.ua

We consider the following boundary value problem

$$y''(x) = \sum_{i=0}^{n}\Bigg(a_i(x)\,y(x-\tau_i(x)) + b_i(x)\,y'(x-\tau_i(x)) \tag{1}$$

$$+\sum_{j=0}^{1}\int_a^b K_{ij}(x,s)\,y^{(j)}(s-\tau_i(s))\,ds\Bigg) + f(x),$$

$$y^{(j)}(x) = \varphi^{(j)}(x),\ j=0,1,\ x\in[a^*;a],\ y(b)=\gamma, \tag{2}$$

where $\tau_0(x) = 0$ and $\tau_i(x),\ i=\overline{1,n}$ are continuous nonnegative functions defined on $[a,b]$, $\varphi(x)$ is a continuously differentiable function given on $[a^*;a]$, $\gamma \in R$, $a^* = \min\limits_{0\le i<n}\left\{\inf\limits_{x\in[a;b]}(x-\tau_i(x))\right\}$.
We introduce the sets of points determined by the delays $\tau_1(x),\dots,\tau_n(x)$:

$$E_i = \Big\{x_j \in [a,b]:\ x_j - \tau_i(x_j) = 0,\ j=1,2,\dots\Big\},\ E = \bigcup_{i=1}^{n} E_i.$$

A function $y(x)$ is called a solution of (1)-(2) if it satisfies the equation (1) on $[a;b]$ (with the possible exception of a set E) and the conditions (2).
In this work coefficient conditions for the existence of a solution of the boundary value problem for linear integro-differential equations with many delays, which are efficient for verification in practice, are researched [1].
Approximation of the boundary value problem (1)-(2) solution using spline functions with defect 2 was investigated in [2].

Bibliography

[1] Cherevko, I., Dorosh, A., *Boundary Value Problem Solution Existence For Linear Integro-Differential Equations With Many Delays*, Carpathian Math. Publ., **10** (2018) 1, pp. 65–70.

[2] Cherevko, I., Dorosh, A., *Approximation of boundary value problem solutions for linear integro-differential equations with many delays*, Bukovynsky Math. J., **5** (2017) 3–4, pp. 77–81. (in Ukrainian)

Methods of solving linear recurrences

Ana Costas

Technical University of Moldova, Chişinău, Republic of Moldova

Recurrent sequences offer ways to solve effectively many problems that arise not only in multiple branches of mathematics but also in various other areas of knowledge. Their applications for the raising to a power of matrices, calculation of determinants, solving the Diophantine and functional equations, counting the polygons, determined by a network of straight lines in the plan etc. are well known. Recurrent sequences offer original ways to solve problems related to sound wave motion, bacterial culture establishment, chromatography, minimal time learning strategies etc. The beauty of recurrent sequences makes them an important chapter of Competitive Mathematics.
Some methods of solving recurrent sequences, including the use of finite-difference methods to solve linear recurrent sequences of the first order with constant and variable coefficients and linear recurrent sequences of second and third order with constant coefficients, will be presented. Some problems, solved using the linear recurrent sequences methods will be presented too.

Optimal Algorithm for Optimization Problems with Special Restriction

Eugeniu Gărlă

ASEM
e-mail: eugeniugarla@yahoo.com

The present paper analyses a class of nonlinear optimization problems with special restrictions, we propose a direct method for solving the auxiliary problem, for which we calculate complexity, we also assesses the maximum number of elementary operations and describe the optimal algorithm for performing numerical calculations. The study builds an optimal algorithm for solving the auxiliary problem of PG model; the complexity of this algorithm is $O(nm^2, N)$, the number of elementary operations is minimal. Matrix inversion does not depend on the size of problem n and always has constant size - $m \times m$ and the operation complexity is $O(m^3)$. In practical situations, if $m << n$, then the value of m^2 is much smaller than the value of n, thus can be considered as a constant, resulting the complexity is $O(n^2)$, in other words it's almost the same as for the approximation methods. Thus, the model PG is fully functional and practically "immune" to the size of the optimization problem.

Key words: algorithm complexity, optimization methods.

Shadowing and Specification in weakly contracting relations

Vasile Glavan[1,2], Valeriu Guţu[1]

[1]*Moldova State University, Chişinău, Republic of Moldova,*
[2]*Siedlce University of Natural Sciences and Humanities, Siedlce, Poland*
e-mail: vgutu@yahoo.com

Set-valued dynamical systems in discrete time as iterations of a multi-function (relation), or dispersive flows in continuous time, have attracted the attention of researchers during the last decades. Most of these works were dedicated to the topology of limit sets, including geometry of the attractors. Much less attention was paid to the dynamics itself on these limit sets.
Chain recurrence in dispersive flows has been considered by I. U. Bronshteyn and A. Y. Kopanski (1984, 1985).
D. N. Cheban (2010) has studied the limit sets in dissipative such flows.
E. Akin (e.g., 1993, 2017) studied various aspects of set valued dynamics and their topological properties, including Shadowing property.
B. E. Raines and T. Tennant (2015) have studied set-valued dynamical systems and their inverse limits, having the Specification property.
The dynamics in its own rights of some (weakly) contracting with respect to the Pompeiu-Hausdorff metric relations in complete metric spaces has been considered by the authors (2003, 2004). More precisely, we stated the "asymptotic phase" property, topological transitivity on the attractor as well as Shadowing property near the attractor.
In our report we strengthen the transitivity and Shadowing properties up to topological mixing and Specification property, respectively. Following M. Hata and I. A. Rus, we relax the contractivity condition for set-valued mappings to a level of weak contractivity, using the notion of comparison function.

Spectral Collocation Solutions to Eigenproblems on Unbounded Domains

Călin-Ioan Gheorghiu

"T. Popoviciu" Institute of Numerical Analysis of Romanian Academy, Cluj-Napoca, Romania
e-mail: ghcalin@ictp.acad.ro

The aim of this communication is to argue that spectral collocation based on Laguerre (LGRC), Hermite (HC) as well as Sinc (SiC) functions offers reliable and accurate solutions to a large class of eigenvalue problems on unbounded domains.
We consider non-standard eigenvalue problems, singular and/or self-adjoint as well as eigenproblems supplied with boundary conditions depending on eigenparameter ([2]). Recently we have obtained important results concerning eigenvalue problems with transmission conditions.
In order to estimate the accuracy of outcomes we display the behavior (the decreasing way) of the expansion coefficients of solutions. Because this method works in physical space, to get these coefficients in the spectral space we make use of the FFT or another polynomial transforms ([1]). To the same aim we compute the relative drift of a specified set of eigenvalues. The orthogonality of eigenvectors is another way we asses the accuracy of our computations.
The communication will be illustrated with a large number of figures and tables. They underline the efficiency of spectral collocation methods used to solve eigenproblems on unbounded domains.

Bibliography

[1] Boyd, J.P., *Traps and Snares in Eigenvalue Calculations with Application to Pseudospectral Computations of Ocean Tides in a Basin Bounded by Meridians*, J. Comput. Phys. **126**, 11-20 (1996)

[2] Gheorghiu, C.I., *Spectral Collocation Solutions to Problems on Unbounded Domains*, Casa Cărţii de Ştiinţă Publishing House, Cluj-Napoca, 2018 (see https://ictp.acad.ro/gheorghiu/papers)

Positive Nonlinear Sem–Group Associated to a Class of Abstract Kinetic Models

Cecil P. Grünfeld

"Gheorghe Mihoc - Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania
e-mail: grunfeld@venus.nipne.ro

We introduce a positive nonlinear semi-group in an ordered space, associated to a class of evolution equations abstracting properties of known kinetic models of interest in applied mathematics and physics. We investigate the basic properties of the semi-group, in particular, its invariants and long-time behavior. In addition, discuss the usefulness of the results in some application examples, and also present some challenging open problems.

On the Global Existence of the Solutions of the Riemann Problem for Shallow Water Equations

Anca-Veronica Ion, Stelian Ion

"Gheorghe Mihoc - Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania
e-mail: ro_diff@yahoo.com, averionro@yahoo.com

In this talk we investigate the Riemann Problem for a shallow water model with vegetation and terrain data. We present a constructive method, that is not dependent on how large data jump is, to solve the problem. Essentially the method involves the resolution of a nonlinear equation that can have multiple solutions or no solution. The method uses a criterion of admissibility to select among multiple possible solutions a physical relevant one. To illustrate the method several examples are presented.

Acknowledgement. *Partially Supported by a grant of Romanian Ministery of Reserach and Innovation, CCCDI-UEFISCDI, project number PN-III-P1-1.2-PCCDI-2017-0721/34PCCDI/2018, within PNCDI III*

On the generalized factorization of functions in weighted spaces

Petru Moloşnic, Vasile Neagu

State University of Moldova, Chişinău, Republic of Moldova
e-mail: vasileneagu45@gmail.com

In this paper the notion of factorization of functions with respect to contour Γ in the spaces $L_p(\Gamma,\rho)$ is presented [1]. The main result of the paper is the determination of some classes of functions that allow a factorization, as well as the application of factorization in studying of singular integral operators with measurable and bounded coefficients.
Let Γ be a closed Lyapunov contour which bounds the domain G^+. By G^- we denote the domain which complements $G\bigcup\Gamma$ to the whole plane. Assume that $0\in G^+$ and $\infty\in G^-$.
Let $L_p^+(\Gamma,\rho) = P(GL_p(\Gamma,\rho))$, $L_p^-(\Gamma,\rho) = Q(GL_p(\Gamma,\rho)) + c$, $c \in \mathbb{C}$. **Definition.** The generalized factorization of function $a \in GL_\infty(\Gamma)$ with respect to contour Γ in the space $L_p(\Gamma,\rho)$ is its representation in the form

$$a(t) = a_-(t)t^\kappa a_+(t),$$

where $\kappa\in\mathbb{Z}$ and the factors $a_\pm$ satisfy the follwing conditions:
1) $a_- \in L_p^-(\Gamma,\rho)$, $a_+ \in L_q^+(\Gamma,\rho^{1-q})$, $a_-^{-1} \in L_q^-(\Gamma,\rho^{1-q})$,
$a_+^{-1} \in L_p^+(\Gamma,\rho)$ $(p^{-1}+q^{-1}=1)$;
2) the operator $a_+^{-1}Pa_-^{-1}I$ is bounded in the space $L_p(\Gamma,\rho)$.
Denote by $R^+L_\infty(\Gamma)$ the set of measurable real functions on the contour Γ verifying the conditions $0 < es\sin_{t\in\Gamma} f\, a(t),\ ess\sup_{t\in\Gamma} a(t) < \infty$.

Theorem 1. $R^+L_\infty(\Gamma)\subset Fact_{p,\rho}(\Gamma)$. *If* $a\in R^+L_\infty(\Gamma)$, *then*

$$a(t) = a_-(t)\cdot a_+(t),$$

where $a_+(t) = \exp(P\ln a)(t)$, $a_-(t) = \exp(Q\ln a)(t)$. *In addition, the functions* $a_\pm$ *verify the conditions:* $a_+^{\pm1}\in L_\infty^+(\Gamma)$ *and* $a_-^{\pm1}\in L_\infty^-(\Gamma)$.

Denote by $Nt_{p,\rho}(\Gamma)$ the set of measurable bounded functions $a(t)$ on the contour Γ for which the operator $A = aP+Q$ is Noetheran in $L_p(\Gamma,\rho)$.

Theorem 2. $Nt_{p,\rho}(\Gamma) = Fact_{p,\rho}(\Gamma)$.

Bibliography

[1] Gohberg, N. Krupnik, *Introduction to the theory of one-dimensional singular integral operators*, Vols. I.- Birkhäuse Verlag, Basel, 1992.

Multi-index transportation problem with non-linear cost functions

Tatiana Paşa

State University of Moldova, Chişinău, Republic of Moldova
e-mail: pasa.tatiana@yahoo.com

In this paper we discuss the multi-index transportation problems [1, 2], which is a widespread and current problem nowadays.
The aim of this paper is to model the multi-index transportation problem which implies solving the non-linear problem with a non-linear objective function and linear restrictions and to propose a method to solve the given problem that was mathematically modeled. We also present results obtained from testing the described method on a diverse test data.

Bibliography

[1] A. Djamel, N. Amel, L. T. H. An, Z. Ahmed *A modified classical algorithm ALPT4C for solving a capacitated four-index transportation problem*, AMV J., **37**, 3(2012), 379-390.

[2] O. Diaz-Parra, J. A. Ruiz-Vanoye, B. B. Loranca, A. Fluentes-Penna, R. A. Barrera-Camara *A survey of ransportation problem*, HPC J. of AM, **V**olum 2014, ID 848129, 17 pages, http://dx.doi.org/10.1155/2014/848129 (visited July 2018).

Set-Valued Almost Periodic Functions and Perfect Mappings

Dorin Pavel

Department of Computer Sciences, Tiraspol State University, Chişinău, Republic of Moldova, MD 2069
e-mail: dorinp@mail.md

Fix a natural number $n \geq 1$. Denote by d the Euclidean distance on the n-dimensional Euclidian space $\mathbb{R}^n$ and by $Com(\mathbb{R}^n\ )$ the space of all non-empty compact subsets of $\mathbb{R}^n$ with the Pompeiu-Hausdorff distance $d_P(A, B)$. The space $\mathbb{R} = \mathbb{R}^1$ is the space of reals and $\mathbb{C} = \mathbb{R}^2$ is the space of complex numbers. The space $(Com(\mathbb{R}^n), d_P)$ is a complete metric space.
Fix a topological space G. By $T(G)$ denote the family of all single-valued continuous mappings of G into G. Relatively to the operation of composition, the set $T(G)$ is a monoid (a semigroup with unity).

A single-valued $\varphi : G \longrightarrow Com(\mathbb{R}^n)$ is called a set-valued function on G. For any two set-valued functions $\varphi, \psi : G \longrightarrow B(\mathbb{R})$ and $t \in \mathbb{R}$ are determined the distance $\rho(\varphi, \psi) = sup\{d_P(\varphi(x), \psi(x)) : x \in G\}$ and the set-valued functions $\varphi + \psi$, $\varphi \cdot \psi$, $-\varphi$, $\varphi \cup \psi$, where $(\varphi \cup \psi)(x) = (\varphi(x) \cup \psi(x)$, and $t\varphi$. We put $(\varphi \circ f)(x) = \varphi(f(x))$ for all $f \in T(G)$, $\varphi \in SF(G)$ and $x \in G$. Let $SF(G, \mathbb{R}^n)$ be the space of all set-valued functions on G with the metric ρ. The space $SF(G, \mathbb{R}^n)$ is a complete metric space.

A set-valued function $\psi : G \longrightarrow \mathbb{R}^n$ is called lower (upper) semicontinuous if the set $\psi^{-1}(H) = \{x \in G : \psi(x) \cap H \neq \emptyset\}$ is an open (a closed) subset of G for any open (closed) subset H of the space R^n. Denote by $LSC(G, \mathbb{R}^n)$ the family of all lower semicontinuous functions and by $USC(G, \mathbb{R}^n)$ the family of all upper semicontinuous functions on the space G.

If $\varphi \in SF(G)$ and $f \in T(G)$, then $\varphi_f = \varphi \circ f$ and $\varphi_f(x) = \varphi(f(x))$ for any $x \in G$). Evidently, $\varphi_f \in SF(G)$.

Fix a submonoid P of the monoid $T(G)$. We say that P is a monoid of continuous translations of G. The set P is called a transitive set of translations of G if for any two points $x, y \in G$ there exists $f \in P$ such that $f(x) = y$. In particular, $1_G \in P$, where $1G$ is the identical translation on the space G.

For any function $\varphi \in SF(G, \mathbb{R}^n)$ we put $P(\varphi) = \{\varphi_f : f \in P\}$.

Definition 1. *A function $\varphi \in SF(G, \mathbb{R}^n)$ is called a P-periodic function on a space G if the closure $\bar{P}(\varphi)$ of the set $P(\varphi)$ in the space $SF(G, \mathbb{R}^n)$ is a compact set.*

Denote by P-$ap_s(G, \mathbb{R}^n)$ the subspace of all P-periodic set-valued functions on the space G.

Let $\mu : A \longrightarrow B$ be a perfect mapping of a space A onto a space B, $P_A \subset T(A)$, $P_B \subset T(B)$ and $h : P_A \longrightarrow P_B$ is a single valued mapping such that $\mu(f(x)) = h(f)(\mu(x)$ for any $x \in A$ and $f \in P_A$. Consider the mapping $\Phi_{(\mu,h)} : SF(A, \mathbb{R}^n) \to SF(B, \mathbb{R}^n)$ and $\Psi_{(\mu,h)} : SF(B, \mathbb{R}^n) \to SF(A, \mathbb{R}^n)$, where $\Phi_{(\mu,h)}(\varphi) = \mu \circ \varphi$ for each $\varphi \in SF(B, \mathbb{R}^n)$ and $\Psi(\psi)_f(y) = \mu(\psi_g(\mu^{-1}(y))$, where $h(g) = f$. We have $\Psi(\psi)_f(y) = \cup\{\mu(\psi_g(\mu^{-1}(y)) : h(g) = f\}$.

Theorem 1. *The mappings $\Phi_{(\mu,h)}$ and $\Psi_{(\mu,h)}$ are continuous.*

Theorem 2. *$\Phi_{(\mu,h)}(P_B$-$ap_s(B, \mathbb{R}^n) \subset P_A$-$ap_s(A, \mathbb{R}^n)$ and*

$Psi_{(\mu,h)}(P_A$-$ap_s(A, \mathbb{R}^n) \subset P_B$-$ap_s(B, \mathbb{R}^n)$.

Limits of the Solutions to the Initial-Boundary Dirichlet Problem for the Semilinear Klein-Gordon Equation with Two Small Parameters

Andrei Perjan, Galina Rusu

Moldova State University, Chişinău, Republic of Moldova
e-mail: aperjan1248@gmail.com, rusugalinamoldova@gmail.com

Let $\Omega \subset \mathbb{R}^n$ be a bounded open set with smooth boundary $\partial\Omega$. We consider the following singularly perturbed initial-boundary problem

$$\begin{cases} \varepsilon\, \partial_t^2 u_{\varepsilon\delta} + \delta\, \partial_t u_{\varepsilon\delta} + A u_{\varepsilon\delta} + |u_{\varepsilon\delta}|^q\, u_{\varepsilon\delta} = f(x,t), (x,t) \in \Omega \times (0,T), \\ u_{\varepsilon\delta}\big|_{t=0} = u_0(x), \quad \partial_t u_{\varepsilon\delta}\Big|_{t=0}(0) = u_1(x), \quad x \in \overline{\Omega}, \\ u_{\varepsilon\delta}\big|_{x\in\partial\Omega} = 0, \quad t \in [0,T), \end{cases} \tag{$P_{\varepsilon\delta}$}$$

where A is a strong elliptic operator, $q > 0$ and ε, δ are two small parameters.
We study the behavior of solutions $u_{\varepsilon\delta}$ to the problem $(P_{\varepsilon\delta})$ in two different cases:

(i) when $\varepsilon \to 0$ and $\delta \geq \delta_0 > 0$;

(ii) when $\varepsilon \to 0$ and $\delta \to 0$.

We obtain some *a priori* estimates of solutions to the perturbed problem, which are uniform with respect to parameters, and a relationship between solutions to both problems. We establish that the solution to the unperturbed problem has a singular behavior, relative to the parameters, in the neighbourhood of $t = 0$. We show the boundary layer and boundary layer function in both cases.

Splines for the Set Functions

Vasile Postolică

Romanian Academy of Scientists;
Vasile Alecsandri University of Bacău, Faculty of Sciences, Department of Mathematics, Informatics and Educational Sciences, Romania
e-mail: vpostolica@ambra.ro

Following the multiple possibilities to approximate any set function using proper sequences of countable additive set-functions and our main conclusions given in [1]–[3] concerning the best approximation, we present the adequate splines for the set-functions.
Keywords: set function, best approximation.

Bibliography

[1] Isac, G., Postolica, V., *The Best Approximation and Optimization in Locally Convex Spaces*, Verlag Peter Lang GmbH, Frankfurt am Main, Germany, 1993.

[2] Postolica, V., *A Method which Generates Splines in H-locally Convex Spaces and Connections with Vectorial Optimization*, Positivity, 2 (4), 1998, 369 - 377.

[3] Postolica V., *On the Best Approximation of Set Functions*, Proceeding of The Fourth International Conference on Functional Analysis and Approximation Theory, Acquafreda di Maratea, Potenza - Italy, September 22 - 28, 2000. Published in Supplemento ai Rendiconti del Circolo Matematico di Palermo, serie II, no 68, 2002, p. 761–775.

The Investigation of Euler Function Preimages

Dmitriy Rudenko, Ruslan Skuratovskii

Institute of Mathematics of NAS of Ukraine, Kiev, Ukraine
e-mail: ruslan@imath.kiev.ua

Summary. The purpose of this work is to study theoretical numerical properties a multivalued function [1, **?**] which is reversed to Euler's function, show the relevance of the examples. Also we want to explore the composition of this function.
Subject of study: explore the composition of the function $\varphi(n)$ with itself and the tasks associated with it, it's properties, the number of prototypes of the function $\varphi(n)$, behavior of the straight OA_n, where $A_n(n; \varphi(n))$ and $O(0;0)$ where $n \to \infty$.
Results.

Example 1. *The set of preimages for 12 is following:* $\phi^{-1}(12) = \{13, 21, 26, 28, 36, 42\}$. *Also we have* $\varphi^{-1}(16) = \{32,\ 48,\ 17,\ 34,\ 40,\ 60\}$, $\varphi^{-1}(18) = \{19,\ 27,\ 38,\ 54\}$. *We remind, that the number of a form* $2^{2^n}+1$, *where n is not-negative integer, is called Fermat number.*

The recursive formula for Fermat numbers [12] was also used: $F_n = F_0 ... F_{n-1} + 2$. Useful for the study of the number of prototypes is Lucas's Theorem: each prime divisor of the Fermat number F_n, where $n > 1$, has a form of $k2^{n+2} + 1$.

Theorem 1. *Let $n \in N$. If $2^{2^n}+1$ is not prime, then for any number of the form 2^{2^n+a}, where $a \in N$, $a < 2^n$ exists exactly 2^t natural numbers m such that $\varphi(m) = 2^{2^n+a}$, where t is amount of prime Fermat numbers lesser than $2^{2^n}+1$.*

Example 2. *For a non-prime Fermat number $2^{32}+1$ number of preimages for subsequent numbers of form 2^{2^n+a}, $a \leq 32-1$ is equal to 2^{32}.*

Theorem 2. *If $\varphi(m) = 2^n$, then $m = 2^s p_1 p_2 ... p_x$, where p_i are different Fermat numbers, $s \in N$.*

Theorem 3. *Right line with a positive coefficient, carried through the beginning because of the origin of the coordinates, is not the lower bound of the Euler function graph.*

Bibliography

[1] Michal Kevek, Florian Luca, Lawrence Somer, *17 Lectures on Fermat Numbers: From Number Theory to Geometry*, Springer, CMS Books 9, ISBN 0-387-95332-9.

[2] Rodney Coleman, *On the image of Euler's totient function.* Journal of Computer mathematics Sci. (2012), Vol.3 (2), P. 185-189.

[3] R. V. Skuratovskii, *Structure and minimal generating sets of Sylow 2-subgroups of alternating groups.* Sao Paulo Journal of Mathematical Sciences. (2018), no. 1, pp. 1-19. Source: https://link.springer.com/article/10.1007/s40863-018-0085-0.

[4] R. V. Skuratovskii, *Structure and minimal generating sets of Sylow 2-subgroups of alternating groups.* Source: https://arxiv.org/abs/1702.05784v2

The Aumann-Pettis-Sugeno integral for vector multifunctions relative to a vector fuzzy multimeasure

Cristina Stamate[1] and Anca Croitoru[2]

[1] *"Octav Mayer" Institute of Mathematics, Romanian Academy, Iaşi Branch, Bd. Carol I, No. 11, Iaşsi 700506, Romania;*

[2] *Faculty of Mathematics, "Alexandru Ioan Cuza" University, Bd. Carol I, no 11, Iaşi 700506, Romania*

e-mail: `cstamate@ymail.com, croitoru@uaic.ro`

In this paper, we define and study the general Aumann-Pettis-Sugeno integral for a vector multifunction relative to a vector fuzzy multimeasure, both taking values in a locally convex space X, ordered by a closed convex pointed cone X_+, with nonempty interior. For the selections of the multifunctions we use the general Pettis-Sugeno integral. Several classic properties of this integral and some comparative results are established.

On modified computing schemes of the spline-collocation method for solving integral equations of the second kind

Vladislav Seichiuc, Eleonora Seichiuc, Gheorghe Carmocanu

Trade Co-operative University of Moldova, Moldova State University
e-mail: seiciuc@mail.ru

The Intelligent Support System (ISS) for approximate solving of the Fredholm and Volterra integral equations (IE) of the second kind (ISS_IE) utilizes the following results (see [1]):
– computing algorithms of spline-collocations method for solving the Fredholm and Volterra IE of the second kind, which essentially use the linear splines as basic functions for modeling and presenting the unknown solutions;
– a theoretical substantiation of the developed computing algorithms, obtained in the space of continuous functions and in the Hölder spaces, which is based on the results of function approximation with its linear polygons;
– the core developed component of ISS_IE, called the Base of Kernel Prototypes of IE (BKP_IE_-COL) and destined for solving IE by spline-collocations method, which directly depends on the used splines in the calculation algorithm.
In this paper, for more efficient use of ISS_IE, we study the possibility to build computational schemes based on certain types of second order fundamental splines. There were obtained:
– computational schemes of spline-collocations method for solving IE of the second kind on the basis of some fundamental splines of second order;
– results on approximation of the function with used second order splines;
– a theoretical substantiation of the new developed computing algorithms in the space of continuous functions based on the results of function approximation with used second order splines;
– the extension of the Base BKP_IE_COL, destined for solving IE with spline-collocations method, using the splines of the second order.

The adapted algorithm of Jose A. Diaz for multi-criteria fractional transportation problem with "bottleneck" criterion

Tkacenko Alexandra

Department of Mathematics, Moldova State University
A. Mateevici str., 60, Chisinau, MD–2009, Moldova
e-mail: alexandratkacenko@gmail.com

The paper proposes an adapted version of Jose A. Diaz algorithm for solving the multi-criteria fractional transportation problem with the same bottleneck denominators, additionally with the same time "bottleneck" criterion separately. It generates for each (feasible) time value the best compromise multi criteria solution. So, finally, we will obtain one finite set of efficient solutions for solving the multi-criteria fractional transportation problem with the same bottleneck denominators, separately including the time "bottleneck" criterion. The mathematical model of the proposed problem is the follows:

$$\min Z^k = \frac{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} c_{ij}^k x_{ij}}{\max\limits_{ij} t_{ij} \, |x_{ij} > 0\}} \tag{1}$$

$$\min Z^{k+1} = \max_{ij}\ t_{ij}\,|x_{ij} > 0\,\} \tag{2}$$

$$\min Z^{k+1} = \max_{ij}\ t_{ij}\,|x_{ij} > 0\,\} \tag{3}$$

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1, 2, \ldots, m; \quad \sum_{i=1}^{n} x_{ij} = a_i, j = 1, 2, \ldots, n; \tag{4}$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, r. \tag{5}$$

In order to solve the model (1)-(4) we proposed an iterative algorithm, inspired of Jose A. Diaz algorithm [1]. It generates for every time possible value the corresponding " best compromise solution " of the first k criteria [2]. The algorithm was tested on several examples and was found to be quite effective.

Bibliography

[1] Jose A. Diaz, Solving multiobjective transportation problems, Econ-math overview, 15, pp. 62-73, 1979.

[2] A. I. Tkacenko, *Multiple Criteria Fuzzy Cost Transportation Model of SBottleneck T type,* Journal of Economic Computation and Economic Cybernetics Studies and Research, ISI Thomson Reuter Serv., V.48, No.2, 2014, Bucuresti, Romania, pp. 215-232.

5. Probability Theory, Mathematical Statistics, Operations Research

A Markov chain approach to stock model analysis and inference

Vlad Stefan Barbu[1], Guglielmo D'Amico[2], Riccardo De Blasis[3]

[1] *Université de Rouen–Normandie, Laboratoire de Mathématiques Raphaël Salem, France;*
[2] *Department of Pharmacy, University "G. d'Annunzio" of Chieti-Pescara, Chieti, Italy;*
[3] *Doctoral School in Accounting, Management and Finance, University "G. d'Annunzio" of Chieti-Pescara, Chieti, Italy and Capital Markets Cooperative Research Centre, Wollongong University, Wollongong, Australia*
e-mail: `vladstefan.barbu@univ-rouen.fr, g.damico@unich.it, riccardo.deblasis@unich.it`

In this presentation, based on Barbu et al., 2017, we are interested in applications of statistical techniques for Markov chains in financial mathematics. We have modelled through a Markov chain the time evolution of the dividend growth factor of a stock. We were interested in estimating the first two moments of the price of the stock and also in forecasting the price of the stock within n time units. This work represents further advancements of the Markov chain stock model proposed in Ghezzi and Piccardi, 2003. We give theoretical results about the consistency and asymptotic normality of the estimated quantities and apply our findings to real dividend data. The statistical techniques for Markov chains are mainly based on Sadek and Limnios, 2002.
These results were integrated into a semi-Markov framework as provided by D'Amico, 2013, where the semi-Markov hypothesis was advanced and validated on real data. A further generalization was given in D'Amico, 2016, where a continuous state space semi-Markov model is considered for the computation of the fundamental price and risk of the stock.

Acknowledgement. *The research work of Vlad Stefan Barbu was partially supported by the projects* XTerM – Complex Systems, Territorial Intelligence and Mobility *(2014–2018) and* MOUSTIC – Random Models and Statistical, Informatics and Combinatorics Tools *(2016–2019).*
The research work of Guglielmo D'Amico was partially supported by the Federation Normandy-Mathematics, France, by offering the opportunity of spending several periods as visiting professor in the Laboratory of Mathematics Raphaël Salem, Department of Mathematics, University of Rouen and in Laboratory of Mathematics Nicolas Oresme, Department of Mathematics, University of Caen, France

Parallel algorithm to determine the solutions of the bimatrix informational extended games

Cataranciuc Emil, Hâncu Boris

State University of Moldova, Chişinău, Republic of Moldova
e-mail: `ecataranciuc@gmail.com, boris.hancu@gmail.com`

Consider the bimatrix game $\Gamma = \langle I, J, A, B \rangle$ in complete and $(1 \leftrightarrows 2)-$ perfect information over the sets of pure strategies. To solve games of this type we propose the following methodology [1]. The first, for *game in complete and perfect information over the sets of pure strategies* Γ we will build the *game in perfect information over the sets of informational extended strategies* $Game\,(1 \leftrightarrows 2)$. Second, for the game $Game\,(1 \leftrightarrows 2)$ we will build the *incomplete and imperfect information game generated by the informational extended strategies* $\widetilde{\Gamma} = \left\langle \{1,2\}, I, J, \left\{ AB(\mathbf{i},\mathbf{j}) = \left\| \left(a_{i_j j_i}, b_{i_j j_i}\right) \right\|_{i\in I}^{j\in J} \right\}_{\mathbf{i}\in\mathbf{I}}^{\mathbf{j}\in\mathbf{J}} \right\rangle$. Third, for the game $\widetilde{\Gamma}$ we will generate the associated *Bayesian game in the non informational extended strategies*

$\Gamma_{Bayes} = \langle\{1,2\},\{\Delta_1,\Delta_2\},\mathbf{L},\mathbf{C},\mathbf{A},\mathbf{B}\rangle$. Finaly any fixed $\alpha \in \Delta_1$ and $\beta \in \Delta_2$ we will determine the solutions of the following sub games $sub\Gamma_{Bayes} = \langle\{1,2\},\mathbf{L}(\alpha),\mathbf{C}(\beta),\mathbf{A}(\alpha),\mathbf{B}(\beta)\rangle$ of the game Γ_{Bayes}. Using the MPI-OpenMP programming model and ScaLAPACK -BLACS packages we have elaborated the parallel algorithm to find the all equilibrium profiles $(\mathbf{l}^*,\mathbf{c}^*)$ in the game Γ_{Bayes}. We can demonstrate the following theorem that estimate the run time performance and communication complexity of the parallel algorithm.

Theorem 1. *The run time complexity of the parallel algorithm is*

$$T_{comput} = \sum_{k=2}^{7} T_p^k = O(\max(n,m)) + O\left(\max(\varkappa_1,\varkappa_2)\right) +$$
$$+O\left(\max\left(|\mathcal{I}|,|\mathcal{J}|\right)\right) + O\left(\max\left(\left|\widehat{\mathcal{I}}\right|,\left|\widehat{\mathcal{J}}\right|,|grBr_1|\cdot|grBr_2|\right)\right)$$

and communication complexity is

$$T_{comm} = O\left(t_s + \left[\max\left(|\mathcal{I}|\times|\mathcal{J}|, m\times n\right)\right] * t_b + t_h\right).$$

Bibliography

[1] Boris Hâncu, Anatolie Gladei. *Parallel algorithm to find Bayes-Nash solution to the bimatrix informational extended game.* Computer Science Journal of Moldova (CSJM). V. 26, n.1(76), 2018. Pages 39-58.

Synthesis of the minimum variance control law for the linear time variant processes

Irina Cojuhari, Ion Fiodorov, Bartolomeu Izvoreanu, Dumitru Moraru

Technical University of Moldova, Faculty of Computers, Informatics and Microelectronics, Department of Software Engineering and Automatics, Chişinău, Republic of Moldova
e-mail: irina.cojuhari@ati.utm.md, ion.fiodorov@ati.utm.md, bartolomeu.izvoreanu@ati.utm.md, dumitru.moraru@ati.utm.md

Above the linear process can act the disturbance signals, so kind of processes can be described by the parametrical models, where the most used take part from the ARMAX class (Auto-Regressive Moving Average with eXogenous control). The general model of the class is the ARMAX model $[na, nb, bc, nk]$, which in fact represents that the output signal is obtained as a result of the superposition between a useful signal obtained by filtering the input signal and a parasitic signal obtained by filtering the white noise :

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})}u(k) + \frac{C(q^{-1})}{A(q^{-1})}e(k) = y^u(k) + y^e(k).$$

where $y(k)$ is the output of the noisy system, $u(k)$ - control signal, $e(k)$ is a sequence of independent normal variables with zero mean value and variance one (white noise) and the polynomials $A(q^-1), B(q^-1), C(q^-1)$ are

$$A(q^{-1}) = 1 + a_1q^{-1} + ... + a_{na}q^{-na},$$
$$B(q^{-1}) = b_1q^{-1} + ... + b_{nb}q^{-nb},$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + ... + c_{nc} q^{-nc},$$

where q^{-1} is backshift operator.

To determine the optimal control, it is used the performance criterion that provides the minimum variance value of the output The purpose of the minimum variance control is to determine the control signal $u(k)$ in such a way that the loss function

$$J = E\{y^2(k)\}.$$

is as small as possible and the control law that ensures the minimization of the given criteria is called the minimum variance control.

The component $y^e(k)$ represents the influence of the environment on the process and it is characterized by the stochastic disturbance signals and is given by

$$y^e(k) = \frac{C(q^{-1})}{A(q^{-1})} e(k).$$

If the output at the k and $k-1$ tact are observed, then the output at the m tact is

$$y(k+m) = \frac{C(q^{-1})}{A(q^{-1})} e(k+m) = F(q^{-1}) e(k+m) + \frac{q^{-m} G(q^{-1})}{A(q^{-1})} e(k+m).$$

To obtain the polynomials $F(q^{-1})$ and $G(q^{-1})$ it is necessary to be solved the diophantine equation:

$$C(q^{-1}) = A(q^{-1}) F(q^{-1}) + q^{-m} G(q^{-1}).$$

In this way, the control law is represented by the following equation:

$$u(k) = -\frac{G(q^{-1})}{A(q^{-1}) B(q^{-1})}.$$

The output of the system under the control of the minimum variance in the stationary regime regime is:

$$y(k) = F(q^{-1}) e(k) = e(k) + f_1 e(k-1) + ... + f_d e(k-d).$$

And the variance of the estimator error can be determinate by the

$$\sigma_y^2 = (1 + f_1^2 + ... + f_d^2) \sigma_e^2.$$

It is given the thermic process of temperature variation in a oven. The mathematical model that approximates the temperature variation in a oven was obtained based on the MATLAB software and it is

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} u(k) + \frac{C(q^{-1})}{A(q^{-1})} e(k) =$$
$$= \frac{0.0000021 q^{-1} + 0.000002158 q^{-2}}{1 - 1.991 q^{-1} + 0.991 q^{-2}} u(k) + \frac{1 - 1.229 q^{-1} + 0.2402 q^{-2}}{1 - 1.991 q^{-1} + 0.991 q^{-2}} e(k).$$

The diofantic equation for solving the polimomials $F(q^{-1})$ and $G(q^{-1})$ is

$$(1 - 1.229 q^{-1} + 0.2402 q^{-2}) = (1 - 1.991 q^{-1} + 0.991 q^{-2})(1 + f_1 q^{-1} + f_2 q^{-2}) + \\ + q^{-3}(g_0 + g_1 q^{-1}).$$

The control law can be presented in the following way:

$$u(k) = -\frac{(0.77058 - 0.7594 q^{-1})}{(0.0000021 q^{-1} + 0.000002158 q^{-2})(1 + 0.762 q^{-1} + 0.766 q^{-2})}.$$

The variance of the estimator error is $\sigma_y = 2.1674$.

Bibliography

[1] Dumitrache I. (coord.), *Automatica.* Editura Academiei Române, vol. II, 2009.

[2] Jelali M., *Control Performance Management in Industrial Automation.Assessment, Diagnosis and Improvement of Control Loop Performance.* Springer, 2013.

Egalitarian Allocations and the Inverse Problem for the Shapley Value

Irinel Dragan

University of Texas, Mathematics, Arlington, TX 76019, USA
e-mail: `dragan@uta.edu`

In a cooperative transferable utilities game, the allocation of the win of the grand coalition is an Egalitarian Allocation if this win is divided into equal parts among all players. The Inverse Set relative to the Shapley Value of a game is a set of games in which the Shapley Value is the same as the initial one. In the Inverse Set we determined a family of games for which this Shapley Value is a coalitional rational value. The Egalitarian Allocation of the game is efficient, so that in the Inverse Set relative to the Shapley Value, the allocation is the same as the initial one, but may not be coalitional rational. In this paper, we shall be finding out in the same family of the Inverse Set, a subfamily of games for which the Egalitarian Allocation is also coalitional rational. We show some relationship between the two sets of games, where our values are coalitional rational. Finally, we discuss the possibility that our procedure may be used for solving the same problem for other efficient values. Numerical examples show the procedure to get solutions for the efficient values.
Key Words. Egalitarian Allocation, Coalitional Rationality, Inverse Problem.

Kernels in Transitively Orientable Graphs

Grigoriu Nicolae

State University of Moldova, Chişinău, Republic of Moldova

Kernel represents an abstract generalization of a concept of solution for cooperative games. These structures have many applications in game theory.[1] We will recall that kernel is a subset of vertices K of the directed graph $\overrightarrow{G} = (X;U)$ when K does not contain adjacent vertices and every vertex in $X \setminus K$ has a successor in K.[2]

Definition 1. *[3] Graph $F = (X_F;U_F)$ is called B-stable subgraph of the undirected graph $G = (X;U)$ if F is stable subgraph of G and for every stable subgraph M of G one of the following conditions is satisfied:*

1. $X_F \cap X_M = \emptyset$;

2. $X_F \subseteq X_M$.

Theorem 1. *If K is a kernel of the transitively oriented graph $\overrightarrow{G} = (X;\overrightarrow{U})$ and $x_i, x_j \in K$ then $x_i \in X_{F_i}$ and $x_j \in X_{F_j}$, $i \neq j$, where F_i, F_j are B-stable directed subgraphs of the graph G.*

Bibliography

[1] E. Boros, V.Gurvich. *Perfect graphs, kernels, and cores of cooperative games*, Discrete Mathematics, Volume 306, Issues 19-20, 6 October 2006, p. 2336-2354.

[2] T. Fleiner. *Stable and crossing structures*, Ph.D. Thesis, Alferd Renyi Mathematical Institute of the Hungarian Academy of Science, 2000.

[3] N. Grigoriu. *B-stable subgraphs in undirected graphs*, The third conference of Mathematical Society of the Republic of Moldova, Chişinău, 2014, p. 354-357.

Methods of solving perfect informational games

Hâncu Boris

State University of Moldova, Chişinău, Republic of Moldova
e-mail: `boris.hancu@gmail.com`

We consider a two persons game in complete and "$1 \leftrightarrows 2$"$-$perfect information with the normal form $\Gamma = \langle X, Y, H_1, H_2 \rangle$. The "$1 \leftrightarrows 2$"$-$perfect information permits us to use other types of strategies, which represents "programs of action". We call these strategies "informationally extended strategies" and denote the sets of these strategies by $\Theta_1 = \{\theta_1 : Y \to X \ \forall y \in Y, \theta_1(y) \in X\}$, $\Theta_2 = \{\theta_2 : X \to Y \ \ \forall x \in X, \theta_2(x) \in Y\}$. We shall remark the following ways to solve games in informationally extended strategies.

1. For any strategies profile (θ_1, θ_2) it is constructed the normal forms of game on set of informationally nonextended strategies X, Y. Thus the set of games $\{\Gamma(\theta_1, \theta_2)\}_{\theta_1 \in \Theta_1}^{\theta_2 \in \Theta_2}$ is generated. In this case only the form of utility functions is chainged $\widetilde{H}_1(x, y) \equiv H_i(\theta_1(y)), \theta_2(x))$ and $(x^*, y^*) \in$ $NE(\Gamma(\theta_1, \theta_2)) \rightleftarrows \begin{cases} \max\limits_{x \in X} \widetilde{H}_1(x, y^*), \\ \max\limits_{y \in Y} \widetilde{H}_2(x^*, y). \end{cases}$

2. The case when $\widehat{H}_i : \Theta_1 \times \Theta_2 \to R$ are not functions, but functionals and we operate not with elements $x \in X$ and $y \in Y$, but with the functions $\theta_1 \in \Theta_1$ and $\theta_2 \in \Theta_2$. Equilibrium profiles are defined on the set $\Theta_1 \times \Theta_2$.

3. The case when the utility of players is described by the functions H_1 and H_2 respectively, but solutions are defined on the set $\Theta_1 \times \Theta_2$. Let $(x^*, y^*) \in NE(\Gamma)$, then as a solution one can take the strategy profile $(\theta_1^*, \theta_2^*) \in \Theta_1 \times \Theta_2$ for which is verified $\begin{cases} \theta_1^*(y) = x^* \ \forall y \in Y, \\ \theta_2^*(x) = y^* \ \forall x \in X. \end{cases}$

4. The case when it is "extended" the number of players introducing "$1 \leftrightarrows 2$"informational type players. It is considered the game with the following normal form $\widetilde{\Gamma} = \left\langle I, J, \Theta_1, \Theta_2, \widetilde{H}_i, \widetilde{H}_j \right\rangle$, where I is the set of θ_1^i-informational type players "generated" by the strategy $\theta_1^i \in \Theta_1$, J is the set of θ_2^i-informational type players "generated" by the strategy $\theta_2^i \in \Theta_2$, $\widetilde{H}_i\left(\theta_1^i, \theta_2^j\right)$, $i \in I$, respectively $\widetilde{H}_j\left(\theta_1^i, \theta_2^j\right)$, $j \in J$, is the utility function of the θ_1^i-informational type players, respectively of the θ_2^j-informational type players. Here it is possible to use Harsanyi principle in solving such types of games.

On the Asymptotic Structure of the Stabilizing Solution of the Riccati Equation Arising in Connection with the Linear Quadratic Regulator Problem for a System Described by Itô Differential Equations with Two Fast Time Scales

V. Drăgan[1], I.-L. Popa[2], S. Aberkane[3,4]

[1] *Institute of Mathematics "Simion Stoilow" of the Romanian Academy*
P.O.Box 1-764, RO-014700, Bucharest, Romania
and The Academy of the Romanian Scientists
[2] *Department of Exact Sciences and Engineering,*
"1 Decembrie 1918" University of Alba Iulia, Alba Iulia, 510009, Romania
[3] *Université de Lorraine, CRAN, UMR 7039, Campus Sciences,*
BP 70239 Vandoeuvre-les-Nancy Cedex, 54506, France
[4] *CNRS, CRAN, UMR 7039, France*
e-mail: `Vasile.Dragan@imar.ro`

We consider a stochastic optimal control problem described by a quadratic performance criterion and a linear controlled system modeled by a system of singularly perturbed Itô differential equations with two fast time scales.
Our goal is to analyse the asymptotic structure with respect to the small parameters $\varepsilon_j > 0, j = 1, 2$ associated to the two fast time scales of the stabilizing solution of the matrix Riccati equation associated to the optimal control problem under consideration. The results derived in this stochastic framework cannot be obtained mutatis-mutandis from the already existing ones in the deterministic case, as those from [1].
The knowledge of the asymptotic structure of the stabilizing solution of the Riccati equation allows us to avoid the ill conditioning of the numerical computations required for obtaining the gain matrix of the optimal control. Also, the analysis performed in this work may be used for the design of a near optimal control for many practical applications in which the values of the small parameters are not precisely known.

Bibliography

[1] V. Dragan, A. Halanay, *Suboptimal Stabilization of Linear Systems with Several Time Scales*, Int. J. Control, 36, 1, 109-126, (1982).

Some limits theorems for lifetime's distributions and their applications in Network's Reliability

Alexei Leahu

Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: `alexei.leahu@ati.utm.md`

In this paper it was presented limits theorems for lifetime distributions as a limits of distributions of random variables $\min(X_1, X_2, ..., X_n)$ and $\max(X_1, X_2, ..., X_n)$, where $X_1, X_2, ..., X_n$ are independent identically distributed random variables such that $X_k = X_{k1} + X_{k2} + ...X_{kN}$, X_{k1}, X_{k2}, ..., X_{kN} are nonnegative independent identically distributed random variables and N is a

Pascal distributed random variable independent of random variables X_{k1}, X_{k2}, ... We connect this results with some mathematical models in Network's Reliability and show their effectiveness to approximate and simplify research of reliability characteristics of different types of Networks.
Key words: lifetime distributions, Pascal's distribution, Limits Theorem, series and parallel network systems, reliability.

Monte Carlo simulation for risk approach

Ludmila Novac

State University of Moldova, Chişinău, Republic of Moldova
e-mail: novac-ludmila@yandex.com

Foreign exchange is a risk factor that is often overlooked by enterprises that wish to enter, grow, and succeed in the global marketplace. The currency rate depends on factors that affect the economy such as trade, inflation, employment, interest rates, growth rate and others. One of the best predictions of losses could be done by using the time series of daily exchange rates for some period and make financial forecasts for the near futures, taking in the view the parameters that have influences to the exchange rates. We also could apply some statistical simulation. At present, a widely used method is the value-at-risk (VaR) model. To calculate the VaR, there exists a variety of models. Among them, the more widely-used are: the historical simulation, the variance-covariance model, and Monte Carlo simulation, which assumes that future currency returns will be randomly distributed. Monte Carlo Methods are used for portfolio evaluation. A similar approach is used in calculating value at risk.

Applications of the KKM property to coincidence theorems, equilibrium problems, minimax inequalities and variational relation problems

Monica Patriche

Faculty of Mathematics and Computer Science, University of Bucharest, 14 Academiei Street, Bucharest, Romania
e-mail: monica.patriche@yahoo.com

The aim of this talk is twofold: firstly, to establish a Fan type geometric result and to apply it in order to obtain some coincidence-like theorems for the case when the images of the correspondences are not convex. Further, new theorems concerning the existence of solutions for equilibrium problems are provided. For coincidence theorems, the reader is referred, for instance, to [2], [3], [10], [11], [13], [19]. The equilibrium problems have been studied, for example, in [1], [2], [3], [6], [7], [9], [12], [18]. Our goal is also to investigate whether the class of minimax inequalities can be extended. In fact, we obtain a new general minimax inequality of the following type: $\inf_{x\in X}\sup_{y\in Y}t(x,y) \leq \frac{\sup_{y\in Y}\inf_{z\in Z} q(y,z)}{\inf_{z\in Z}\sup_{x\in X} p(x,z)}$. Its study is motivated and inspired by the results obtained, for instance, in [1], [2], [3], [19], which concern the three-function inequality: $inf_{x\in X}t(x,x) \leq \sup_{y\in Y}\inf_{z\in Z} q(y,z) + \sup_{z\in Z}\inf_{y\in Y} p(z,y)$. We intend to connect, in forthcoming papers, the present results with the new ones, which consider the equilibrium in games, and are established in [15] or [16]. Another recent result, a contribution of the author, regarding minimax inequalities for discontinuous correspondences, is [14]. In this first part, the originality consists of introducing a new type of properly quasi-convex-like correspondences, which proved to play an important role in our results. The method of proof is based on the well known KKM property.

There exists a large literature containing applications of the KKM property to coincidence theorems, equilibrium theorems, maximal element theorems and minimax inequalities. We refer the reader, for instance, to M. Balaj [2], Lin, Ansari and Wu [10], Lin and Wan [11] or Park [13]. Secondly, we explore how the KKM principle can promote new more theorems which show the existence of solutions for some classes of variational relation problems. We emphasize that, here, the method of application of the KKM property is new and provides new hypotheses for our research. The presented results are mainly published in [17].

Bibliography

[1] M. Balaj, An intersection theorem with applications in minimax theory and equilibrium problem, *J. Math. Anal. Appl.* 336 (2007) 363–371

[2] M. Balaj, Coincidence and maximal element theorems and their applications to generalized equilibrium problems and minimax inequalities, Nonlinear Anal. 68 (2008), 3962-3971.

[3] M. Balaj and L. J. Lin, Fixed points, coincidence points and maximal elements with applications to generalized equilibrium problems and minimax theory, *Nonlinear Anal.* 70 (2009), 393–403.

[4] M. Balaj, Solution existence for two types of mixed variational relation problems, Applicable Analysis, DOI: 10.1080/00036811.2015.1006204

[5] A. Dhara and D.T. Luc, A solution method for linear variational relation problems, J. Global Optimization 59 (2014) 729-756

[6] X. P. Ding, T. M. Ding, KKM type theorems and generalized vector equilibrium problems in noncompact FC-spaces, J. Math. Anal. Appl. 331 (2007) 1230–1245

[7] B. V. Dinh, P. G. Hung and L. D. Muu, Bilevel Optimization as a Regularization Approach to Pseudomonotone Equilibrium Problems, Numerical Functional Analysis and Optimization 35 (2014) 539-563

[8] M. Fakhar, M. Lotfipour, J. Zafarani, On the Br´ezis Nirenberg Stampacchia-type theorems and their applications, Journal of Global Optimization 55 (2013), 751–770

[9] S.-M. Guu and J. Li, Vector quasi-equilibrium problems: separation, saddle points and error bounds for the solution set, Journal of Global Optimization, 58, (2014), 751-767

[10] L.J. Lin, Q. H. Ansari and J. Y. Wu, Geometric properties and coincidence theorems with applications to generalized vector equilibrium problems, J. Optim. Theory Appl. 117(2003) 121-137

[11] L. J. Lin, W.P. Wan, KKM type theorems and coincidence theorems with applications to the existence of equilibrium, J. Optim. Theory Appl. 123 (2004) 105-122.

[12] L. J. Lin, H.L. Chen, The study of KKM theorems with applications to vector equilibrium problems and implict vector variational inequalities problems. J. Glob. Optim. 32 (2005), 135-157

[13] S. Park, Coincidence points and maximal elements of multifunctions on convex spaces, Comment. Math. Univ. Carolina 36 (1995) 57-67.

[14] M. Patriche, Minimax theorems for set-valued maps without continuity assumptions, Optimization, 65 (2016), 957-976

[15] M. Patriche, Existence of equilibrium for generalized games in choice form and applications, Optimization, 65 (2016), 2135–2151

[16] M. Patriche, Existence of the equilibrium in choice, J. Optim. Theory Appl., 175 (2017), 58–171

[17] M. Patriche, Applications of the KKM property to coincidence theorems, equilibrium problems, minimax inequalities and variational relation problems, Numerical Functional Analysis and Optimization, 38 (2017), 4, 523-548

[18] P. H. Sach and N. B. Minh, New Results on Henig Proper Generalized Vector Quasiequilibrium Problems, Numerical Functional Analysis and Optimization, 36 (2015), 387-403

[19] M. Yang and N. Huang, Coicidence theorems for noncompact RC-maps in abstract convex spaces with applications, Bull. Korean Math. Soc. 49 (2012), 1147–1161

The broken stick model. The optimality ptoperty of the uniform distribution

Gheorghiță Zbăganu

"Gheorghe Mihoc - Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania

e-mail:

The problem. Let $I = [0, l]$ be a stick of length l. Let $(X_n)_{1 \le n \le N+1}$ be iid absolutely continuous random variables valued into I and let F be their distribution. Let $(X_{n:N})_{1 \le n \le N+1}$ be their order statistic and $Y_n = X_{n:N} - X_{(n-1):N}, 1 \le n \le N+1$ be the corresponding spacings. Here $X_{0:N} = 0$. Let $G_n = G_n(F) = \frac{1}{N} \sum \delta_{\frac{NY_n}{l}}$ be the normalized empirical distributions of the spacings. It is known that if F is the uniform distribution, then G_n weakly converges to the exponential distribution $Exp(1)$ hence the Lorenz curves $L_N(x) = \int_0^x G_n^{-1}(t)\,dt$ converge to the Lorenz curve of the exponential distribution $L(x) = x + (1-z)\ln(1-x)$. See for example *Towards understanding the Lorenz curve using the Uniform distribution* ,Chris J. Stephens,Gini-Lorenz Conference, University of Siena, Italy, May 2005).
Results. Let us say that a distribution on an interval I has the property (D) if the distributions $G_n(F)$ weakly converge to some limit distribbution $H = H(F)$. We prove

Theorem 1. *Let $a_0 < a_1 < a_2 < ... < a_k$ and $I_j = [a_{j-1}, a_j)$. Suppose that $F = \sum_{j=1}^{k} p_j F_j$ where*

F_j are distributions on I_j having the property (D) and $p_j > 0, \sum_{j=1}^{k} p_j = 1$.

Then F has the property (D) ,too. Precisely, if $\lim G_n(F_j) = H_j$ then $\lim G_n(F) = \sum_{j=1}^{k} p_j H_j \circ h_{\frac{\pi_j}{p_j}}^{-1}$

where $\pi_j = \frac{a_j - a_{j-1}}{a_k - a_0}$ and $h_\alpha(x) = \alpha x$ is the homothethy.

Corollary 1. *Suppose that $F_j =$Uniform(I_j) . Then $H = \sum_{j=1}^{k} p_j Exp\left(\frac{p_j}{\pi_j}\right)$*

Corollary 2. *COROLLARY 3. If F has a density f of the form $f(x) = \sum_{j=1}^{k} \alpha_j 1_{I_j}$ then the Lorenz curve of H is under the graph of $x + (1-x)\ln x$*

Conclusion. The uniform distribution is the most egalitarian.

6. Algebra, Logic, Geometry (with applications)

Rota-Baxter operators and elliptic curves

Cristian Anghel

"Simion Stoilov" Institute of Mathematics of Romanian Academy, Bucharest, Romania
e-mail: Cristian.Anghel@imar.ro

The Rota-Baxter operators have a long history, with manny applications in both pure mathematics and theoretical physics. After a short review of this subject, I will present a class of Rota-Baxter operators comming from the world of vector bundles over elliptic curves. If time permits we will see also some connections with modular forms/functions.

On almost contact metric 2- and 3-hypersurfaces in W_4-manifolds

Mihail Banaru[1], Galina Banaru[2]

[1] *Chair of Mathematics and Informatics, Smolensk State University, 4, Przhevalsky Street, Smolensk V 214 000 RUSSIA;*
[2] *Chair of Applied Mathematics, Smolensk State University, 4, Przhevalsky Street, Smolensk V 214 000 RUSSIA*
e-mail: mihail.banaru@yahoo.com,

The class of W_4-manifolds is one of so-called "small" Gray-Hervella classes [1] of almost Hermitian manifolds. Some specialists identify this class with the class of locally conformal Kählerian (LCK-) manifolds that is not absolutely correct. In fact, the class contains all locally conformal Kählerian manifolds, but coincides with the class of LCK-manifolds only for dimension at least six [2]. W_4-manifolds were studied in detail by such outstanding mathematicians as A. Gray (USA), V.F. Kirichenko (Russian Federation) and I. Vaisman (Israel).
As it is known, almost contact metric structures are induced on oriented hypersurfaces of an almost Hermitian manifold. We remind that the almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi, \xi, \eta, g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1, 1) and $g = \langle\cdot,\cdot\rangle$ is a Riemannian metric [5], [6]. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta \circ \Phi = 0,\ \Phi^2 = -id + \xi \otimes \eta,$$

$$\langle \Phi X, \Phi Y \rangle = \langle X, Y \rangle - \eta(X)\,\eta(Y),\ X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of the smooth vector fields on N.
In [4] it has been proved that if the almost Hermitian manifold is of class W_4 and the type number of its hypersurface is equal to one, then the almost contact metric structure on such a hypersurface is identical the structure on totally geodesic hypersurface. The similar results were obtained for 0- and 1-hypersurfaces of W_1- and W_3-manifolds [5], [6].
The main result of the present communication is the following:
Theorem. *3-hypersurfaces of W_4-manifolds admit an almost contact metric structure that can be identical to the structure induced on 2-hypersurfaces of such manifolds.*
Taking into account the above mentioned fact that the class of W_4-manifolds contains all LCK-manifolds, we get the following:
Corollary. *3-hypersurfaces of locally conformal Kählerian manifolds admit an almost contact metric structure that can be identical to the structure induced on 2-hypersurfaces of such manifolds.*
We remark that the structure induced on 2- and 3-hypersurfaces of W_4-manifolds does not belong to any well-studied classes of almost contact metric structures (cosymplectic, nearly cosymplectic, Kenmotsu, Sasaki structures etc).

Bibliography

[1] A. Gray and L.M. Hervella, *The sixteen classes of almost Hermitian manifolds and their linear invariants*, Ann. Mat. Pura Appl., 123 (4), (1980), 35–58.

[2] V.F. Kirichenko, *Differential-geometric structures on manifolds*, Pechatnyi Dom, Odessa, (2013) (in Russian).

[3] M.B. Banaru and V.F. Kirichenko, *Almost contact metric structures on the hypersurface of almost Hermitian manifolds*, Journal of Mathematical Sciences (New York), 207(4), (2015), 513–537.

[4] M.B. Banaru, *On almost contact metric hypersurfaces with small type numbers in W_4-manifolds*, Moscow University Mathematics Bulletin, 73(1), (2018), 38–40.

[5] M.B. Banaru, *Almost contact metric hypersurfaces with type number 0 or 1 in nearly-Kählerian manifolds*, Moscow University Mathematics Bulletin, 69(3), (2014), 132–134.

[6] M.B. Banaru, *A note on geometry of special Hermitian manifolds*, Lobachevskii Journal of Mathematics, 39(1), (2018), 20–24.

Sous-catégories $\mathcal{L}$-semi-reflexives

Dumitru Botnaru

L'Université d'Etat de Tiraspol
e-mail: dumitru.botnaru@gmail.com

On étudie les sous-catégories $\mathcal{L}$-semi-reflexives.
Key words. The semi-reflexif product of two subcategories, the left exact functors, semireflexive subcategories.

About Cartesian Product of Two Subcategories

Dumitru Botnaru, Olga Cerbu

State University from Tiraspol, State University Moldova
e-mail: dumitru.botnaru@gmail.com, olga.cerbu@gmail.com

Summary. We examine a categorial construction which permit to obtained a new reflective subcategories with a specal properties.
Key words: Reflective subcategories, the pairs of conjugated subcategories, the right product of the two subcategories.

Results. Let $\mathcal{K}$ be a coreflective subcategory, and $\mathcal{R}$ a reflective subcategory of the category of locally convex topological vector Hausdorff spaces $\mathcal{C}_2\mathcal{V}$ with respective functors $k : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K}$ and $r : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{R}$.

Concerning of the terminology and notation see [1]. Note by $\mu\mathcal{K} = \{m \in \mathcal{M}ono \mid k(m) \in \mathcal{I}so\}$, $\varepsilon\mathcal{R} = \{e \in \mathcal{E}pi \mid r(e) \in \mathcal{I}so\}$. Further for an arbitrary object X of the category $\mathcal{C}_2\mathcal{V}$ we examine the follows construction: let $k^X : kX \longrightarrow X$ is $\mathcal{K}$-coreplique, and $r^{kX} : kX \longrightarrow rkX$-replique of the respective objects. On the morphism k^X and r^{kX} we construct the cocartesian square

$$\overline{v}^X \cdot k^X = u^X \cdot r^{kX}. \tag{1}$$

Definition 1. *1. The full subcategory of all isomorphic objects with the type of objects is called $\overline{v}X$ cartesian product of the subcategories $\mathcal{K}$ and $\mathcal{R}$, noted by $\overline{v} = \mathcal{K} *_{dc} \mathcal{R}$.*

2.The diagram of cartesian product is called the diagram of cartesian product of the pair of conjugate subcategories $(\mathcal{K}, \mathcal{R})$ (Diagram (RCP)).

Definition 2. *The full subcategory of all isomorphic objects with the objects of type $\overline{v}X$ is called cartesian product of the subcategories $\mathcal{K}$ and $\mathcal{R}$, note by $\overline{\mathcal{W}} = \mathcal{K} *_{sc} \mathcal{R}$.*

Lemma 1. $\mathcal{R} \subset \mathcal{K} *_{dc} \mathcal{R}$.

Theorem 1.*The application $X \mapsto \overline{v}X$ defined a functor*

$$\overline{v} : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K} *_{dc} \mathcal{R}.$$

We examine the following condition:

(RCP) For any object X of the category $\mathcal{C}_2\mathcal{V}$ in the diagram (RCP) the morphism u^X belongs to the class $\mu\mathcal{K}$.

Theorem 2. *Let it be a pairs of the subcategories $(\mathcal{K}, \mathcal{R})$ verify the condition (RCP). Then $\overline{v}$ it is a reflector functor.*

Theorem 3. *Let $\mathcal{K}$ be a coreflective subcategory, but $\mathcal{R}$ is a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$, $\widetilde{\mathcal{M}}$ - the subcategory of the spaces with Mackey topology, $\mathcal{S}$ is the subcategory of the spaces with weak topology. If $\mathcal{K} \subset \widetilde{\mathcal{M}}$, but $\mathcal{S} \subset \mathcal{R}$, then the pair of subcategories $(\mathcal{K}, \mathcal{R})$ verify condition (RCP) the cartesian product is a reflective subcategory.*

Examples. 1. *For any coreflective subcategory $\mathcal{K}$ we have $\mathcal{K} *_{dc} \Pi = \Pi$, Π-reflective subcategory of the complete space with weak topology.*

2. *For any coreflective subcategory $\mathcal{K}$ we have $\mathcal{K} *_{dc} \mathcal{S} = \mathcal{S}$, $\mathcal{S}$-reflective subcategory of the space with weak topology.*

Theorem 4. *Let $(\mathcal{K}, \mathcal{L})$ a pair of conjugate subcategories, and $\mathcal{R}$ a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$. Then:*

1. *$\mathcal{K} *_{dc} \mathcal{R} = \mathcal{Q}_{\varepsilon\mathcal{L}}(\mathcal{R})$, where $\mathcal{Q}_{\varepsilon\mathcal{L}}(\mathcal{R})$ is the full subcategory of all $\varepsilon\mathcal{L}$-factorobjects of objects of the subcategory $\mathcal{R}$.*
2. *$\mathcal{K} *_{dc} \mathcal{R}$ is a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$.*
3. *The subcategory $\mathcal{K} *_{dc} \mathcal{R}$ is closed in relation to $\varepsilon\mathcal{L}$-factorobjects.*
4. *$\overline{v} \cdot k = r \cdot k$.*
5. *If $r(\mathcal{K}) \subset \mathcal{K}$, then the coreflector functor $k : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K}$ and the reflector $\overline{v} : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K} *_{dc} \mathcal{R}$ commute: $k \cdot \overline{v} = \overline{v} \cdot k$.*

Theorem 5. *Let $\mathcal{K}$ (respective $\mathcal{R}$) a coreflective subcategory (respective: reflective)of the category $\mathcal{C}_2\mathcal{V}$, those functors $k : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K}$ and $r : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{R}$ commute: $k \cdot r = r \cdot k$. Then*

$$\mathcal{K} *_{dc} \mathcal{R} = \mathcal{K} *_{d} \mathcal{R}.$$

Bibliography

[1] Botnaru D.,*Structures bicatégorielles complémentaires*, ROMAI Journal, v.5, N.2, 2009, p. 5-27.

[2] Botnaru D., Turcanu A., *The factorization of the right product of two subcategories*, ROMAI Journal, v.6, N.2, 2010, p. 41-53.

Hausdorff extensions

Laurenţiu I. Calmuţchi

Tiraspol State University, Republic of Moldova
e-mail: lcalmutchi@gmail.com

Any space is considered to be a Hausdorff space. We use the terminology and notations from [3, 1, 2].

Let τ be an infinite cardinal.

A point $x \in X$ is called a $P(\tau)$-point of the space X if for any non-empty family γ of open subsets of X for which $x \in \cap\gamma$ and $|\gamma| < \tau$ there exists an open subset U of X such that $x \in U \subset \cap\gamma$. If any point of X is a $P(\tau)$-point, then we say that $P(\tau)$-space.

Fix a set Φ of almost disjoint τ-centered families of subsets of the set E. We put $e_\Phi E = E \cup \Phi$. On $e_\Phi E$ we construct two topologies.

Topology $T^s(\Phi)$. The basis of the topology $T^s(\Phi)$ is the family $\mathcal{B}^s(\Phi) = \{U_L = L \cup \{\eta \in \Phi : H \subset L \textit{ for some } H \in \eta\} : L \subset E\}$.

Topology $T_m(\Phi)$. For each $x \in E$ we put $B_m(x) = \{\{x\}\}$. For every $\eta \in \Phi$ we put $B_m(\eta) = \{V_{(\eta,L)} = \{\eta\} \cup L : L \in \eta\}$. The basis of the topology $T_m(\Phi)$ is the family $\mathcal{B}_m(\Phi) = \cup\{B_m(x) : x \in e_\Phi E\}$.

Theorem 1. *The spaces* $(e_\Phi E, T^s(\Phi))$ *and* $(e_\Phi E, T_m(\Phi))$ *are Hausdorff zero-dimensional extensions of the discrete space* E*, and* $T^s(\Phi) \subset T_m(\Phi))$*. In particular,* $(e_\Phi E, T^s(\Phi)) \leq (e_\Phi E, T_m(\Phi))$.

Theorem 2. *The spaces* $(e_\Phi E, T^s(\Phi))$ *and* $(e_\Phi E, T_m(\Phi))$ *are* $P(\tau)$*-spaces.*

Corollary 3. *If* $T^s(\Phi) \subset \mathcal{T} \subset T_m(\Phi))$*, then* $(e_\Phi E, \mathcal{T})$ *is a Hausdorff extension of the discrete space* E*, and* $(e_\Phi E, T^s(\Phi)) \leq (e_\Phi E, \mathcal{T}) \leq (e_\Phi E, T_m(\Phi))$.

Theorem 4. *The space* $(e_\Omega E, T^s(\Omega))$*, where* Ω *is the set of well-ordered almost disjoint* τ*-centered families, is a zero-dimensional paracompact space with character* $\chi(e_\Omega E, T^s(\Omega)) = \tau$ *and weight* $\Sigma\{|E|^m : m < \tau\}$.

Consider the Hausdorff extension rE of the space E. We put $e_{rE}X = X \cup (rE \setminus E)$. In $e_{rE}X$ we construct the topology $\mathcal{T} = T(\gamma, E, \xi_\mu, \tau)$.

Theorem 5. *The space* $(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau))$ *is a Hausdorff extension of the space* X.

Theorem 6. *If* rE *is a* $P(\tau)$*-space, then* $(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau))$ *is a* $P(\tau)$*-space too. Moreover,* $\chi(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau)) = \chi(X) + \chi(rE)$ *and* $w(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau)) = w(X) + w(rE)$.

Theorem 7. *Assume that the spaces* rE *and* X *are zero-dimensional, and the sets* $H_{(\mu,\alpha)}$ *are open-and-closed in* X*. Then: 1.* $(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau))$ *is a zero-dimensional space.*

2. The space $(e_{(E,Y)}X, T(\gamma, E, \xi_\mu, \tau))$ *is paracompact if and only if the spaces* rE *and* X *are paracompact.*

Bibliography

[1] L.Calmuţchi. *Methods of construction of Hausdorff extensions.* The Fourth Conference of Mathematical Society of the Republic of Moldova (dedicated to the centenary of Vladimir Andrunachevici(1917-1997)). Chişinau 2017, p.193-196

[2] L.Calmuţchi. *Construction of Hausdorff extensions.* MITI2018 International conference on mathematics, informatics and information technologies dedicated to the illustrious scientist Valentin Belousov. 19-21 april, 2018. Bălţi, p.25-26.

[3] R. Engelking, *General Topology*, PWN, Warszawa, 1977.

Extreme Points in the Complex of Multy-ary Relations

Sergiu Cataranciuc, Galina Braguța

State University of Moldova, Chişinău, Republic of Moldova
e-mail: s.cataranciuc@gmail.com, gallinna@yahoo.com

Let $\mathcal{R}^{n+1} = (R^1, R^2, ..., R^{n+1})$ be a complex of multy-are relations, defined in the work [1]. We denote by d_k^m the distance function defined on the set R^k [2]. Let $d_k^m - conv(A)$ be the convex hull of a subset A from the metric space (R^k, d_k^m).

Definition 1. The cortege $r = (x_{i_1}, x_{i_2}, ...x_{i_k}) \in R^k$ is called m-extreme point of the set $A \subset R^k$, $1 \leq k < m \leq n+1$ if:

a) $r \in A$;

b) $r \notin d_k^m - conv(A - r)$.

Knowing m-extreme points of a set often simplifies the procedure of convex hull construction and the study of its properties. Let denote by $ext^m(A)$ the set of all m-extreme points of the set A.

Lemma 1. If A is a subset from m-ary relation R^k and $r \in ext^m(A)$ then $r \in ext^m(d_k^m - conv(A))$.

From Lemma 1 results that $ext^m(A) \subset ext^m(d_k^m - conv(A))$.

Let be $r \in A \subset R^k$. We denote by $\Gamma_A^m(r) = \{z \in A : z \cup r \in R^m\}$ the set of all elements from A that are joined with r through a m-dimensional chain of length one. Such a set may be named m-dimensional neighborhood of the element r in A.

A complex of multi-ary relations $\mathcal{R}^{n+1} = (R^1, R^2, ..., R^{n+1})$, defined on a set of elements X is complete, if $R^1 = X$ and $R^s = X^s$, $2 \leq s \leq n+1$. The complex $\mathcal{R}^{n+1}$ is named m-complete, if $R^m = X^m$, $2 \leq m \leq n+1$.

Lemma 2. If the complex $\mathcal{R}^{n+1} = (R^1, R^2, ..., R^{n+1})$ is m-complete, then it is and t-complete, for any $2 \leq t \leq m$.

Definition 2. The cortege $r \in A \subset R^k$ is named m-simplicial cortege in A, if the set $\Gamma_A^m(r)$ generates a m-complete subcomplex.

Theorem 1. If the cortege $r \in A \subset R^k$ is m-simplicial in A, then it is m-simplicial and in the set $d_k^m - conv(A)$.

It follows conditions in which an arbitrary cortege r from the set $A \subset R^k$ is m-extreme point in A.

Theorem 2. The cortege $r \in A \subset R^k$ is m-extreme point in A, if and only if r is m-simplicial cortege in A.

Theorem 3. If $d_k^m - conv(A)$ is the convex hull of a set $A \subset R^k$, then any m-extreme point from $d_k^m - conv(A)$ is m-extreme point in A.

Extreme Points in the Complex of Multy-ary Relations

Sergiu Cataranciuc, Galina Braguța

State University of Moldova, Chişinău, Republic of Moldova
e-mail: s.cataranciuc@gmail.com, gallinna@yahoo.com

Let $\mathcal{R}^{n+1} = (R^1, R^2, ..., R^{n+1})$ be a complex of multy-are relations, defined in the work [1]. We denote by d_k^m the distance function defined on the set R^k [2]. Let $d_k^m - conv(A)$ be the convex hull of a subset A from the metric space (R^k, d_k^m).

Definition 1. The cortege $r = (x_{i_1}, x_{i_2}, ...x_{i_k}) \in R^k$ is called m-extreme point of the set $A \subset R^k$, $1 \leq k < m \leq n+1$ if:

a) $r \in A$;

b) $r \notin d_k^m - conv(A - r)$.

Knowing m-extreme points of a set often simplifies the procedure of convex hull construction and the study of its properties. Let denote by $ext^m(A)$ the set of all m-extreme points of the set A.
Lemma 1. If A is a subset from m-ary relation R^k and $r \in ext^m(A)$ then $r \in ext^m(d_k^m - conv(A))$.
From Lemma 1 results that $ext^m(A) \subset ext^m(d_k^m - conv(A))$.
Let be $r \in A \subset R^k$. We denote by $\Gamma_A^m(r) = \{z \in A : z \cup r \in R^m\}$ the set of all elements from A that are joined with r through a m-dimensional chain of length one. Such a set may be named m-dimensional neighborhood of the element r in A.
A complex of multi-ary relations $\mathcal{R}^{n+1} = \left(R^1, R^2, ..., R^{n+1}\right)$, defined on a set of elements X is complete, if $R^1 = X$ and $R^s = X^s$, $2 \leq s \leq n+1$. The complex $\mathcal{R}^{n+1}$ is named m-complete, if $R^m = X^m$, $2 \leq m \leq n+1$.
Lemma 2. If the complex $\mathcal{R}^{n+1} = \left(R^1, R^2, ..., R^{n+1}\right)$ is m-complete, then it is and t-complete, for any $2 \leq t \leq m$.
Definition 2. The cortege $r \in A \subset R^k$ is named m-simplicial cortege in A, if the set $\Gamma_A^m(r)$ generates a m-complete subcomplex.
Theorem 1. If the cortege $r \in A \subset R^k$ is m-simplicial in A, then it is m-simplicial and in the set $d_k^m - conv(A)$.
It follows conditions in which an arbitrary cortege r from the set $A \subset R^k$ is m-extreme point in A.
Theorem 2. The cortege $r \in A \subset R^k$ is m-extreme point in A, if and only if r is m-simplicial cortege in A.
Theorem 3. If $d_k^m - conv(A)$ is the convex hull of a set $A \subset R^k$, then any m-extreme point from $d_k^m - conv(A)$ is m-extreme point in A.

Bibliography

[1] Cataranciuc S., *Topologia algebrică a relaţiilor multi-are.* Chişinău: CEP USM, 2015, 228 p.

[2] S. Cataranciuc, G. Braguţa, *The convexity in the complex of multi-ary relations*, ROMAI Journal, v. 11, **2** (2015), 51-62.

On non-isomorphic quasigroups of small order

L. Chiriac, A. Danilov, N. Lupasco, N. Josu

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: llchiriac@gmail.com, a.danilov@gmail.com, nlupashco@gmail.com, nbobeica1978@gmail.com

A non-empty set G is said to be a *groupoid* relatively to a binary operation denoted by $\{\cdot\}$, if for every ordered pair (a, b) of elements of G there is a unique element $ab \in G$.
A groupoid $(G, \cdot)$ is called a *quasigroup* if for every $a, b \in G$ the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions.
A quasigroup $(G, \cdot)$ is called a *Ward quasigroup* if it satisfies the law $(a \cdot c) \cdot (b \cdot c) = a \cdot b$ for all $a, b, c \in G$.
A quasigroup $(G, \cdot)$ is called a *Cote quasigroup* if it satisfies the law $a \cdot (ab \cdot c) = (c \cdot aa) \cdot b$ for all $a, b, c \in G$.
A groupoid $(G, \cdot)$ is called a *Manin quasigroup* if it satisfies the law $a \cdot (b \cdot ac) = (aa \cdot b) \cdot c$ for all $a, b, c \in G$.

We consider the following problem:
Problem 1. How many non-isomorphic Ward quasigroups, Cote quasigroups and Manin quasigroups of order 3, 4, 5, 6 do there exist?
We elaborated algorithms for generating non-isomorphic Ward quasigroups, Cote quasigroups and Manin quasigroups of small order.The results established here are related to the work in ([1,2,3,4,5]). Applying the algorithms elaborated, we prove the following results:

Theorem 1. *There are exactly:*
- 1 non-isomorphic Ward quasigroup of order 3;
- 2 non-isomorphic Ward quasigroups of order 4;
- 1 non-isomorphic Ward quasigroup of order 5;
- 2 non-isomorphic Ward quasigroups of order 6.

Theorem 2. *There are exactly:*
- 3 non-isomorphic Cote quasigroups of order 3;
- 4 non-isomorphic Cote quasigroups of order 4;
- 2 non-isomorphic Cote quasigroups of order 5;
- 3 non-isomorphic Cote quasigroups of order 6.

Theorem 3. *There are exactly:*
- 3 non-isomorphic Manin quasigroups of order 3;
- 4 non-isomorphic Manin quasigroups of order 4;
- 4 non-isomorphic Manin quasigroups of order 5;
- 3 non-isomorphic Manin quasigroups of order 6.

Bibliography

[1] Belousov V.D., *Foundations of the theory of quasigroups and loops*, Moscow, Nauka, 1967, 223 pp.

[2] Chiriac L., Bobeica N., Pavel D., *Study on properties of non-isomorphic finite quasigroups using the computer.* Proceedings of the Third Conference of Mathematical Society of Moldova IMCS-50, August 19-23, 2014, Chisinau, Republic of Moldova, 2014, p. 44-47.

[3] Kiriyak L., Bobeica N., Guzun S., Gropa A., Covalschi A. *Identification of nonisomorphic quasigroups.* Second Conference of the Mathematical Society of the Republic of Moldova, Chisinau, August,17-19, 2004, p.109-110.

[4] Cote B., Harvill B., Huhn M., Kirchman A., *Classification of loops of generalized Bol-Moufang type*, Quasigroups Related Systems, 19, 2, (2011), p. 193-206.

[5] Chernov V., Moldovyan N., Shcherbacov V., *On some groupoids of small order.* Proceedings of the 4th Conference of Mathematical Society of Moldova CMSM4, 2017, June 28-July 2, 2017, Chisinau, Republic of Moldova, p.51-54.

On the algebraic properties of the ring of Dirichlet convolutions

Mircea Cimpoeaş

"Simion Stoilov" Institute of Mathematics of Romanian Academy, Bucharest, Romania
e-mail: mircea.cimpoeas@imar.ro

Let $(\Gamma,\cdot)$ be a commutative monoid of finite type and let R be a commutative ring with unity. In the set of functions defined on Γ with values in R, $\mathcal{F}(\Gamma,R) := \{\alpha : \Gamma \to R\}$, we consider the operations

$$(\alpha+\beta)(n) := \alpha(n)+\beta(n),\ (\forall)n \in \Gamma \text{ and } (\alpha\cdot\beta)(n) := \sum_{ab=n} \alpha(a)\beta(b),\ (\forall)n \in \Gamma.$$

It is well known that $(\mathcal{F}(\Gamma,R),+,\cdot)$ is a commutative ring with unity, see for instance [3]. Let M be a R-module. Let $\mathcal{F}(\Gamma,M) := \{f : \Gamma \to M\}$. For any $f,g \in \mathcal{F}(\Gamma,M)$ and $\alpha \in \mathcal{F}(\Gamma,R)$, we define:

$$(f+g)(n) := f(n)+g(n),\ (\forall)n \in \Gamma \text{ and } (\alpha\cdot f)(n) := \sum_{ab=n} \alpha(a)f(b),\ (\forall)n \in \Gamma.$$

We show that $\mathcal{F}(\Gamma,M)$ has a natural structure of a $\mathcal{F}(\Gamma,R)$-module and we discuss certain properties of the functor $M \mapsto (\Gamma,M)$. Given a morphism of monoids $L : (\Gamma,\cdot) \to (M,+)$, we show that the induced map $\Phi_{L,M} : \mathcal{F}(\Gamma,M) \to \mathcal{F}_L(\Gamma,M)$, $\Phi_{L,M}(f)(n) := L(n)f(n)$ is a morphism of $\mathcal{F}(\Gamma,R)$-modules.
Let A be a commutative ring with unity and let $i : A \to R$ be a morphism of rings with unity. Let M be an R-module. An A-derivation $D : R \to M$ is an A-linear map, satisfying the Leibniz rule, i.e. $D(fg) = fD(g)+gD(f), (\forall)f,g \in R$. The set of A-derivations $\operatorname{Der}_A(R,M)$ has a natural structure of a R-module. Let $D \in \operatorname{Der}_A(R,M)$ and let $\delta : \Gamma \to (M,+)$ be a morphism of monoids. We prove that

$$\widetilde{D} : \mathcal{F}(\Gamma,R) \to \mathcal{F}(\Gamma,M),\ \widetilde{D}(\alpha)(n) = D(\alpha(n)) + \alpha(n)\delta(n),\ (\forall)n \in \Gamma, \text{ is an } A\text{-derivation.}$$

Assume that the monoid $(\Gamma,\cdot)$ is cancellative, i.e. $xy = xz$ implies $y = z$. Let $G(\Gamma)$ be the Grothendieck group associated to Γ, see [2]. In the set

$$\mathcal{F}^f(G(\Gamma),R) := \{\alpha : G(\Gamma) \to R\ :\ (\exists)d \in \Gamma \text{ such that } (\forall)q \in G(\Gamma), \alpha(q) \neq 0 \Rightarrow dq \in \Gamma\}.$$

we consider the operations

$$(\alpha+\beta)(q) := \alpha(q)+\beta(q),\ (\forall)q \in G(\Gamma) \text{ and } (\alpha\cdot\beta)(q) := \sum_{q'q''=q} \alpha(q')\beta(q''),\ (\forall)q \in G(\Gamma).$$

We prove that $\mathcal{F}^f(G(\Gamma),R)$ is an extension of the ring $\mathcal{F}(\Gamma,R)$. Let M be an R-module. We consider the set

$$\mathcal{F}^f(G(\Gamma),M) := \{f : G(\Gamma) \to M\ :\ (\exists)d \in \Gamma \text{ such that } (\forall)q \in G(\Gamma), f(q) \neq 0 \Rightarrow dq \in \Gamma\}.$$

For $f,q \in \mathcal{F}^f(G(\Gamma),M)$ we define $(f+g)(q) := f(q)+g(q), (\forall)q \in G(M)$. Given $\alpha \in \mathcal{F}^f(G(\Gamma),R)$ and $f \in \mathcal{F}^f(G(\Gamma),M)$ we define $(\alpha\cdot f)(q) := \sum_{q'q''=q} \alpha(q')f(q'')$, $(\forall)q \in G(\Gamma)$. We prove that $\mathcal{F}^f(G(\Gamma),M)$ has a structure of an $\mathcal{F}^f(G(\Gamma),R)$-module and we study the connections between

the associations $M \mapsto \mathcal{F}(\Gamma, M)$ and $M \mapsto \mathcal{F}^f(\Gamma, M)$.
In particular, we show that if $D \in \mathrm{Der}_A(R, M)$ is an A-derivation and $\delta : \Gamma \to (M, +)$ is a morphism of monoids, then we can construct an A-derivation on $\overline{D} : \mathcal{F}^f(G(\Gamma), R) \to \mathcal{F}^f(G(\Gamma), M)$ which extend $\widetilde{D}$.
The most important case, largely studied in analytic number theory [1], is the case when R is a domain (or even more particullary, when $R = \mathbb{C}$) and $\Gamma = \mathbb{N}^*$ is the multiplicative monoid of positive integers. Cashwell and Everett showed in [4] that $\mathcal{F}(\mathbb{N}^*, R)$ is also a domain. Moreover, if R is an UFD with the property that $R[[x_1, \ldots, x_n]]$ are UFD for any $n \geq 1$, then $\mathcal{F}(\mathbb{N}^*, R)$ is also an UFD, see [5]. It is well known that the Grothendieck group associated to $\mathbb{N}^*$ is $\mathbb{Q}_+^* :=$ the group of positive rational numbers. We show that

$$\mathcal{F}^f(\mathbb{Q}_+^*, R) \cong R[[x_1, x_2, \ldots]][x_1^{-1}, x_2^{-1}, \ldots],$$

and, in particular, if $R[[x_1, x_2, \ldots]]$ is UFD, then $\mathcal{F}^f(\mathbb{Q}_+^*, R)$ is an UFD.
We make some remarks in the following case: Let $U \subset \mathbb{C}$ be an open set and let $\mathcal{O}(U)$ be the ring of holomorphic functions defined in U with values in $\mathbb{C}$. It is well known that $\mathcal{O}(U)$ is a domain. We consider

$$\widetilde{D} : \mathcal{F}(\mathbb{N}^*, \mathcal{O}(U)) \to \mathcal{F}(\mathbb{N}^*, \mathcal{O}(U)),\ \widetilde{D}(\alpha)(n)(z) := \alpha(n)'(z) - \alpha(n)(z) \log n,\ (\forall) n \in \mathbb{N}, z \in U.$$

We note that $\widetilde{D}$ is a $\mathbb{C}$-derivation on $\mathcal{F}(\mathbb{N}^*, \mathcal{O}(U))$. Assume that the series of functions $F_\alpha(z) := \sum_{n=1}^{\infty} \frac{\alpha(n)(z)}{n^z}$, $z \in U$, and and $G_\alpha(z) = \sum_{n=1}^{\infty} \left(\frac{\alpha(n)(z)}{n^z}\right)'$, $z \in U$, are uniformly convergent on the compact subsets $K \subset U$. It is well known that, in this case, F defines a derivable (holomorphic) function on U and, moreover, $F' = G$. It is easy to see that $F'_\alpha = F_{\widetilde{D}(\alpha)}$. Further connections between the $\mathbb{C}$-linear independence of $\alpha_1, \ldots, \alpha_m \in \mathcal{F}(\mathbb{N}^*, \mathcal{O}(U))$, with some suplimentary conditions, and their associated series $F_{\alpha_1}, \ldots, F_{\alpha_m}$ were made in [6].

Bibliography

[1] T. M. Apostol, *Introduction to analytic number theory*, UTM, Springer-Verlag, (1976).

[2] M. F. Atiyah, *K-Theory*, (Notes taken by D.W.Anderson, Fall 1964), published in 1967, W.A. Benjamin Inc., New York.

[3] S. K. Berberian, *Number-Theoretic Functions via Convolution Rings*, Mathematics Magazine, **Vol. 65, No. 2**, (1992), 75-90

[4] E. D. Cashwell, C. J. Everett, *The ring of number-theorethic functions*, Pacific J. Math. **9**, (1959), 975-985.

[5] E. D. Cashwell, C. J. Everett, *Formal power series*, Pacific J. Math. **13(1)**, (1963), 45-64.

[6] M. Cimpoeaş, *A note on the linear independence of a class of series of functions*, https://arxiv.org/pdf/1802.03377.pdf, Preprint (2018).

Subnormalizing Extensions and D-structures

E. P. Cojuhari[1] and B. J. Gardner[2]

[1] *Department of Mathematics, Technical University of Moldova, and Institute of Mathematics and Computer Science, Chişinău, Moldova;*
[2] *Discipline of Mathematics, University of Tasmania, Hobart, Australia*
e-mail: elena.cojuhari@mate.utm.md, Barry.Gardner@utas.edu.au

A ring S with identity 1 is a *subnormalizing extension* of a subring R with the same identity if for some finite subset $\{x_1, x_2, \ldots, x_n\}$ or some countable subset $\{x_i \mid i = 1, 2, \ldots\}$ of elements of S, we have $S = \sum x_i R = \sum R x_i$ and $\sum_{j=1}^{i} x_j R = \sum_{j=1}^{i} R x_j$ for each i. A D-*structure* on a ring A with identity 1 is a family of self-maps σ_{gh} indexed by pairs of elements of a monoid G with identity e satisfying a large collection of rather natural conditions which allow the construction of a generalized monoid ring $A\langle G; \sigma\rangle$ with multiplication given by $(a \cdot x)(b \cdot y) = a \sum_{z \in G} \sigma_{x,z}(b) \cdot zy$.
In this talk all rings have identity but need not be commutative.
In many cases the ring $A\langle G; \sigma\rangle$ is a subnormalizing extension of A generated by G. On the other hand, if S is a subnormalizing extension of R, the x_i form a multiplicative submonoid of S, and generate S as a free left R-module, then as for each i and each $r \in R$ we have $x_i r = \sum_{j=1}^{i} r_{ij} x_j$ for unique elements r_{ij} of R, we can define $\sigma_{ij}(r) = r_{ij}$ when $i \geq j$ and 0 when $i < j$ and thereby obtain a collection of self-maps of R which turn out to satisfy most of the requirements of a D-structure. For several types of monoid they do form a D-structure. Whether this is so in all cases remains an open question.

Bibliography

[1] E. A. Whelan, *Finite subnormalizing extensions of rings*, J. Algebra 101(1986), 418-432.

[2] E. P. Cojuhari, *Monoid algebras over non-commutative rings*, Int. Electron. J. Algebra 2(2007), 28-53.

[3] E. P. Cojuhari and B. J. Gardner, *Generalized higher derivations*, Bull. Aust. Math. Soc. 86(2012), 266-281.

[4] E. P. Cojuhari and B. J. Gardner, *Skew ring extensions and generalized monoid rings*, Acta Math. Hungar. 154 (2018), no. 2, 343-361.

On generalization of expressibility in 5-valued logic

Ion Cucu

State University of Moldova, Chişinău, Republic of Moldova
e-mail: cucuion2012@gmail.com

The function f (the principal derivate operation) of algebra $\mathfrak{A}$ is called parametrically expressible via a system of functions Σ of $\mathfrak{A}$, if there exists the functions $g_1, h_1, ..., g_m, h_m$ which are expressed explicitly via Σ using superpositions, such that the predicate $f(x_1...x_n) = x_{n+1}$ is equivalent to the predicate $\exists t_1...\exists t_k ((g_1 = h_1) \wedge ... \wedge (g_m = h_m))$ on the algebra $\mathfrak{A}$.
Let us consider the pseudo-Boolean algebra $\langle M; \wedge, \vee, \supset, \neg \rangle$, where $\supset$ is relative pseudo-complement, and $\neg$ is pseudo-complement.
We say that the system of pseudo-Boolean terms on the set of variables $\mathcal{X}$ (Ω – words over $\mathcal{X}$) is parametrically complete in algebra $\langle M; \Omega \rangle$, if we can parametrically express the operations from Ω via functions expressed by terms over Σ. The function $f(x_1, ..., x_n)$ of M preserves the predicate (relation) $R(x_1, ..., x_m)$ if for any possible values $x_{ij} \in M$ $(i = 1, ..., m; j = 1, ..., n)$ from the truth of $R(x_{11}, x_{21}, ..., x_{n1}), ..., R(x_{1n}, x_{2n}, ..., x_{mn})$ follows the truth of
$R(f(x_{11}, x_{12}, ..., x_{1n}), ..., f(x_{n1}, x_{n2}, ..., x_{nm}))$.
The centralizer $\langle f(x_1, ..., x_n) \rangle$ coincides with the set of all functions of M, which preserve the predicate $f(x_1, ..., x_n) = x_{n+1}$, where the variable x_{n+1} differs from $x_1, ..., x_n$.
We examine the 5-valued pseudo-Boolean algebra $Z_5 = \langle \{0, \rho, \tau, \omega, 1\}; \wedge, \vee, \supset, \neg \rangle$, where $0 < \rho < \omega < 1$, $0 < \tau < \omega < 1$, ρ and τ are incomparable elements. The algebra $Z_3 = \langle \{0, \omega, 1\}; \wedge, \vee, \supset, \neg \rangle$ is a subalgebra of Z_5.
Let us define the function $\varphi(p)$ on Z_5 as follows:

$$\varphi(0) = 0, \varphi(\rho) = \tau, \varphi(\tau) = \rho, \varphi(\omega) = \varphi(1) = 1.$$

The logic of the algebra $\mathfrak{A}$ is defined as the set of all formulas that are true on $\mathfrak{A}$, i.e. formulas identically equal to the greatest element 1 of this algebra.

Theorem 1. *A system of formulas Σ is parametrically complete in the logic of algebra Z_5 iff Σ is parametrically complete in the logic of subalgebra Z_3 and the system Σ is not included into the centralizer $\langle \varphi(p) \rangle$ on algebra Z_5.*

Intermediate representation of hyperbolic manifolds by equidistant polyhedra

F. Damian[1], V.S. Makarov[2], P.V. Makarov[3]

[1]*Moldova State University,* [2]*Moscow State University,*
[3]*Moscow Mining State University*
e-mail: fl_damian@yahoo.com

The n-dimensional hyperbolic manifold is usually considered as a homogeneous complex. One compact polytope is sufficient to describe the manifold by pairwise identifying its faces. We discuss an "intermediate" way of representing the manifold by an equidistant (generalized) polyhedron over compact basis as a submanifold of codimension one.
First such representation was proposed for a symmetric 3-submanifold of the Davis hyperbolic

4-manifold on regular 120-cells. A totally geodesic 2-submanifold (or border), which is a surface of genus 4 with a Platonic map $\{5, 5\}$, serves as a compact basis for equidistant polyhedra. We call them also lens hyperbolic polytopes. Remark that from the combinatorial point of view, the above Platonic surface $\{5, 5\}$ of genus 4 is exactly the large star dodecahedron $\{5, 5/2\}$.
For some equidistant polyhedra with elliptic, parabolic and hyperbolic incidences of hyperfaces at vertices, respectively over Platonic maps $\{4, 5\}$, $\{5, 4\}$, $\{5, 5\}$ on surface of genus 4, we construct examples that lead to compact or non-compact hyperbolic 3-manifolds. The geometry of such manifolds is described. In dimension 4 the star regular polytope $\{5, 3, 5/2\}$, or 3-submanifold locally geodesic immersed in Davis 4-manifold, can be considered as a compact basis for an equidistant 4-dimensional polyhedron over Platonic map $\{5, 3, 5\}$.
In a general case, we start with cells complexes over regular (semiregular or k-regular) maps on totally geodesic hyperbolic submanifolds and indicate pairs of faces of the lens polytope that lead to hyperbolic manifolds. Thus, using the proposed method, we construct manifolds starting from their submanifolds not, as usually, from fundamental polytopes. Algebraic aspects of this approach (embedding and extensions of the fundamental group) are discussed.

Some properties of Neumann quasigroups

Natalia Didurik

University "Dmitry Cantemir", Republic of Moldova
e-mail:

Main concepts and definitions can be found in [1, 4, 6].

Definition 1. *Quasigroup $(Q, \cdot)$ is unipotent if and only if $x \cdot x = a$ for all $x \in Q$ and some fixed element $a \in Q$.*

Definition 2. *Quasigroup $(Q, \cdot)$ has right unit element (a right unit) if there exists element e (unique) such that $x \cdot e = x$ for all $x \in Q$.*

Definition 3. *A quasigroup $(Q, \cdot)$ is said to be Neumann quasigroup if in this quasigroup the identity*

$$x \cdot (yz \cdot yx) = z \tag{1}$$

holds true [3, 5, 2], [7, p. 248].

In the articles [3, 7, 2] the following result is pointed out.

Theorem 1. *If quasigroup $(Q, \cdot)$ satisfies the following identity*

$$xy \cdot z = y \cdot zx, \tag{2}$$

then (13)-parastrophe of this quasigroup satisfies Neumann identity (1).

Notice that the identity (2) has the following identity as its (13)-parastrophe : $(x \cdot yz) \cdot xy = z$.

Theorem 2. *If quasigroup $(Q, \cdot)$ satisfies the identity (2), then this quasigroup is an abelian group.*

Theorem 3. *Any Neumann quasigroup $(Q, \cdot)$ is isotope of an abelian group $(Q, +)$ of the form $x \cdot y = x - y$.*

Corollary 1. *Any Neumann quasigroup $(Q, \cdot)$ is unipotent and has right unit element.*

Bibliography

[1] V.D. Belousov. *Foundations of the Theory of Quasigroups and Loops.* Nauka, Moscow, 1967. (in Russian).

[2] A.A. Gvaramiya and M.M. Glukhov. A solution of the fundamental algorithmic problems in certain classes of quasigroups with identities. *Sibirsk. Mat. Zh.*, 10:297–317, 1969.

[3] G. Higman and B. H. Neumann. Groups as groupoids with one law. *Publ. Math. Debrecen*, 2:215–221, 1952.

[4] H.O. Pflugfelder. *Quasigroups and Loops: Introduction.* Heldermann Verlag, Berlin, 1990.

[5] A. Sade. Quasigroupes obéissant á certaines lois. *Rev. Fac. Sci. Univ. Istambul*, 22:151–184, 1957.

[6] Victor Shcherbacov. *Elements of Quasigroup Theory and Applications.* CRC Press, Boca Raton, 2017.

[7] Sh. K. Stein. On the foundations of quasigroups. *Trans. Amer. Math. Soc.*, 85(1):228–256, 1957.

Application of formal groups to reciprocity laws

O. Demchenko, S. Vostokov, R.P. Vostokova

St.Petrsburg State University, Rusia
e-mail: vasja@eu.spb.ru, sergei.vostokov@gmail.com

Formal groups are easily defined algebraic objects that have a wide range of applications in many field of mathematics from cobordism theory to number theory with the present talk being devoted to the latter. They are defined as formal power series F in two variables such that $F(x,0) = x$; $F(F(x,y),z) = F(x,F(y,z))$ and $F(x,y) = F(y,x)$. A relation between formal groups and reciprocity laws is investigated following the approach by Honda. Let ξ denote an m-th primitive root of unity. For a character χ of order m, we define two one-dimensional formal groups over $\mathbb{Z}[\xi]$ and prove the existence of an integral homomorphism between them with linear coefficient equal to the Gauss sum of χ. This allows us to deduce a reciprocity formula for the m-th residue symbol which, in particular, implies the cubic reciprocity law.

Some properties of a permutation representation of a group by cosets to its included subgroups

Kuznetsov Eugene

Institute of Mathematics and Computer Science, Academy of Sciences of Moldova
e-mail: kuznet1964@mail.ru

All necessary definitions and notations it can be found in [1,2].

Theorem 1. *Let G be a group and $H \subseteq K \subseteq G$ be its two included subgroups. Let set $T = \{t_{i,j}\}_{i\in E_1, j\in E_2}$ be a loop transversal in G to H and set $T_1 = \{t_{0,j}\}_{j\in E_2}$ be a corresponding loop transversal in K to H. So there exist the loop transversal operation $L = \langle E, \cdot\rangle$, corresponding to the transversal T, and its subloop - loop transversal operation $L_1 = \langle E_2, \cdot\rangle$, corresponding to the transversal T_1. Also there exist folllowing three permutation representations:*

1. a permutation representation $\hat{G}$ of the group G by the left cosets to its subgroup H;

2. a permutation representation $\check{G}$ of the group G by the left cosets to its subgroup K;

3. a permutation representation $\breve{L}$ of the loop L by the left cosets to its subloop L_1.

Then the following afirmations are true:

a The kernel $Core_G(H)$ of the permutation representation $\hat{G}$ is a multiplication group of the loop $Core_L(L_1)$ - the kernel of the permutation representation $\breve{L}$;

b For every $g \in G$:

$$\hat{g}(\langle x, y\rangle) = \langle u, v\rangle \Leftrightarrow \breve{g}(x) = u,\ \breve{g}(y) = v.$$

Bibliography

[1] Kuznetsov E., *Transversals in loops.1.Elementary properties*, Quasigroups and related systems, No. 1 **18**(2010), 43–58.

[2] Kuznetsov E., *Transversals in groups.1.Elementary properties*, Quasigroups and related systems, No. 1, **1**(1994), 22–42.

Left coquotient with respect to join in the class of preradicals in modules

Ion Jardan

Technical University of Moldova;
Institute of Mathematics and Computer Science, Chişinău, Republic of Moldova
e-mail: ion.jardan@mate.utm.md, jordanion79@gmail.com

The purpose of this communication is the definition and investigation of a new inverse operation in the class of preradicals $\mathbb{PR}$ of the category R-Mod of left R-modules.
In [1] and [2] two inverse operations, the left quotient with respect to join and the left coquotient with respect to meet, are defined and studied. These operations exist for any pair of preradicals of $\mathbb{PR}$. In [3] another inverse operation is introduced and investigated, namely the left quotient with respect to meet which, in contrast to the preceding cases, is partial.
In the present lecture in a similar manner as in [3], a new partial inverse operation is defined. We will show the criteria of existence of this operation, its main properties, the relations with the lattice operations of $\mathbb{PR}$ and some particular cases.
We remind that *a preradical* r in the category R-Mod is a subfunctor of identity functor of R-Mod, i.e. $r(M) \subseteq M$ and $f(r(M)) \subseteq r(M')$ for any R-morphism $f : M \to M'$ ([4]).
Definition. Let $r, s \in \mathbb{PR}$. The *left coquotient with respect to join of* r *by* s is defined as the greatest preradical among $r_\alpha \in \mathbb{PR}$ with the property $r_\alpha \# s \leq r$. We denote this preradical by $r \,{}^{\vee}\!\!/_{\#}\, s$ and we will say that r is the *numerator* and s is the *denominator* of the left coquotient $r \,{}^{\vee}\!\!/_{\#}\, s$.
Lemma 1.(Criteria of existence) Let $r, s \in \mathbb{PR}$. The left coquotient $r \,{}^{\vee}\!\!/_{\#}\, s$ of r by s with respect to join exists if and only if $r \geq s$ and it can be presented in the form
$r \,{}^{\vee}\!\!/_{\#}\, s = \vee\{r_\alpha \in \mathbb{PR} \mid r_\alpha \# s \leq r\}$.
Proposition 1. If $r, s \in \mathbb{PR}$, then for any preradical $t \in \mathbb{PR}$ we have:

$$r \geq t \# s \Leftrightarrow r \,{}^{\vee}\!/_{\#}\, s \geq t.$$

Some basic properties of this operation are elucidated. In particular:
1) the left coquotient $r \,{}^{\vee}\!/_{\#}\, s$ is monotone in the numerator and antimonotone in the denominator;
2) $(r \,{}^{\vee}\!/_{\#}\, s) \,{}^{\vee}\!/_{\#}\, t = r \,{}^{\vee}\!/_{\#}\, (t \# s)$; 3) $(r \# s) \,{}^{\vee}\!/_{\#}\, t \geq r \# (s \,{}^{\vee}\!/_{\#}\, t)$;
4) $(r \,{}^{\vee}\!/_{\#}\, t) \,{}^{\vee}\!/_{\#}\, (s \,{}^{\vee}\!/_{\#}\, t) \geq r \,{}^{\vee}\!/_{\#}\, s$; 5) $(r \# t) \,{}^{\vee}\!/_{\#}\, (s \# t) \geq r \,{}^{\vee}\!/_{\#}\, s$;
6) $\left(\underset{\alpha \in \mathfrak{A}}{\wedge} r_\alpha\right) \,{}^{\vee}\!/_{\#}\, s = \underset{\alpha \in \mathfrak{A}}{\wedge} (r_\alpha \,{}^{\vee}\!/_{\#}\, s)$; 7) $\left(\underset{\alpha \in \mathfrak{A}}{\vee} r_\alpha\right) \,{}^{\vee}\!/_{\#}\, s \geq \underset{\alpha \in \mathfrak{A}}{\vee} (r_\alpha \,{}^{\vee}\!/_{\#}\, s)$.

In continuation we consider the left coquotient $r \,{}^{\vee}\!/_{\#}\, s$ in some particular cases ($r = s$, $s = 1$, $r = 0$):
1) $r \,{}^{\vee}\!/_{\#}\, r = c(r)$; 2) $r \,{}^{\vee}\!/_{\#}\, 0 = r$; 3) $1 \,{}^{\vee}\!/_{\#}\, s = 1$,
where $c(r) = \vee \{ r_\alpha \in \mathbb{PR} \mid r_\alpha \# r = r \}$ is co-equalizer of r ([5]).

The co-equalizer of every preradical r is a radical. It is known that $r \in \mathbb{PR}$ is a radical if and only if $c(r) = r$ ([5]). We have $c(r) \leq r \,{}^{\vee}\!/_{\#}\, s \leq r$ and if r is a radical, then $r \,{}^{\vee}\!/_{\#}\, s = r$.
Further we indicate some properties of co-equalizer relative to the studied operation:

$$c(r) \# (r \,{}^{\vee}\!/_{\#}\, s) = r \,{}^{\vee}\!/_{\#}\, s; \quad (r \,{}^{\vee}\!/_{\#}\, s) \# c(s) = r \,{}^{\vee}\!/_{\#}\, s; \quad (r \,{}^{\vee}\!/_{\#}\, s) \,{}^{\vee}\!/_{\#}\, c(s) = r \,{}^{\vee}\!/_{\#}\, s.$$

The following statement show the behaviour of $r \,{}^{\vee}\!/_{\#}\, s$ in the case of some special types of preradicals (prime, $\wedge$-prime, irreducible ([6])).

Proposition 2. Let $r, s \in \mathbb{PR}$. Then:
1) If r is prime, then $r \,{}^{\vee}\!/_{\#}\, s$ is prime;
2) If r is $\wedge$-prime, then $r \,{}^{\vee}\!/_{\#}\, s$ is $\wedge$-prime;
3) If r is irreducible and $r = t \# s$ for some preradical $t \in \mathbb{PR}$, then $r \,{}^{\vee}\!/_{\#}\, s$ is irreducible.

The preradicals obtained by this inverse operation are arranged in the following order:

$$r \,{}^{\vee}\!/_{\#}\, s \leq (r \,{}^{\vee}\!/_{\#}\, s) \# s \leq r \leq (r \# s) \,{}^{\vee}\!/_{\#}\, s \leq r \# s.$$

Bibliography

[1] JARDAN I. *On the inverse operations in the class of preradicals of a module category, I.* Bul. Acad. Ştiinţe a Repub. Moldova. Mat., 2017, No. 1(83), 57-66.

[2] JARDAN I. *On the inverse operations in the class of preradicals of a module category, II.* Bul. Acad. Ştiinţe a Repub. Moldova. Mat., 2017, No. 2(84), 77-87.

[3] JARDAN I. *The left quotient with respect to meet.* Inter. Conf. of Math., Inf. and Inf. Tehn.(abstracts), Bălţi, Repub. of Moldova, 2018, 47-48.

[4] BICAN L., KEPKA T., NEMEC P. *Rings, modules and preradicals.* Marcel Dekker, New York, 1982.

[5] RAGGI F., RIOS J., RINCON H., FERNANDES-ALONSO R., SIGNORET C. *The lattice structure of preradicals II.* J. of Algebra and its Applications, Vol. 1., No. 2, 2002, 201-214.

[6] RAGGI F., RIOS J., RINCON H., FERNANDES-ALONSO R., SIGNORET C. *Prime and irreducible preradicals.* J. of Algebra and its Applications, 2005, Vol. 4, No. 4, 451-466.

About the generalized symmetry of geometric figures weighted regularly and easily by "physical" scalar tasks

Alexandru Lungu

Moldova State University, Chisinau, Republic of Moldova
e-mail:

Let us have geometrical figure F with discrete group of symmetry G and finite set $N = \{1, 2, ..., m\}$ of "indexes", which mean a non-geometrical feature. On fix a certain transitive group P of permutations over N. We will note with the symbol F_i the intersection of geometric figure F with the fundamental domain S_i of the group G. Ascribe to each interior point M of F_i the same "index" r from the set N . We obtain one figure $F^{(N)}$, weighted regularly and easily with summary load N.

Let each "index" r from the set N have a scalar nature (temperature, density, color). The mixed transformation $\tilde{g}$ of the "indexed" figure $F^{(N)}$ is composed of two independent components: $\tilde{g} = gw$, where g is pure geometrical isometric transformation and w is certain complex rule which describes the transformation of the "indexes". If the rule w is the same for every "indexed" point of $F^{(N)}$, then the mixed transformation $\tilde{g}$ is exactly a transformation of Zamorzaev's P-symmetry. The set of transformations of P-symmetry of "indexed" figure $F^{(N)}$ forms a minor or semi-minor group of P-symmetry , where is subgroup of the direct product of the group P with generating group G [1,3].

The "indexes" r_i and r_j, ascribed to the points which belong to distinct domains F_i and F_j, are transformed, in general, by different permutations p_i and p_j from group P. In this case the rule w is composed exactly from $|G|$ components-permutations $p \in P$. In conditions of this case the transformation $\tilde{g} = gw$ is exactly a transformation of W_p-symmetry [2-5]. The set of transformations of W_p-symmetry of the given "indexed" figure $F^{(N)}$ forms a semi-minor or pseudo-minor group of W_p-symmetry, where is subgroup of the left standard Cartaisian wreath product of groups P and G.

Bibliography

[1] A.M. Zamorzaev. *On the groups of quasi-symmetry (P-symmetry).* (Russian). Soviet. Phys. Crystallogr. vol. 12, no 5 (1967), pp. 819-825.

[2] V.A. Koptsik and I.N. Kotsev, *To the theory and classification of color symmetry Groups. II. W-symmetry.* (Russian). OIYI Reports, R4-8068, Dubna, 1974.

[3] A.M. Zamorzaev, Yu.S. Karpova, A.P. Lungu and A.F. Palistrant. *P-symmetry and its further development.* (Russian). Chisinau: Shtiintsa, 1986.

[4] A.P. Lungu, *Classification of groups of W-symmetry*, (Russian). Studies in modern algebra and geometry. p. 79-84, Chisinau: Shtiintsa, 1983.

[5] Alexandru Lungu, *Cristallografic punctual groups of cyclical W_p-symmetries*, (Romanian). Studia Universitatis Moldaviae, Seria: Shtiintse exacte shi economice, nr.2(102) (2017), pp.3-12.

Acknowledgement. *This work was partially supported by the project 15.817.02.26F.*

Some Properties of a Lattice Generated by Implicational Logics

Ion Negru

Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: ion.negru1941@gmail.com

Consider the following implicational formulae:

$$\boldsymbol{A}_1 = (\boldsymbol{p} \supset \boldsymbol{p}), \boldsymbol{A}_2 = (((\boldsymbol{p} \supset \boldsymbol{p}) \supset \boldsymbol{p}) \supset \boldsymbol{p}) = ((\boldsymbol{A}_1 \supset \boldsymbol{p}) \supset \boldsymbol{p}), \cdots,$$
$$\boldsymbol{A}_{i+1} = ((\boldsymbol{A}_i \supset \boldsymbol{p}) \supset \boldsymbol{p}), \cdots, (i = 1, 2, 3, \cdots)$$

Using these formulae (axioms), we may construct the following logics:

$$\boldsymbol{L}_1 =< \boldsymbol{A}_{2i} >, \boldsymbol{L}_2 =< (\boldsymbol{A}_{2i} \supset \boldsymbol{A}_{2i+1}) >,$$
$$\boldsymbol{L}_3 =< \boldsymbol{A}_{2i-1} >, \boldsymbol{L}_4 =< (\boldsymbol{A}_{2i-1} \supset \boldsymbol{A}_{2i}) >, i = 1, 2, 3, \cdots$$

viz. the logic $\boldsymbol{L}_1$ is generated by the axioms $\boldsymbol{A}_{2i}$, $i = 1, 2, 3, ...$; the process is analogous for logics $\boldsymbol{L}_2, \boldsymbol{L}_3, \boldsymbol{L}_4$.
The rule of deduction for these logics is unique - modus ponens: $\boldsymbol{A}, (\boldsymbol{A} \supset \boldsymbol{B}) \vdash \boldsymbol{B}$ (if the formulae $\boldsymbol{A}$ and $(\boldsymbol{A} \supset \boldsymbol{B}) \in$ to the given logic, then formula $\boldsymbol{B}$ also $\in$ to this logic).
Let $\boldsymbol{S}$ be the lattice generated by the logics $\boldsymbol{L}_1, \boldsymbol{L}_2, .\boldsymbol{L}_3, \boldsymbol{L}_4$. The following results are obtained:

1. Lattice $\boldsymbol{S}$ is infinite.
2. If logics $\boldsymbol{L}_1, \boldsymbol{L}_2, .\boldsymbol{L}_3, \boldsymbol{L}_4$ possess a finite number of axioms (viz. i = 1, 2, 3, ..., n), then the respective lattice $\boldsymbol{S}$ is finite.
3. For lattice $\boldsymbol{S}$ the problem of the equality of any two lattice elements is solvable.
4. If the rule of deduction - the substitution is added to the above logics, then statements 1)–3) are also true. (The rule of deduction the substitution means: if formula $\boldsymbol{A} \in$ to the given logic, then the result of the substitution in formula $\boldsymbol{A}$ of any implicational formula of the variable $\boldsymbol{p}$ for the same variable $\boldsymbol{p}$ also $\in$ to the same logic.)

On a Certain Property of the Elements of a Finitely Generated Lattice

Ion Negru

Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: ion.negru1941@gmail.com

Firstly, a reminder that a lattice is a set $\boldsymbol{S}$ of elements; it is partially ordered, closed in relation to two lattice operations: the reunion $\boldsymbol{a} + \boldsymbol{b}$ and the intersection $\boldsymbol{a} \cdot \boldsymbol{b}$ of any two elements $\boldsymbol{a}$ and $\boldsymbol{b}$ from set $\boldsymbol{S}$. (The reunion $\boldsymbol{a} + \boldsymbol{b}$ is the smallest element of the lattice containing both elements $\boldsymbol{a}$ and $\boldsymbol{b}$; the intersection $\boldsymbol{a} \cdot \boldsymbol{b}$ is the greatest element of the lattice contained in both elements $\boldsymbol{a}$ and $\boldsymbol{b}$. Obviously, $\boldsymbol{a} \leq \boldsymbol{a} + \boldsymbol{b}, \boldsymbol{b} \leq \boldsymbol{a} + \boldsymbol{b}, \boldsymbol{a} \geq \boldsymbol{a} \cdot \boldsymbol{b}, \boldsymbol{b} \geq \boldsymbol{a} \cdot \boldsymbol{b}$)
A lattice may also be defined thusly: the generating elements of the lattice are given. Other elements, different from the generators, are obtained via the two lattice operations, applied to the generators.

Statement. Let $\boldsymbol{a1, a2, a3, \cdots ,}$, an be the lattice generators. And let $\boldsymbol{T}$ be any element (term) of the lattice. The following takes place:

$$\boldsymbol{T \geq a_1 + a_2 + \cdots + a_{i-1} + a_{i+1} + \cdots + a_n}$$

or:

$$\boldsymbol{T \geq a_i, (i = 1, 2, 3, \cdots , n)}.$$

The statement is proven through the method of mathematical induction in relation to the length of element (term) $\boldsymbol{T}$. (Elements of length 1 are, evidently, the lattice generators. Any other element $\boldsymbol{T}$ of a length greater than 1 is presented as $\boldsymbol{T = T_1 + T_2}$ or $\boldsymbol{T = T_1 \cdot T_2}$, where the lengths of $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ are less than the length of element $\boldsymbol{T}$ - this presentation is deployed within the application of the method of mathematical induction.)

Example of ternary non-commutative Moufang loop

Vasilii I. Onoi

Moldova State Technical University
e-mail: `matematica@mail.utm.md`

It is demonstrated that there exist ternary Moufang loops that are different from ternary groups. Let $(K, +, \cdot, 1)$ be an associative ring (not necessary commutative) which has characteristic 3, i.e., $x + x + x = 0$ for all $x \in K$.
By $K'(\cdot)$ we denote an abelian subgroup of the group $K^*(\cdot)$. Here $K^* = K \setminus \{0\}$. The map $x \to s \cdot x$ for all $x \in K$ is a permutation for any $s \in K'$. Moreover, we require that $s^2 = 1$ for all $s \in K'$.
In particular case $K = Z_3$ is a ring of residues modulo 3.
On the set $Q = K' \times K = \{< s, k > \mid\ s \in K', k \in K\}$ we define the following ternary operation

$$A(< s_1, x_1 >, < s_2, x_2 >, < s_3, x_3 >) =< s_1s_2s_3, s_2x_1 + s_3x_2 + s_1x_3 > \qquad (1)$$

for all $s_1, s_2, s_3 \in K', x_1, x_2, x_3 \in K$.
Algebra $Q(A)$ with operation defined on the set $Q = K' \times K$ by the formula (1) is a ternary non-commutative Moufang loop that is not a ternary group.
Example of ternary commutative Moufang loop that is not a ternary group is also constructed.

Bibliography

[1] V.D. Belousov. *n-Ary Quasigroups*, Stiintsa, Kishinev, (1971).

On topological endomorphism rings with no more than two non-trivial closed ideals

Valeriu Popa

Institute of Mathematics and Computer Science, Moldova
e-mail: `vpopa@math.md`

Let $\mathcal{L}$ be the class of locally compact abelian groups. For $X \in \mathcal{L}$, we denote by $t(X)$ the torsion subgroup of X and by $E(X)$ the ring of continuous endomorphisms of X, taken with the compact-open topology. If X is topologically torsion, then $S(X)$ stands for the set of primes p such that the corresponding topological p-primary component of X is non-zero. Given a positive integer n, we set $nX = \{nx \mid x \in X\}$ and $X[n] = \{x \in X \mid nx = 0\}$.

Theorem 1. *Let n be a positive integer, and let X be a group in $\mathcal{L}$ such that $\overline{nX}$ is densely divisible and $t(X) = X[n]$. If $E(X)$ has no more than two non-trivial closed ideals, then X is either topologically torsion or topologically isomorphic with the topological direct product of a topologically torsion group by a group of the form $\mathbb{R}^d$, $\mathbb{Q}^{(\mu)}$, or $(\mathbb{Q}^*)^\mu$, where d is a positive integer and μ is a non-zero cardinal*

Theorem 2. *Let X be a group in $\mathcal{L}$ such that $E(X)$ has no more than two non-trivial closed ideals. If X is topologically torsion, then $|S(X)| \leq 2$. If X is topologically isomorphic with a group of the form $S \times T$, where T is topologically torsion and S is either $\mathbb{R}^d$ for some positive integer d, or $\mathbb{Q}^{(\mu)}$ or $(\mathbb{Q}^*)^\mu$ for some non-zero cardinal number μ, then $|S(X)| \leq 1$.*

On weak expressibility of formulas in the simplest non-trivial propositional provability logic

Andrei Rusu[1], Elena Rusu[2]

[1] *Ovidius University of Constanta, Romania*
[2] *Technical University of Moldova*
e-mail: agrusu@univ-ovidius.ro, elena.rusu@mate.utm.md

We consider the simplest non-trivial extension of the propositional provability logic, denoted by $G4$, which is based on variables and logical connectives $\&, \vee, \supset, \neg, \Delta$, axioms of the classical logic of propositions and Δ-axioms:

$$\Delta(p \supset q) \supset (\Delta p \supset \Delta q),\ \Delta p \supset \Delta\Delta p,$$

$$\Delta(\Delta p \supset p) \supset \Delta p,\ \Delta(p \supset p) \supset (p \supset p).$$

$$\Delta\Delta p,\ (\Box p \supset \Box q) \vee (\Box q \supset \Box p),$$

where $\Box p$ denotes $(p\&\Delta p)$. Rules of inference of $G4$ are the rules of the classical logic of propositions and the rule $\frac{A}{\Delta A}$.
They say formula F is weak-expressible by formulas of the system Σ in the logic L if F can be obtained from unary formulas of L and from formulas of Σ by applying the rule of weak substitution (which allows to pass from the formulas A and B to the result of the substitution of one of them in another one instead of all occurrences of the same variable, say p) and by the rule of replacement by an equivalent formula (which permit to pass from the formula A to an equivalent to it in L formula B).
They say the system of formulas Σ is complete with respect to weak-expressibility in the logic L if any formula of the calculus of L is weak-expressible via Σ in L.
In the present paper we found out the conditions for a system of formulas Σ to be complete as to weak-expressibility in the logic $G4$. **Acknowledgement.** *The work was partially supported by the research projects 18.50.07.10A/PS and 15.817.06.13A.*

The order of projective Edwards curve over $\mathbb{F}_{p^n}$ and embedding degree of this curve in finite field

Ruslan Skuratovskii

Institute of Mathematics of NAS of Ukraine, Kiev, Ukraine
e-mail: ruslan@imath.kiev.ua

Summary. We consider algebraic affine and projective curves of Edwards [9, 12] over a finite field F_{p^n}. Most cryptosystems of the modern cryptography [2] can be naturally transform into elliptic curves [11]. We research Edwards algebraic curves over a finite field, which at the present time is one of the most promising supports of sets of points that are used for fast group operations. We find not only a specific set of coefficients with corresponding field characteristics, for which these curves are supersingular but also a general formula by which one can determine whether a curve $E_d[\mathbb{F}_p]$ is supersingular over this field or not.
The embedding degree of the supersingular curve of Edwards over $\mathbb{F}_{p^n}$ in a finite field is investigated, the field characteristic, where this degree is minimal, was found.
The criterion of supersungularity of the Edwards curves is found over $\mathbb{F}_{p^n}$. Also the generator of crypto stable sequence on an elliptic curve with a deterministic lower estimate of its period is proposed.
Key words: finite field, elliptic curve, Edwards curve, group of points of an elliptic curve.
Results. We calculate the genus of curve according to Fulton citeF $\rho^*(C) = \rho_\alpha(C) - \sum\limits_{p\in E}\delta_p = \frac{(n-1)(n-2)}{2} - \sum\limits_{p\in E}\delta_p = 3-2 = 1$ because $n = 4$, where $\rho_\alpha(C)$ - the arithmetic type of the curve C, parameter $n = degC = 4$.
In order to detect supersingular curves, according to Koblitsa's study [10, 11], one can use the search for such parameters for which the curve and its corresponding twisded curve have the same number of solutions.

Theorem 1. *If* $p\ \equiv 3\ (mod\ 4)$ *and* p *is a prime number and* $\sum\limits_{j=0}^{\frac{p-1}{2}} (C^j_{\frac{p-1}{2}})^2 d^j \equiv 0(\bmod p)$ *then the order of the curve* $x^2+y^2 = 1+dx^2y^2$ *coincides with order of the curve* $x^2+y^2 = 1+d^{-1}x^2y^2$ *over* F_p *and equal to* $N_{E_d} = p+1$ *if* $p \equiv 3(\bmod 8)$*, and it equals to* $N_E = p-3$ *if* $p\equiv 7(\bmod 8)$*. Over the extended field* F_{p^n}*, where* $n \equiv 1(mod2)$ *order of this curve is* $N_E = p^n+1$*, if* $p \equiv 3(\bmod 8)$*, and it is* $N_E = p^n - 3$*, if* $p \equiv 7(\bmod 8)$*.*

Example 3. *A number of points for* $d = 2$ *and* $p = 31$ $N_{E_2} = N_{E_2^{-1}} = p-3 = 28$*.*

Corollary 1. *If coefficient* d *of* E_d *is such that* $\sum\limits_{j=0}^{\frac{p-1}{2}} (C^j_{\frac{p-1}{2}})^2 d^j \equiv 0(\bmod p)$*, then* E_d *has* $p-1-2(\frac{d}{p})$ *points over* F_p *and birational equivalent [1] curve* E_M *has* $p+1$ *points over* F_p*.*

Corollary 2. *If the coefficient of the curve satisfies the supersingularity equation* $\sum\limits_{j=0}^{\frac{p-1}{2}} (C^j_{\frac{p-1}{2}})^2 d^j \equiv 0(\bmod p)$ *studied in Theorem 1, then* E_d *has* $p-1-2(\frac{d}{p})$ *points over* F_p *a boundary-equivalent [8] curve with* $p+1$ *points over* F_p*.*

Theorem 2. *The number of points of the affine Edwards curve is equal to*

$$N_{E_d} = (p+1+(-1)^{\frac{p+1}{2}} \sum_{j=0}^{\frac{p-1}{2}} (C^j_{\frac{p-1}{2}})^2 d^j) \equiv ((-1)^{\frac{p+1}{2}} \sum_{j=0}^{\frac{p-1}{2}} (C^j_{\frac{p-1}{2}})^2 d^j + 1)(\mathrm{mod} p).$$

Theorem 3. *The number of points of the projective Edwards curve is equal to* $N_{E_d} = (p+1+2+(-1)^{\frac{p+1}{2}} \sum_{j=0}^{\frac{p-1}{2}} (C^j_{\frac{p-1}{2}})^2 d^j) \equiv ((-1)^{\frac{p+1}{2}} \sum_{j=0}^{\frac{p-1}{2}} (C^j_{\frac{p-1}{2}})^2 d^j + 3)(\mathrm{mod} p).$

Let curve contains a subgroup C_r of order r.

Definition 1. *We call the embedding degree a minimal power k of finite field extention such that can embedded in multiplicative group of* $\mathbb{F}_{p^k}$.

Let us obtain conditions of embedding [7] the group of supersingular curve $E_d[\mathbb{F}_p]$ of order q in multiplicative group of field $\mathbb{F}_{p^k}$ with embedding degree $k = 12$ [5]. For this goal we use Zigmondy theorem. This theorem implies that suitable characteristic of field $\mathbb{F}_p$ is an arbitrary prime q, which do not divide 12 and satisfy the condition $q\,|{}_{12}(p)$, where ${}_{12}(x)$ is the cyclotomic polynom. This p will satisfy the necessary conditions namely $(x^n - 1) \not\vdots p$ for an arbitrary $n = 1, ..., 11$.

Corollary 3. *The embedding degree [7] of the supersingular curve* $E_{1,d}$ *is equal to 2.*

Theorem 4. *If Edwards curve over finite field* F_p*, where* $p \equiv 7(mod 8)$ *is supersingular and* $p - 3 = 4q$*, where* $p,\ q \in P$*, then it has minimal cofactor* 4.

Theorem 5. *An arbitrary point of a twisted Edwards curve (1), which is not a point of the 2nd or 4th order, admits divisibility [4] if and only if* $\left(\frac{1-aX^2}{p}\right) \neq -1$.

We propose the generator of pseudo random sequence [13].

Take the elliptic curve of a given large simple order q [3], where $p \neq q$. As a one-sided, take the function: $P_i = f(P_{i-1}) = \phi(P_{i-1})G$, where $\phi(P_{i-1}) = x$, if $P_{i-1} = (x,\ y)$
and p, if $P_{i-1} = O$.

Apply the generation formula $P_i = f(P_{i-1}) = \phi(P_{i-1})G$. Therefore, the complexity of the inverse of this function is equivalent to the problems of a discrete logarithm.

A possible modification is the choice of the coordinate of the point $_i$ which gcd with $|E_d|$ is lesser. Otherwords, let $t := \underset{z\in\{x,y\}}{Argmin}$ $(\gcd(x, |E_d|), \gcd(y, |E_d|))$ and as a factor we take:

$$\phi(P_{i-1}) = \begin{cases} t, & P_{i-1} = (x,\ y) \\ p, & P_{i-1} = O. \end{cases}$$

Conclusions. Apply the generation formula $P_i = f(P_{i-1}) = \phi(P_{i-1})G$. Therefore, the complexity of the inverse of this function is equivalent to the problems of a discrete logarithm.

Bibliography

[1] Bernstein Daniel J., Birkner Peter, Joye Marc, Lange Tanja, Peters Christiane. *Twisted Edwards Curves.* IST Programme ECRYPT, and in part by grant ITR-0716498, 2008. 1-17.

[2] Skuratovskii R. V., *Modernized Pohlig-Hellman and Shanks algorithm*, Vol. 1 Visnuk of KNU. Cybernetics. pp. 56., 2015.

[3] Skuratovskii R. V., Movchan P. V.,*Normalizatsiya skruchenoyi kryvoyi Edvardsa ta doslidzhennya yiyi vlastyvostey nad Fp T*, Zbirnyk prats 14 Vseukrayinskoyi. FTI NTUU "KPI" 2016, Tom 2, S. 102-104.

[4] Skuratovskii R. V., Kvashuk D. M., *Vlastyvosti skruchenoyi kryvoyi Edvardsa, mozhlyvist podilu yiyi tochky na dva i zastosuvannya*, Zbirnyk naukovyx prac, Problemy informatyzaciyi ta upravlinnya.. 2017.4(60).S. 61-72.

[5] R. V. Skuratovskii, *Structure and minimal generating sets of Sylow 2-subgroups of alternating groups*, Sao Paulo Journal of Mathematical Sciences. (2018), no. 1, pp. 1-19. Source: https://link.springer.com/article/10.1007/s40863-018-0085-0.

[6] R. V. Skuratovskii, U. V. Skruncovich, *Twisted Edwards curve and its group of points over finite field F_p*, Akademgorodok, Novosibirsk, Russia. Conference. Graphs and Groups, Spectra and Symmetries. http://math.nsc.ru/conference/g2/g2s2/exptext/SkruncovichSkuratovskii-abstract-G2S2.pdf

[7] Paulo S. L. M. Barreto Michael Naehrig, *Pairing-Friendly Elliptic Curves of Prime Order*, International Workshop on Selected Areas in Cryptography SAC 2005: pp. 319-331.

[8] W. Fulton, *Algebraic curves. An Introduction to Algebraic Geometry*, 2008.

[9] H. Edwards, *A normal form for elliptic curves. American Mathematical Society.*, 2007, Volume 44, Number 3, July, pp. 393-422.

[10] Koblitz N., *Eliptic Curve Cryptosystems*, Mathematics of Computation, **48**(177), 1987, pp.203-209.

[11] A. A. Bolotov, S. B. Gashkov, A. B. Frolov, A. A. Chasovskikh, *Elementarnoye vvedeniye v ellipticheskuyu kriptografiyu*, KomKnika. Tom 2., 2006. p. 328.

[12] Deepthi P.P., Sathidevi P.S., *New stream ciphers based on elliptic curve point multiplication*, Computer Communications (2009). pp 25-33.

[13] Shafi Goldwasser, Mihir Bellare., *Lecture Notes on Cryptography*, Cambridge, Massachusetts, July 2008. p. 289.

Minimal generating set and properties of commutator of Sylow subgroups of alternating and symmetric groups

Ruslan Skuratovskii

Institute of Mathematics of NAS of Ukraine, Kiev, Ukraine
e-mail: ruslan@imath.kiev.ua

Summary. Given a permutational wreath product sequence of cyclic groups [12, 6] of order 2 we research a commutator width of such groups and some properties of its commutator subgroup. Commutator width of Sylow 2-subgroups of alternating group A_{2^k}, permutation group S_{2^k} and $C_p \wr B$ were founded. The result of research was extended on subgroups $(Syl_2 A_{2^k})'$, $p > 2$. The paper presents a construction of commutator subgroup of Sylow 2-subgroups of symmetric and alternating groups. Also minimal generic sets of Sylow 2-subgroups of A_{2^k} were founded. Elements presentation of $(Syl_2 A_{2^k})'$, $(Syl_2 S_{2^k})'$ was investigated. We prove that the commutator width [14] of an arbitrary element of a discrete wreath product of cyclic groups C_{p_i}, $p_i \in \mathbb{N}$ is 1.
Let G be a group. The commutator width of G, $cw(G)$ is defined to be the least integer n, such

that every element of G' is a product of at most n commutators if such an integer exists, and $cw(G) = \infty$ otherwise. The first example of a finite perfect group with $cw(G) > 1$ was given by Isaacs in [9].

A form of commutators of wreath product $A \wr B$ was briefly considered in [7]. For more deep description of this form we take into account the commutator width ($cw(G)$) which was presented in work of Muranov [14]. This form of commutators of wreath product was used by us for the research of $cw(Syl_2 A_{2^k})$, $cw(Syl_2 S_{2^k})$ and $cw(C_p \wr B)$. As well known, the first example of a group G with $cw(G) > 1$ was given by Fite [4]. We deduce an estimation for commutator width of wreath product $B \wr C_p$ of groups C_p and an arbitrary group B taking into the consideration a $cw(B)$ of passive group B.

A research of commutator-group serves to decision of inclusion problem [5] for elements of $Syl_2 A_{2^k}$ in its derived subgroup $(Syl_2 A_{2^k})'$.

Results. We consider $B \wr (C_p, X)$, where $X = \{1, .., p\}$, and $B' = \{[f, g] \mid f, g \in B\}$, $p \geq 1$. If we fix some indexing $\{x_1, x_2, ..., x_m\}$ of set the X, then an element $h \in H^X$ can be written as $(h_1, ..., h_m)$ for $h_i \in H$.

The set X^* is naturally a vertex set of a regular rooted tree, i.e. a connected graph without cycles and a designated vertex v_0 called the root, in which two words are connected by an edge if and only if they are of form v and vx, where $v \in X^*$, $x \in X$. The set $X^n \subset X^*$ is called the n-th level of the tree X^* and $X^0 = \{v_0\}$. We denote by $v_{j,i}$ the vertex of X^j, which has the number i. Note that the unique vertex $v_{k,i}$ corresponds to the unique word v in alphabet X. For every automorphism $g \in AutX^*$ and every word $v \in X^*$ define the section (state) $g_{(v)} \in AutX^*$ of g at v by the rule: $g_{(v)}(x) = y$ for $x, y \in X^*$ if and only if $g(vx) = g(v)y$. The subtree of X^* induced by the set of vertices $\cup_{i=0}^{k} X^i$ is denoted by $X^{[k]}$. The restriction of the action of an automorphism $g \in AutX^*$ to the subtree $X^{[l]}$ is denoted by $g_{(v)}|_{X^{[l]}}$. A restriction $g_{(v)}|_{X^{[1]}}$ is called the vertex permutation (v.p.) of g in a vertex v.

The set X^* is naturally a vertex set of a regular rooted tree, i.e. a connected graph without cycles and a designated vertex v_0 called the root, in which two words are connected by an edge if and only if they are of form v and vx, where $v \in X^*$, $x \in X$. The set $X^n \subset X^*$ is called the n-th level of the tree X^* and $X^0 = \{v_0\}$. We denote by $v_{j,i}$ the vertex of X^j, which has the number i.

The commutator length of an element g of the derived subgroup of a group G is denoted *clG(g)*, is the minimal n such that there exist elements $x_1, ..., x_n, y_1, ..., y_n$ in G such that $g = [x_1, y_1]...[x_n, y_n]$. The commutator length of the identity element is 0. The commutator width of a group G, denoted $cw(G)$, is the maximum of the commutator lengths of the elements of its derived subgroup $[G, G]$.

Let us make some notations. The commutator of two group elements a and b, denoted

$$[a, b] = aba^{-1}b^{-1},$$

conjugation by an element b as

$$a^b = bab^{-1},$$

$\sigma = (1, 2, \dots, p)$. Also $G_k \simeq Syl_2 A_{2^k}$, $B_k = \wr_{i=1}^{k} C_2$. The structure of G_k was investigated in [6]. For this research we can regard G_k and B_k as recursively constructed i.e. $B_1 = C_2$, $B_k = B_{k-1} \wr C_2$ for $k > 1$, $G_1 = \langle e \rangle$, $G_k = \{(g_1, g_2)\pi \in B_k \mid g_1 g_2 \in G_{k-1}\}$ for $k > 1$.

The following Lemma follows from the corollary 4.9 of the Meldrum's book [7].

Lemma 1. *An element of form $(r_1, \dots, r_{p-1}, r_p) \in W' = (B \wr C_p)'$ iff product of all r_i (in any order) belongs to B', where B is an arbitrary group.*

Proof. Analogously to the Corollary 4.9 of the Meldrum's book [7] we can deduce new presentation of commutators in form of wreath recursion

$$w = (r_1, r_2, \ldots, r_{p-1}, r_p),$$

where $r_i \in B$. □

Lemma 2. *For any group B and integer $p \geq 2$, $p \in \mathbb{N}$ if $w \in (B \wr C_p)'$ then w can be represented as the following wreath recursion*

$$w = (r_1, r_2, \ldots, r_{p-1}, \prod_{j=1}^{k}[f_j, g_j]),$$

where $r_1, \ldots, r_{p-1}, f_j, g_j \in B$, and $k \leq cw(B)$.

Lemma 3. *An element $(g_1, g_2)\sigma^i \in G'_k$ iff $g_1, g_2 \in G_{k-1}$ and $g_1 g_2 \in B'_{k-1}$.*

Lemma 4. *For any group B and integer $p \geq 2$ inequality*

$$cw(B \wr C_p) \leq \max(1, cw(B))$$

holds.

Corollary 1. *If $W = C_{p_k} \wr \ldots \wr C_{p_1}$ then for $k \geq 2$ $cw(W) = 1$.*

Corollary 2. *Commutator width $cw(Syl_p(S_{p^k})) = 1$ for prime p and $k > 1$ and commutator width $cw(Syl_p(A_{p^k})) = 1$ for prime $p > 2$ and $k > 1$.*

Theorem 1. *Elements of $Syl_2 S'_{2^k}$ have the following form $Syl_2 S'_{2^k} = \{[f, l] \mid f \in B_k, l \in G_k\} = \{[l, f] \mid f \in B_k, l \in G_k\}$.*

For the group G''_k we denote by s_{ij} vertex permutation of automorphism in v_{ij}.

Lemma 5. *The group G''_k has equal permutation in vertices of X^2, viz $s_{21} = s_{22} = s_{23} = s_{24}$.*

Theorem 2. *Commutator width of the group $Syl_2 A_{2^k}$ equal to 1 for $k \geq 2$.*

Proposition 1. *The subgroup $(syl_2 A_{2^k})'$ has a minimal generating set of $2k - 3$ generators.*

Conclusion . The commutator width of Sylow 2-subgroups of alternating group A_{2^k}, permutation group S_{2^k} and Sylow p-subgroups of $Syl_2 A_p^k$ ($Syl_2 S_p^k$) is equal to 1. Commutator width of permutational wreath product $B \wr C_n$, were B is an arbitrary group, was researched.

Bibliography

[1] R. V. Skuratovskii, *Structure and minimal generating sets of Sylow 2-subgroups of alternating groups.* Sao Paulo Journal of Mathematical Sciences. (2018), no. 1, pp. 1-19. Source: https://link.springer.com/article/10.1007/s40863-018-0085-0.

[2] R. Skuratovskii, *Generators and relations for Sylows p-subgroup of group S_n.*(in ukranian) Naukovi Visti KPI. 2013, N. 4 pp. 94 -105.

[3] Alexey Muranov, *Finitely generated infinite simple groups of infinite commutator width.* arXiv:math/0608688v4 [math.GR] 12 Sep 2009.

[4] W. B. Fite, *On metabelian groups*, Trans. Amer. Math. Soc, 3 (1902), pp. 331-353.

[5] Roger C. Lyndon Paul E. Schupp, *Combinatorial group theory.* Springer-Verlag Berlin Heidelberg New York 1977. 447 p.

[6] R. V. Skuratovskii, *Structure and minimal generating sets of Sylow 2-subgroups of alternating groups.* Source: https://arxiv.org/abs/1702.05784v2

[7] J.D.P. Meldrum, *Wreath Products of Groups and Semigroups.* Pitman Monographs and Surveys in Pure and Applied Mathematic English. 1st Edition. Jun (1995). 425 p.

[8] Nikolay Nikolov, *On the commutator width of perfect groups.* Bull. London Math. Soc. 36 (2004) p. 30 V36.

[9] I. M. Isaacs, *QCommutators and the commutator subgroup R*, Amer. Math. Monthly 84 (1977) P. 720 V722.

[10] I.V. Bondarenko, I.O. Samoilovych, *On finite generation of self-similar groups of finite type.* Int. J. Algebra Comput. February (2013), Volume 23, Issue 01, pp. 69-77

[11] R. Skuratovskii, *Corepresentation of a Sylow p-subgroup of a group S_n.* Cybernetics and systems analysis, (2009), N. 1, pp. 27-41.

[12] R. Skuratovskii, *Generators and relations for sylows o-subgroup of group S_n.* Naukovi Visti KPI. **4** (2013), pp. 94–105. (in Ukrainian)

[13] R.V. Skuratovskii, Y.A. Drozd, *Generators and relations for wreath products of groups.* Ukr Math J. (2008), vol. 60. Issue 7, pp. 1168 V1171.

[14] Alexey Muranov, *Finitely generated infinite simple groups of infinite commutator width.* arXiv:math/0608688v4 [math.GR] 12 Sep 2009.

On generalized multiplication groups of the commutative Moufang loops

Syrbu P., Grecu I.

State University of Moldova, Chişinău, Republic of Moldova
e-mail: syrbuviv@yahoo.com

The multiplication groups of quasigroups, i.e. the groups generated by all left and right translations, represent an efficient tool in the theory of quasigroups (loops). For example, using the multiplications groups, the nilpotency of loops, in particular of commutative Moufang loops, is studied in [1,3,4], some invariants under the isostrophy of Bol loops are found in [5].
Belousov considered in [2] the groups, generated by all left, right and middle translations of a quasigroup, called the generalized multiplication group. He remarked that these groups are invariant under parastrophy of quasigroups, and found a set of generators for the stabilizer of a fixed element in the generalized multiplication group. The generalized multiplication groups and the generalized inner mapping groups are invariant under the isostrophy of loops [6]. Also, it was shown in [6] that the center $\overline{Z}(Q)$ of the generalized multiplication group GM(Q) of a loop Q defines a normal subloop, which consists of those elements of the loop, which are invariant under all mappings from the generalized inner mapping group $\overline{J}(Q)$ of Q.
The generalized multiplication group and the generalized inner mapping group of a commutative Moufang loop Q are considered in the present work. In particular it is proved that:

1. $\overline{J}(Q) \leq Aut(Q)$;

2. $GM(Q/\overline{Z}(Q)) \cong GM(Q)/\overline{Z}(Q)$;

3. $\overline{J}(Q/\overline{Z}(Q)) \cong \overline{J}(Q)/(\overline{Z}^*(Q) \cap \overline{J}(Q))$ and $\overline{Z}^*(Q) \cap \overline{J}(Q) \subseteq Z(\overline{J})$, where $\overline{Z}^*(Q)) = \{\alpha \in GM(Q) \mid \overline{Z}(Q)\alpha(x) = \overline{Z}(Q)x, \forall x \in Q\}$.

Bibliography

[1] Bruck R. H. *A Survey of Binary Systems.* Springer-Verlag, New-York, 1971.

[2] Belousov V. *On the associated groups of quasigroups* (Russian), Matem. issled. 1969, 21-39.

[3] Niemenmaa M., Rytty M. *Centrally nilpotent finite loops.* Quasigroups and Related Systems 19 (2011), 123-132.

[4] Csorgo P. *Abelian inner mappings and nilpotency class grater than two.* European J. Combin., 28(2007), 858-867.

[5] Grecu I., Syrbu P. *On some isostrophy invariants of Bol loops.* Bull. Transylv. Univ. Braşov, Ser. C. – 2012, **5**, 145–154.

[6] Syrbu P. *On the generalized multiplication groups of loops.* (Submitted)

Definition and example of n-ary Moufang loop

Leonid Ursu

Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: matematica@mail.utm.md

Summary. In this work necessary and sufficient conditions that isotope of n-IP-loop ($n \in N^*$, $n > 3$) is also n-IP-loop are proved. Definition of n-ary Moufang loop is given, example of such loop is constructed.

Keywords: n-IP-quasigroup, n-IP-loop, Moufang loop, isotopy, LP-isotopy.

Main concepts and definitions. Quasigroup $Q(A)$ of arity n, $n \geq 2$, is called an n-IP-quasigroup if there exist permutations ν_{ij}, $i, j \in \overline{1,n}$ of the set Q such that the following identities are true:

$$A(\{\nu_{ij}x_j\}_{j=1}^{i-1}, A(x_1^n), \{\nu_{ij}x_j\}_{j=i+1}^{n}) = x_i$$

for all $x_1^n \in Q^n$, where $\nu_{ii} = \nu_{i\,n+1} = \varepsilon$, ε denotes identity permutation of the set Q [1]. The matrix

$$\left[\nu_{ij}\right] = \begin{bmatrix} \varepsilon & \nu_{12} & \nu_{13} & \dots & \nu_{1n} & \varepsilon \\ \nu_{21} & \varepsilon & \nu_{23} & \dots & \nu_{2n} & \varepsilon \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \nu_{31} & \nu_{32} & \varepsilon & \dots & \varepsilon & \varepsilon \end{bmatrix}$$

is called *an inversion matrix* for an n-IP-quasigroup, the permutations $\nu_{i,j}$ are called *inversion permutations.* An element e is called a unit of n-ary operation $Q()$, if the following equality is true $(\overset{i-1}{e}, x, \overset{n-i}{e}) = x$ for all $x \in Q$ and $i \in \overline{1,n}$. n-Ary quasigroup with unit element is called an n-ary loop [1].

Permutations I_{ij} of the set Q are defined by equities

$$(\overset{i-1}{e}, x, \overset{j-i-1}{e}, I_{ij}x, \overset{n-j}{e}) = e$$

for all $x \in Q$ and $i, j \in \overline{1,n}$.

N-ary quasigroup (Q, B) is isotopic to n-ary quasigroup (Q, A) (the number n is the same in both quasigroups) if there exist a tuple of permutations $T = (\alpha_1^{n+1})$ of the set Q such that

$$B(x_1^n) = \alpha_{n+1}^{-1} A(\alpha_1 x_1, \dots, \alpha_n x_n).$$

In this case we write $B = A^T$ [1]. Isotope of the form $T = (\alpha_1^n, \varepsilon)$ is called main isotope.

As usual $\overline{a} = a_1^n$. Main isotope is called LP-isotope, if $\alpha_i = L_i^{-1}(\overline{a})$ for all $i \in \overline{1,n}$, where $L_i(\overline{a})x = A(a_1^{i-1}, x, a_{i+1}^n)$.

LP-isotope of a quasigroup (Q, A) is a loop with unit $f = A(a_1^n)$ [1]. If $A = B$, then the tuple T is called autotopy of n-ary quasigroup A.

Main results

Theorem 1. *LP-isotope* $B = A^{(L_1^{-1}(\overline{a}), L_2^{-1}(\overline{a}), \dots, L_n^{-1}(\overline{a}), \varepsilon)}$ *of n-IP-loop A with unit e is an n-IP-loop if and only if*

$$T_i = (\{I_{ij}^e L_j^{-1}(\overline{a}) I_{ij}^f L_j(\overline{a})\}_{j=1}^{i-1}, L_i(\overline{a}), \{I_{ij}^e L_j^{-1}(\overline{a}) I_{ij}^f L_j(\overline{a})\}_{j=i+1}^{n}, L_i^{-1}(\overline{a})),$$

$i \in \overline{1,n}$, *are autotopies of n-IP-loop* (Q, A) *for any fixed element* $a \in Q$, *where* $[I_{ij}^e]$ *is inversion matrix for* (Q, A), $[I_{ij}^f]$ *is inversion matrix for* (Q, B) *[2].*

If $n = 3$, then we obtain results from [3], if $n = 2$, then autotopies T_i are transformed into the well known Moufang identities.

Definition 1. *n-Loop* (Q, A) $(n > 2)$ *is called n-ary Moufang loop, if any its LP-isotope is an n-IP-loop.*

In contrast to the binary case, for $n > 2$ a Moufang loop is not an IP-loop [1]. If (Q, A) is an n-IP-loop, then we call it n-IP Moufang loop.

Example 4. *Let* $A(x_1^4) = (x_1^4) = x_1 \cdot x_2 \cdot x_3 \cdot x_4$ *be a 4-IP-quasigroup which is defined over a binary Abelian group* $(Q, \cdot)$.

It is possible to check that (Q, A) *is a 4-IP-loop with unit e that coincides with the unit of the group* $(Q, \cdot)$ *and with invertible matrix* $[I_{ij}^e]$. *We suppose that* $I_{ij}^e L_j^{-1}(\overline{a}) I_{ij}^f L_j(\overline{a}) = \varphi_{ij}(\overline{a})$.

Then autotopy T_1 *from Theorem 1 is transformed into the following identity:*

$$(((x_1, a_2, a_3, a_4), x_2, x_3, x_4), a_2, a_3, a_4) = (x_1, \varphi_{12}(\overline{a})x_2, \varphi_{13}(\overline{a})x_3, \varphi_{14}(\overline{a})x_4).$$

This identity is true if we take $\varphi_{12}(\overline{a})x = a_2 \cdot a_3 \cdot a_4 \cdot x$, $\varphi_{13}(\overline{a})x = x$, $\varphi_{14}(\overline{a})x = x \cdot a_2 \cdot a_3 \cdot a_4$.

Similarly we see that permutations T_2, T_3, T_4 *are autotopies of loop* (Q, A). *Therefore* (Q, A) *is a symmetric 4-IP-Moufang-loop.*

In the similar way it is possible to construct an n-IP-Moufang-loop of any arity n.

Bibliography

[1] V.D. Belousov. *n-Ary Quasigroups*, Stiintsa, Kishinev, (1971).

[2] L.A. Ursu. About one special inversion matrix of non-symmetric n-IP-loop. *Proceedings of the 4-th Conference of Mathematical Society of the Republic of Moldova*, Chişinău, June 28-July 2, 2017, p. 165-168.

[3] L.A. Ursu. Definition and example of ternary Moufang loop. *International Conference on Mathematics, Informatics and Information Technologies dedicated to the Illustrious Scientist Valentin Belousov, April 19 - April 21, 2018, Bălţi, Communications, 2018, p. 96-97.*

On free nilpotent commutative automorphic loops

Vasile Ursu

"Simion Stoilov" Institute of Mathematics of Romanian Academy, Bucharest, Romania;
Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: Vasile.Ursu@imar.ro

In [4], there was announced (see in [5] her presentation) about one correspondence between the class of all commutative rings with a unit and one finitely axiomatizable class of loops. Moreover, the correspondence by the ring of integers corresponds to a metabelian commutative free automorphic loop of rank two. We derive some equational properties of this loop and prove, that the elementary theory free n-nilpotent commutative and non-associative automorphic loop are recursively undecidable.

1. As is well known (see [1]), the inner mapping group $J(L)$ of loop L is the group generated by all mappings of the form $T_a = R_a L_a^{-1}$, $L_{a,b} = L_b L_a L_{ab}^{-1}$, $R_{a,b} = R_a R_b R_{ab}^{-1}$, where $aL_b = bR_a = ab$ $(a, b \in L)$.

The subloop H of the loop L is normal in L if one of the following two equivalent conditions is satisfied:

(i) $H \cdot xy = Yx \cdot y$, $xy \cdot H = x \cdot yH$, $xh = Hx$ for any $x, y \in L$;

(ii) $H\alpha = H$ for any $\alpha \in J(L)$.

A loop L is said to be an automorphic loop (or A-loop) if any inner permutation from $J(L)$ is an automorphism on L [1].

2. The equational theory of automorphic loops was studied in [3]. Here the left associator and right associator $[a, b, c]$ of any three elements (a, b, c) of any three elements of the loop, and also the commutator $[a, b]$ of any two elements of the loop by the following equalities:

$$(a, b, c) = a\backslash((ab \cdot c)/(bc)), \quad [a, b, c] = ((ab)\backslash(a \cdot bc))/c, \; [a, b] = a\backslash(b\backslash(ab)).$$

Then the internal mappings of the form $L_{a,b}$, $R_{a,b}$, T_a, of the loop L are expressed in terms of associator and commutator formulas

$$c\,L_{a,b} = c(c, a, b), \; cR_{a,b} = [a, b, c]c, \; aT_b = a \cdot a[a, b].$$

Hence it follows that the class of all automorphic loops is a variety defined by the identities in the class of all loops:

$$xy \cdot [xy, z] = x[x, z] \cdot y[y, z];$$

$$xy \cdot (xy, z, t) = x(x, z, t) \cdot y(y, z, t);$$

$$[x, y, zt] \cdot zt = [x, y, z]z \cdot [x, y, t]t.$$

The subloop L' of the loop L generated by all the associators and commutators is called the associant-commutant of the loop L.

Let L be a free automorphic loop of rank $r \geq 2$ and $L_{(n)}$, $1 \leq n < \omega$ subloop of the loop L defined inductively: $L_{(1)} = L'$, $L_{(n)}$ for the $1 < n < \omega$ – is a subloops generated in a loop L by all associators and commutators of the form (x, y, z), $[y, z, x]$, $[x, y]$, where $x \in L_{(n-1)}$, $y, z \in L$. Obviously, all the subloops $L_{(n)}$ are closed with respect to the inner permutations of the group $J(L)$, therefore they are normal subloops. Loops isomorphic factor-loops $L/L_{(n)}$ of a free automorphic loop L by its normal subloop $L_{(n)}$ are called free automorphic loops nilpotent of class n. Loops

$L/L_{(2)}$, as well as any nonassociative or noncommutative loop, is isomorphic to the homomorphic image of one of these loops (as in groups) we call metabelian automorphic loops. Now it is easy to prove

Proposition 1. *In the class of all loops the basis of identities of a free metabelian automorphic loop $L/L_{(2)}$ consists of the following:*

$$[xy,z] = [x,z][y,z], \quad (xy,z,t) = (x,z,t)(y,z,t), \quad (x,y,zt) = (x,y,z)(x,y,t).$$

It follows from [3] that metabelian loops possess the following equational properties, which we present in the following two sentences.

Proposition 2. *In any metabelian loop, the following commutator identities are true:*

$$[y,x]^{-1} = [y^{-1},x], \quad [x/y,z] = [y\backslash x,z] = [x\cdot y^{-1},z],$$

$$[x\backslash y,z] = [x,z\backslash y] = [xy^{-1},y], \quad [x^m,y] = [x,y]^m \textit{ for any integer } m.$$

Proposition 3. *In any metabelian loop, associative identities are true:*

$$(x,y,z) = (y,x,z)(x,z,y), \quad (x,y,z) = (z,y,x)^{-1},$$

$$(x,y,z)^{-1} = (x^{-1},y,z), \quad (x^m,y,z) = (x,y,z)^m \textit{ for any integer } m.$$

3. In [4], a correspondence is established between commutative rings with a unit and metabelian loops of a singularly finitely axiomatizable class. In particular, the ring of integers Z corresponds to a free metabelian commutative loop of the second rank. An effective method is given that allows for each closed formula φ, the signature language of a ring with unity, obtain the formula φ_1 for commutative loops, such that the truth φ on the ring Z is equivalent to the truth φ_1 on the loop F_2. Since the elementary theory of the ring of integers is unsolvable (see [2]), we immediately obtain

Lemma 6. *The elementary theory of a free metabelian commutative* auto-morphic *loop F_2 of rank 2 is undecidable.*

Naturally, a more general

Lemma 7. *Elementary theory of a free metabelian commutative* auto-morphic *loop F_n with n free generators for $n \geq 3$ undecidable.*

Using the induction method, on the basis of Lemma 1, 2 and Propositions $1-3$, we arrive at the conclusion ehat the following is true:

Theorem 1. *The elementary theory of a free k-nilpotent commutative automorphic loop F_n with n free generators is undecidable for $n \geq 2$, $k \geq 2$.*

Bibliography

[1] R.H. Bruck, *A survey of binary systems*, Berlin - Heidelberg - New York, Springers Verlag, 1958.

[2] Yu.L. Ershov, L.A. Lavrov, A.D. Taimanov and M.A. Taitslin, *Elementary theories*, Uspehi mat. nauk, 1965, T. XX, 4(124), p. 37–107.

[3] A.V. Kovalsky and V.I. Ursu, *Equational theory of nilpotent A-loop*, Algebra and Logic, 49:4 (2010), p. 479–497.

[4] V.I. Ursu, *Undecidability of the elementary theories of finite commutative loops*, Fourt Mile Higs on Neassociative Mathematics, July 30-August 5, 2017, University of Denver, Colorado, SUA, Abstracts of Talks, p. 16–17.

[5] V.I. Ursu, *Undecidability of the elementary theories of finite commutative loops*, Preprint series of the *Simion Stoilow* Institute of Mathematics of the Romanian Academy, nr. 1/2018, ISSN 0250 3638.

On 3-isohedral tilings of sphere for group series $n\times$

Elizaveta Zamorzaeva

State University of Moldova, Chişinău, Republic of Moldova
e-mail: zamorzaeva@yahoo.com

A tiling W of the sphere with disks is called k-isohedral with respect to a discrete isometry group G if G maps the tiling W onto itself and the disks of W fall into k transitivity classes under the action of the group G.
Two pairs (W, G) and (W', G') belong to the same Delone class if there exists a homeomorphic transformation φ of the sphere such that φ maps the tiling W onto the tiling W' and the relation $G = \varphi^{-1}G'\varphi$ holds.
Some methods were developed that make it possible to obtain $(k+1)$-isohedral tilings with disks if the respective k-isohedral tilings with disks are known. In [1] the splitting procedure was applied to isohedral tilings of the sphere with disks resulting in all the fundamental Delone classes of 2-isohedral tilings of the sphere with disks for all 7 infinite series and 7 sporadic discrete isometry groups of the sphere.
The splitting procedure has already been applied to 2-isohedral tilings of the sphere with disks for group series $*nn$, nn, $*22n$, and $n*$.
Now turning to the series $n\times$ of isometry groups (which corresponds to the series $\widetilde{2N}$ of 3-dimensional point groups of isometries) we restrict ourselves to 3-isohedral tilings with disks that have at least 3 vertices, so digonal disks are excluded. Thus the splitting procedure has been applied to all the 20 series of Delone classes of fundamental 2-isohedral tilings of the sphere with disks. As a result we have obtained 105 series of Delone classes of fundamental 3-isohedral tilings of the sphere with disks that have at least 3 vertices, among them 94 series of Delone classes are normal in terminology of Grünbaum and Shephard.

Bibliography

[1] E. A. Zamorzaeva,*Classification of 2-isohedral tilings on the sphere*, Bul. Acad. Şt. Rep. Moldova. Matematica, N 3 (1997), 74–85 (in Russian).

7. Computer Science

Distributed data processing method for extracting knowledge

V. Ababii, V. Sudacevschi, V. Lazu, S. Munteanu and V. Ungureanu

Department of Informatics and Systems Engineering, Technical University of Moldova, Chisinau, Republic of Moldova
e-mail: victor.ababii@calc.utm.md

Distributed computing [1] presents one of the most efficient data processing and storage methods. Examples of applications and development of distributed computing can be mentioned as Cloud Computing [2] and Big Data [3], which offer a set of services and computing applications, access to information and data storage. The research aims to develop a distributed data processing method for data located on a set of servers using mobile program codes that are teleported from Users (Agents) to the Server, where large amounts of data are stored (Big Data), to extract the necessary information or knowledge.

Let's define a set of Agents $A = \{a_1, a_2, \cdots, a_N\}$ which activates in the environment $E(X) \in R^K$, where K is the dimension of space and $X = \{x_1, x_2, \cdots, x_K\}$ - the state of the activity environment. For each Agent a_i is defined the strategy $S_i = \bigcup_{j=1}^{J_i} O_{i,j}, i = \overline{1, N}$, where $O_{i,j}$, $j = \overline{1, J_i}$ is the set of Objects (programs code) that solve the strategy S_i, and $\bigcap_{i=1}^{N} S_i \neq \emptyset$. In the space E is also defined the set of Servers $C = \{c_1, c_2, \cdots, c_L\}$ that have sufficient computing resources and data storage $D = \{d_1, d_2, \cdots, d_L\}$, where $d_l = f_l(X)$ is the quantity of data stored on the Server c_l that determines the state of interest of services offered by server to the activity environment E. Each Server c_l, $l = \overline{1, L}$ is ready to host at necessity, Objects $O_{i,j}$, $i = 1, \cdots, N$, $j = 1, \cdots, J_i$, to execute and to return the result (knowledge) to Agent which are transmitted (submitted) to the Object to execution.

The distributed data processing method based on the mobile program code will be used to solve the problem of searching in large amount of information (Cloud Storage Systems, Big Data Systems) for automatic extraction of new knowledge.

Bibliography

[1] Ghosh, Sukumar, *Distributed systems: an algorithmic approach*, University of Iowa, Chapman & Hall/CRC, 390p., ISBN:1-58488-564-5.

[2] Lewis, Grace, *Basics About Cloud Computing*, http://www.sei.cmu.edu/library/abstracts/whitepapers/cloudcomputingbasics.cfm (2010).

[3] Mark A. Beyer and Douglas Laney, *The Importance of 'Big Data': A Definition*, Gartner, (2012).

Ways of resource virtualization for HPC Cloud Systems

Antohi Ionel

State University of Moldova, Chişinău, Republic of Moldova

e-mail: antohi.ionel@gmail.com

Cloud computing means the convergence of two major trends in today's IT: efficiency - where the power of modern computers is more efficiently used through a high scaling of hardware, software and business agility resources - where information technology can be used as a competitive tool on the market due to fast delivery, parallel processing, use of business intelligence tools that require intensive processing and interactive applications that respond in real time to the user requirements. The concept of Cloud computing has several definitions among the access of computing power and storage used as services by a large community of user is the most common one. Cloud system is a revolutionary method of development and implementation of infrastructure as a service (IaaS) based on the evolution of information technologies integrated in modern business [1].

Resource virtualization is the core of any cloud computing architecture, allowing the use of an abstract logical interface for accessing physical resources (servers, networks, storage media). Simulating methods of the interface to physical objects are: (a) Multiplexing - creating multiple virtual objects from a single instance of a physical object, for example a processor is multiplexed to process multiple chained processes (threads) (b) Emulation - building a virtual object from a physical object of another type, for example a physical hard disk can emulate RAM (through a swap file or swap partitions) (c) Aggregation - creating a single a virtual object from multiple physical objects, for example a number of hard drives can form a RAID aggregate disk (d) Multiplexing combined emulation - for example, the TCP protocol emulates a secure communications channel and multiplexes data transfer between the physical channel of communications and processor. In conclusion, virtualization allows intelligent management of the resources as: hardware independence, resources isolation for the compatibility and simplifying administration, copying, backups and deployment activity.

In the last few years, an increased interest has been detected in the need to introduce virtual systems instead of physical ones. Due to these cost-cutting benefits by using the optimal and efficient use of the physical resources of the server. Cloud technologies has taken the virtualization success by giving the ability to expand virtual machines to their customers, theoretically unlimited, with maintenance operations planned at long intervals. Designers for infrastructure application or cloud virtual machines are trained to configure service availability by allocating resource to different geographic locations.

The concept of virtualization and centralized storage is not exclusive cloud concepts. The difference between a large-scale processing center of a large company and a private form of cloud appears only when the self-service term is properly implemented with information tools based on a catalog of services provided to users [2]. From the point of the agility view, we find a technical difference: the virtualization concept is a horizontal model of extensibility by accumulating computing power and allocating processing resources, compared to the cloud which represent a vertical development model by adding services.

Considering the variety of requirement's for HPC clouds, it is known that the range of the virtualization media options may vary a lot. Solutions varies from remotely shared clusters to fully-fledged cloud-based systems. Each method brings its own set of features that must be tailored to the needs of the users.

Bibliography

[1] L. M. Vaquero, L. Rodero-Merino, J. Caceres, M. Lindner, *Abreak in the clouds: towards a cloud definition*, ACM SIGCOMM Computer Communication Review, vol. 39, nr. 1, pp. 50-55, 2008.

[2] M. Armbrust, A. Fox, R. Griffith, I. Stoica, M. Zaharia, *Above the Clouds: A Berkeley View of Cloud*, Electrical Engineering and Computer Sciences, California, 2009

Genetic algorithm for optimization of software process

Vladimir Bondarenco[1], Andrei Rusu[2], Elena Rusu[3]

[1] *Orange Moldova SA*
[2] *Ovidius University of Constanta,*
[3] *Technical University of Moldova*
e-mail: vladimir.bondarenco@orange.md, agrusu@univ-ovidius.ro,elena.rusu@mate.utm.md

The study conducted in this paper started from a real-life problem encounted by a software test manager. Testing a software consist in passing a set of tests, many of them are interdependent, they have different weights and the software testing team have a limited time resource. It is clear that it takes different times to pass different tests. The goal of the team is to conduct tests in the allowed time interval following a path such that the overall weight of passed tests is maximal.
In the present paper we present a mathematical model of the problem and propose a solution to it based on developed evolutionary algorithm.
Acknowledgement. *The work was partially supported by the research projects 18.50.07.10A/PS and 15.817.06.13A.*

Revitalizing the folkloric text of RM from the second half of the 20th century and their diachronic analysis

Bumbu Tudor, Caftanatov Olesea, Malahov Ludmila

Institute of Mathematics and Computer Science of ASM, Chişinău, Republic of Moldova
e-mail: bumbutudor10@gmail.com, olesea.caftanatov@math.md, lmalahov@gmail.com

Summary. The aim of our work is revitalizing the folkloric texts of Republic of Moldova from the second half of the 20th century, furthermore actualizing the folkloric texts that are in Cyrillic writing for their future use in education. In addition, we intend to make a diachronic analysis between two periods (1960-1995 and 1996 - 2018).

We assume that folklore reflects a certain vision of the population's life, beliefs and their feelings that can reach us through history. Our folk art that is manifested through songs, poetry, fairy tales, legends, proverbs and sayings, customs and traditions presents an invaluable wealth of treasure for all people who really love their homeland. For our purpose, we used as base resource the book *Folclor din părţile Codrilor* [1]. Given the fact that, our main resources are books, we needed an OCR tool to convert image text into editable text. Thus, Optical Character Recognition performed by using U Finereader Professional 12 (FR). It is important to mention that in that period the Romanian language was using Cyrillic Alphabet. Because, this alphabet isn't integrate in FR, we created templates and added word dictionaries. In addition, we trained around 100 templates, since; the most of letters can be find in Russian language with embedded templates in FR. The only exception is the letter that produce the sound "gi". The dictionary that we added has about 5000 words, many of them are from other Cyrillic scripts that we recognized previously.

Regarding, recognition accuracy is over 97% words the remaining 3% errors we corrected them manually. In order to convert from Cyrillic alphabet into Latin alphabet, we also used the AAConv Tool. This tool was developed within Institute of Mathematics and Computer Science. The transliteration accuracy is about 99%. For reediting text style such as, font, bolt italic, capital letters etc. we needed a bit of manual work. The obtained resource we intend to make a book. For its illustration were involved few volunteers. Finally, by using Machine-learning techniques such as, LDA latent dirichlet allocation and Euclidean distance, we analyzed the diachronic aspect of folklore text.

Bibliography

[1] G. G. Botezatu, H.M. Băeşu, E.V. Junghientu, M.G. Savina, E.V. Tolstenco, A.S. Hîncu, V.A. Cirimpei şi I.D. Ciobanu, *Folclor din părţile Codrilor*, Academia de Ştiinţe a RSS Moldoveneşti. 1967. p.356

Improvement of the administrative-territorial structure using mixed integer linear programming

Buzatu Radu

State University of Moldova, Chişinău, Republic of Moldova
e-mail: radubuzatu@gmail.com

Efficient public administration at local and national level is an important condition for the sustainable development of the country. There are known some mathematical models for generating scenarios of administrative territorial organization depending on various criteria and restrictions [1]. Most of them are not acceptable in terms of time and space complexity.
A mixed integer linear programming model for optimization of administrative territorial organization is proposed. The model is based on the research results published in [2] and [3]. Due to special compromise restrictions, the model can serve as a flexible and efficient tool for obtaining and evaluating potential administrative territorial scenarios in reasonable time. Undoubtedly this model can be easily adjusted to the conditions of different regions and countries throughout the world.

Bibliography

[1] C. Piccolo, *Mathematical Models to support territorial reorganization decisions in the public sector.* University of Naples Federico II, 2014.

[2] R. Buzatu, *A binary linear programming model for minimum convex partition of a graph.* Proceedings of the International summer mathematical school in memoriam V. A. Plotnikov, June 11-16, 2018, Odessa, Ukraine.

[3] R. Buzatu, C. Rabei, M. Roşcovan, *A mathematical model for optimization of administrative-territorial organization of the Republic of Moldova.* Studia Universitatis Moldaviae, 2018, under publication.

Application of Symbolic Calculations for Wick's Theorem

L. A. Dohotaru

Department of Mathematics, Technical University of Moldova, Chişinău, Moldova
e-mail: leonid.dohotaru@mate.utm.md

Wick's theorem for chronological products or its generalized version are used for the calculation of the scattering matrix in each order of perturbation theory [1-2]. The procedure is reduced to calculation of the vacuum expectation of chronological products of the field operators in the interaction representation. As factors in these products a number of operators a_i of the Fermi fields and the same number of their "conjugate" operators $\bar{a}_i$ are considered. Here all continuous and discrete variables are included in the index. In the interaction representation the operators a_i and $\bar{a}_i$ correspond to free fields and satisfy the commutation relationships of the form $[a_i, a_j]_+ = [\bar{a}_i, \bar{a}_j]_+ = 0, [a_i, \bar{a}_j]_+ = D_{ij}$.

Since we may rearrange the order of the operators inside of T-products taking into account the change of the sign, which arises when the order of the Fermi operators is changed, we present our vacuum expectation value of the chronological product of the Fermi operators in the form

$$\pm\langle T[(a_{i_1}\bar{a}_{j_1})(a_{i_2}\bar{a}_{j_2})\cdots(a_{i_n}\bar{a}_{j_n})]\rangle_0. \tag{1}$$

To calculate (1) we can use Wick's theorem for chronological products. However, while considering the higher-order perturbation theory, the number of pairs $a_i\bar{a}_i$ of the operators a_i and $\bar{a}_i$ becomes so large that the direct application of this theorem begins to represent certain problems because it is very difficult to sort through all the possible contractions between a_i and $\bar{a}_i$.

A consistent use of generalized Wick's theorem would introduce a greater accuracy in our actions. However, in this case we expect very cumbersome and tedious calculations. Hereinafter we show that the computation of (1) can be easily performed using a simple formula

$$\langle T[(a_{i_1}\bar{a}_{j_1})(a_{i_2}\bar{a}_{j_2})\cdots(a_{i_n}\bar{a}_{j_n})]\rangle_0 = det(\Delta_{i_\alpha j_\beta}), \tag{2}$$

$$\Delta_{i_\alpha j_\beta} = \langle T(a_{i_\alpha}\bar{a}_{j_\beta})\rangle_0, (\alpha, \beta = 1, 2, \cdots, n). \tag{3}$$

This result does not depend on the way how we divide the operators on the left hand of (2) into pairs. The proof of this theorem is by induction.

Obviously, the similar formula to (2) can be obtained also in the case of Bose fields.

The above theorem has an important consequence. In fact, it establishes a perfect coincidence between the vacuum expectation values of the chronological products of n pairs of field operators and the n-order determinant. If to present this determinant as the sum of the elements and cofactors of one by any row or column, and thereafter to use again the indicated coincidence for the $(n-1)$-order determinant included in each summand, we will return to the generalized Wick's theorem. Alternatively, we can select arbitrary m rows or columns $(1 < m < n)$ in our n-order determinant and use the Generalized Laplace's Expansion [3] for its presentation as the sum of the products of all m-rowed minors using these rows (or columns) and their algebraic complements. Then, taking into account our theorem, we obtain a representation of the vacuum expectation values of the chronological products of n pairs of field operators as the sum of the products of vacuum expectation values of the chronological products of m pairs of operators and vacuum expectation values of the chronological products of $n - m$ pairs. The number of terms in this sum is equal to $n!/m!(n-m)!$. This decomposition can be useful for the summation of blocks of diagrams.

In quantum statistics the n- body thermal, or imaginary-time, Green's functions in the Grand Canonical Ensemble are defined as the thermal trace of a time-ordered product of the field operators in the imaginary-time Heisenberg representation [4-5]. To calculate them in each order of perturbation theory, Wick's theorem is also used. Obviously, in this case the theorem also may be formulated in the form (2) convenient for practical calculation.

Representation (2) not only greatly simplify all calculation, but also allow one to perform them using a computer with programs of symbolic mathematics [6].

Bibliography

[1] S. Weinberg, *The quantum theory of fields: vol. 1, Foundations*, Cambridge University Press, Cambridge, 1995.

[2] N. N. Bogoliubov and D. V. Shirkov, *Introduction to the theory of quantized fields*, John Willey & Sons, New York, 1980.

[3] G. A. Korn and T. M. Korn, *Mathematical handbook for scientists and engineers*, McGraw-Hill Book Co., New York, 1961.

[4] J. W. Negele and H. Orland, *Quantum many-particle systems*, Westview Press, 1998.

[5] A. A. Abrikosov, L.P. Gor'kov and I.E. Dzyaloshinski, *Methods of quantum field theory in statistical physics*, Dover Publications, Inc., New York, 1963.

[6] S. Wolfram, *The Mathematica Book*, 5th ed., Wolfram Media, Champaign, USA, 2003.

Evaluation of the evacuation parameters from multi-storey buildings by Hierarhical Petri Nets

Titchiev Inga

Institute of Mathematics and Computer Science,
Tiraspol State University, Cisinau, Republic of Moldova
e-mail: inga.titchiev@gmail.com

The aim of this research is to evaluate the evacuation parameters from multi-storey buildings by using Hierarchical Petri Nets[1], for these the specificity of building construction will be taken into account. Extensions of the Petri Nets are applied successfully in various fields. Especially in the area of emergency and disaster management [2]. For modeling of the movement of human flows in the process of evacuation from multi-storey buildings the norms in construction will be applied. These norms are developed by The Normative Supervision Section of Buildings and Fire Department [3] in the Republic of Moldova. The parameters related to traffic intensity in rooms, formation of cluster of people, diffusion of human flows and reforming of the human flows will be evaluated.

Acknowledgement. *The research is performed in the frame of the project "Development of a toolkit for modeling strategies to mitigate social disasters caused by catastrophes and terrorism", (17.80013.5007.01/Ua)*

Bibliography

[1] X. He, T. Murata, *High-Level Petri Nets' Extensions, Analysis, and Applications*, Electrical Engineering Handbook (ed. Wai-Kai Chen), Elsevier Academic Press, pp.459-476, 2005.

[2] S.Cojocaru, M.Petic, I.Titchiev, *Adapting Tools for Text Monitoring and for Scenario Analysis Related to the Field of Social Disasters*, The proceedings of The 18th International Conference on Computer Science and Electrical Engineering (ICCSEE 2016), October 6-7, Prague, Czech Republic, pp. 886-892, 2016.

[3] Gh. Cojusneanu, *Normativ in constructii*, Siguranta la incendii, NCM E.03.02-2001, p. 55, 2001.

Agent, Intelligent Agents, Definition, Classification and Utilization

Inga Lisnic, Sergiu Scrob

Technical University of Moldova
e-mail: inga.lisnic@ati.utm.md, sergiu.scrob@ati.utm.md

At the moment it is obvious that the systems need more and more autonomy, and here is the point where the need for intelligent agents first appears. By the time agents were needed in many application, there have appeared many architectures for their development. As the uses of agents grew in number there have been described some that are different from the others. In this way, Intelligent Agents,with many publications with Intelligent Behavior were described and built. This article is considered to be an introduction to the field of Intelligent Agents giving the reader a structured presentation.
Keywords: agent, intelligent agents.

8. Education

Cu privire la unele probleme referitoare la dreapa şi planul în spaţiu

Iurie Baltag

Universitatea Tehnică din Moldova
e-mail: iubaltag@mail.ru

Se examinează următoarele probleme:

1. **Poziţia reciprocă a două drepte în spaţiu.**
2. **Determinarea distanţei minime dintre două drepte şi a punctelor de pe ele ce asigură această distanţă.**
3. **Poziţia reciprocă a unei drepte şi a unui plan.**
4. **Determinarea punctului simetric în raport cu un plan sau cu o dreaptă.**
5. **Determinarea proecţiei unei drepte pe un plan.**

1. Fie daţi vectorii directori ai dreptelor şi câte un punct pe fiecare din dreptele (a) şi (b). Mai întâi verificăm, dacă vectorii daţi sunt colineari. În caz afirmativ dreptele sunt paralele. Dacă vectorii nu sunt colineari, atunci dreptele se intersectează sau sunt neconcurente.

Pentru a determina poziţia lor scriem ecuaţiile parametrice ale dreptelor şi egalăm părţile drepte ale ecuaţiilor variabilelor corespunzătoare. În rezultat obţinem un sistem ce conţine trei ecuaţii liniare cu două necunoscute t şi r.

Sunt posibile următoarele două cazuri:

1. *Sistemul dat are soluţie unică, atunci dreptele se intersectează şi determinăm punctul lor de intersecţie.*

2. *Sistemul este incompatibil, atunci dreptele sunt neconcurente.*

2. Fie date două drepte neconcurente. Punem problema să aflăm distanţa minimă dintre drepte şi a punctelor de pe ele ce asigură această distanţă. Cu acest scop, utilizând ecuaţiile parametrice ale dreptelor, alcătuim funcţia de două variabile $d(t, r)$ ce descrie distanţa dintre orice două puncte situate pe dreptele (a) şi (b).

Din considerente geometrice este clar, că această funcţie poate avea doar un singur punct de extrem şi anume un punct de minim. Pentru a-l afla aplicăm condiţia necesară de extrem ce se reduce la un un sistem cu 2 ecuaţii liniare şi 2 necunoscute. Rezolvând acest sistem, determinăm coordonatele punctului de minim (t_0, r_0) şi atunci $d(t_0, r_0)$ va fi distanţa minimă dintre drepte. Pentru a determina punctele de pe drepte ce asigură această distanţă înlocuim în ecuaţiile lor parametrice $t = t_0, r = r_0$.

Notă. Dacă dreptele sunt paralele, putem lua pe una din ele un punct fix şi atunci funcţia distanţei $d(\cdot, \cdot)$ va fi de o singură variabilă.

3. Fie dată ecuaţia generală a unui plan Π. Înlocuim ecuaţiile parametrice ale dreptei (a) în ecuaţia planului şi obţinem o ecuaţie de gradul 1 cu o necunoscută t.

Sunt posibile următoarele trei cazuri:

1. *Ecuaţia are soluţie unică $t = t_0$, atunci dreapta intersectează planul într-un punct. Pentru a afla coordonatele lui înlocuim această valoare a lui t în ecuaţiile parametrice ale dreptei.*

2. Ecuaţia nu are soluţii. Atunci dreapta este paralelă planului.

3. Ecuaţia are forma $0 \cdot t = 0$, adică estre adevărată pentru orice t. În acest caz dreapta aparţine planului.

4. Fie dată ecuaţia unui plan Π şi un punct $M_1(x_1, y_1, z_1)$ în afara lui. Pentru a determina punctul lui simetric în raport cu planul dat alcătuim ecuaţiile parametrice ale dreptei (a) ce trece prin punctul M_1 perpendicular pe plan şi aflăm punctul de intersecţie al acestei drepte cu planul.

Procedând ca în cazul 3), aflăm punctul $M_0(x_0, y_0, z_0)$ de intersecţie al dreptei (a) cu planul. Apoi aplicăm formulele mijlocului unui segment şi determinăm punctul M_2 simetric lui în raport cu planul dat.

Dacă trebuie să aflăm punctul simetric lui M_1 în raport cu o dreaptă (b) ce are ecuaţiile parametrice date, atunci alcătuim ecuaţia planului ce trece prin M_1 perpendicular pe (b) şi aflăm punctul de intersecţie al acestui plan cu dreapta dată. Apoi, aplicând formulele mijlocului unui segment, la fel ca şi în cazul precedent aflăm punctul căutat.

5. Fie dată ecuaţia unui plan Π şi o dreaptă (a) cu vectorul director dat ce trece prin punctul A. Pentru a determina ecuaţiile dreptei ce reprezintă proecţia dreptei (a) pe plan, deducem mai întâi ecuaţia planului $\Pi_1 \perp \Pi$ ce trece prin dreapta (a). Cu acest scop aflăm vectorul normal al acestui plan, calculând produsul vectorial dintre vectorul director al dreptei (a) şi vectorul normal al planului şi alcătuim ecuaţia planului căutat aplicând formula ecuaţiei planului ce trece prin punctul $A(x_1, y_1, z_1)$ cu vectorul normal $\boldsymbol{n}_1$.

Intersecţia celor două plane va fi dreapta ce reprezenta proiecţia dreptei (a) pe planul Π, adică sistemul ce conţine ecuaţiile celor două plane.

Bibliography

[1] Năstăsescu C., Niţă C., Stănescu I., *Elemente de algebră superioară*, Bucureşti, 1993.

[2] Baltag I., Baltag Iu., *Elemente de algebră liniară şi geometrie analitică*, Chişinău, 2008.

[3] Baltag Iu., *Elemente de algebră liniară şi vectorială*, Chişinău, 2008.

[4] Chletenic D., *Culegere de probleme la geometria analitică*, Moscova,1976.

Abordări didactice în predarea algoritmilor pentru determinarea arborelui parţial de cost minim

Marina Bostan

Universitatea de Stat din Tiraspol
e-mail: marina.bostan13@gmail.com

Este un fapt bine cunoscut că Teoria Grafurilor se regăseşte într-o mulţime de aplicaţii în diverse sfere ale activităţii umane: construcţii şi sociologie, electrotehnică şi geografie, etc. Implementarea tehnologiilor informaţionale moderne în procesul de predare-învăţare-evaluare cursului dat oferă profesorilor şi studenţilor noi posibilităţi de a facilita predarea şi învăţarea.
În predarea cursului Teoria Grafurilor implementarea activă în cadrul orelor a softurilor Maple18, Delphi, C/C++, Teoria Grafurilor [4] au un rol important în dezvoltarea capacităţii de a gândi creativ şi a elabora algoritmi optimi în scopul obţinerii soluţiilor eficiente a problemelor examinate. Studiul algoritmilor pentru determinarea arborilor de cost minim este justificat de existenţa în practică a unui număr mare de probleme care pot fi rezolvate cu ajutorul acestora, cum ar fi:

1. construirea unor reţele de aprovizionare cu apă potabilă (sau cu energie electrică sau termică etc) a unor puncte de consum, de la un punct central;

2. construirea unor căi de acces între mai multe puncte izolate.

Din algoritmii pentru găsirea arborelui de valoare optimă au fost abordate algoritmul lui Prim şi algoritmul lui Kruskal. Algoritmul Kruskal a fost elaborat de Joseph Kruskal în anul 1956. Algoritmul Prim a fost descoperit în 1930 de către matematicianul Vojtich Jarnik şi apoi, independent, de informaticienii Robert C. Prim în 1957 şi redescoperit de Edsger Dijkstra în 1959. De aceea mai este numit Algoritmul DJP, algoritmul Jarnik sau algoritmul Prim-Jarnik.
Propunerea spre rezolvare problemelor din viaţa reală în cadrul cursului dat îndeamnă studenţii să se implice în propriul proces de formare, să elaboreze modele matematice creative şi să creeze programe pentru acestora, să creeze un mediul de învăţare personalizat. Studierea, testarea şi analiza algoritmilor studiate cu ajutorul softurilor specializate da studenţilor facilitează procesul de alegere algoritmului potrivit în soluţionarea problemelor din viaţa reală.
Eficacitatea şi funcţionalitatea algoritmului utilizat se evidenţiază prin aplicarea softurile specializate (Maple 18, Teoria grafurilor), ceea ce permite vizualizarea interactivă algoritmilor aplicate, verificarea soluţiilor obţinute, testarea algoritmului prin modificarea datelor de intrare, compararea diferitor algoritmi, individualizarea algoritmilor. Toate aceste momente contribuie la dezvoltarea gândirii logice şi critice a studenţilor şi le permite programarea algoritmilor aplicate cu ajutorul limbajelor de programare.

Bibliography

[1] Corlat Sergiu, Gremalschi Anatol, *Grafuri : Metodologia predării în cadrul instruirii de performanţă la disciplinile Matematică & Informatică : pentru uzul studenţilor*, Acad. De Ştiinţe a Moldovei, Univ. de Stat din Tiraspol, 2014, 158 p, ISBN 978-9975-76-122-2

[2] Bang-Jensen, G. Gutin, *Digraphs Theory, Algorithms and Applications*, Springer-Verlag, 2007

[3] J.A. Bomdy, U.S.R. Murty, *Graph Theory*, Springer, 2007

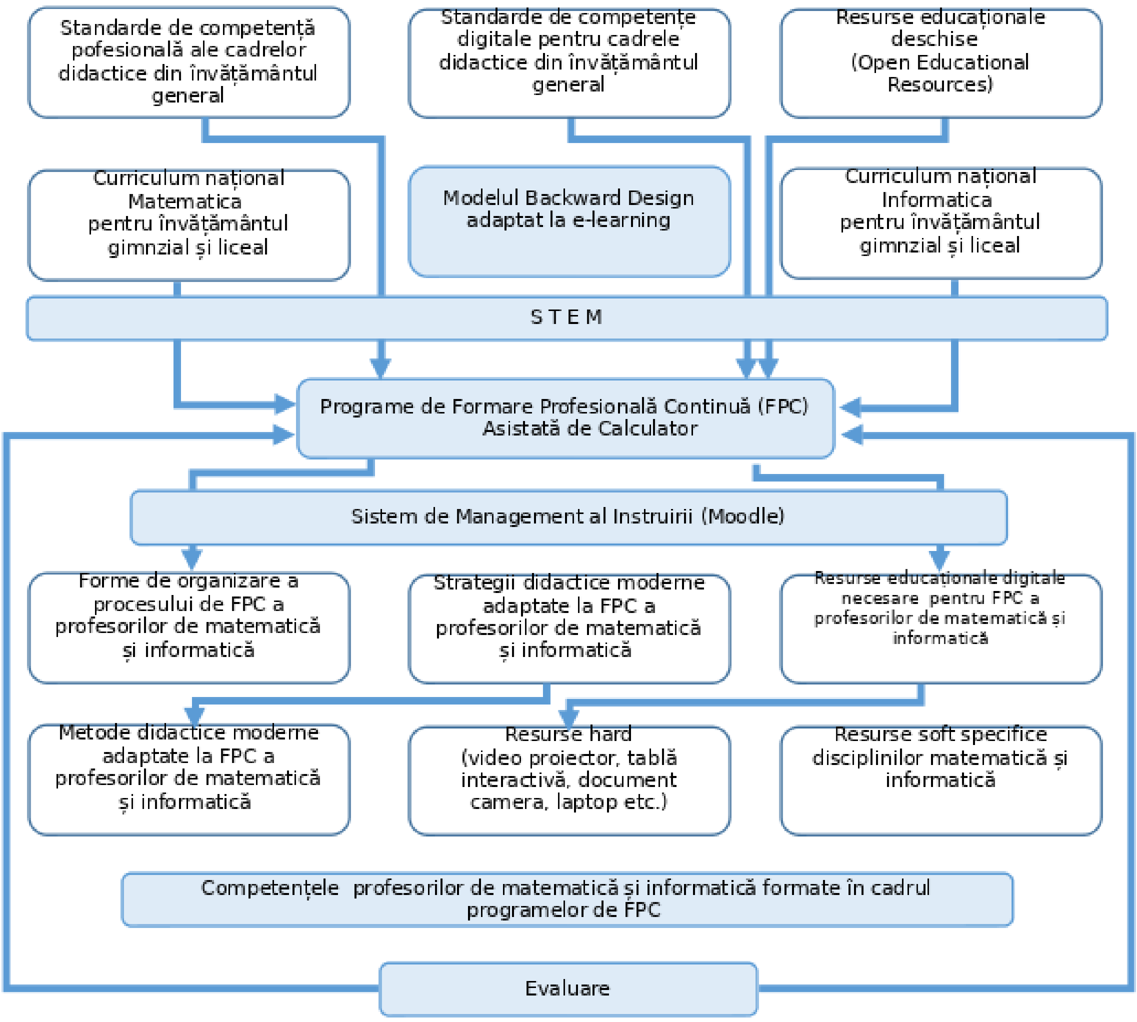
[4] ***, *Teoria grafurilor, Software educaţional*, online `http://campion.edu.ro/arhiva/www/arhiva_2009/seds/17/index.htm`

Modelul de formare profesionala continuă asistataă de calculator a profesorilor de matematică şi informatică

Andrei Braicov, Larisa Sali, Maria Pavel

Universitatea de Stat din Tiraspol, Chişinău, Republica Moldova
e-mail: braicov.andrei@ust.md, sali.larisa@ust.md, pavel.maria@ust.md

Transformările în Educaţie din ultimii ani, impuse de evoluţia tehnologiilor, au creat premize esenţiale pentru cercetări în vederea actualizării procesului de Formare Profesională Continuă (FPC) a cadrelor didactice. Se propune un model validat experimental de FPC asistată de calculator a profesorilor de matematică şi informatică (figura 1), care pune accent pe: proiectarea curriculară în cheia Backward Design, adaptat la e-Learning; instruirea integratoare STEM; strategii didactice moderne; medii de management al instruirii; format mixt de organizare a învăţării.



Modelul de formare profesionala continuă asistataă de calculator a profesorilor de matematică şi informatică

Eficientizarea procesului de studiu al structurilor de date prin abordarea învăţării în bază de problemă

Andrei Braicov, Angela Globa

Universitatea de Stat din Tiraspol, Chişină, Republica Moldova
e-mail: abraicov@gmail.com, angelagloba@gmail.com

Structurile de date au un rol important în optimizarea algoritmilor. Se ştie că organizarea reuşită a datelor poate înbunătăţi esenţial algoritmul. De fapt, această constatare a provocat apariţia programării modulare, apoi şi a programării orientate pe obiecte. Structurile de date sunt studiate şi în cursurile preuniversitare de informatică, examinându-se în această ordine tipurile de date simple (întreg, real, boolean, character, subdomeniu), tipurile de date structurate (tablou, şir de caractere, articol, mulţime, fişier) şi tipul referinţă. Curricula preuniversitară la informatică [1, 2] stipulează (de exemplu, [2, pag. 8]) că elevul trebuie să poată argumenta necesitatea structurării datelor, ceea ce urmează să-l motiveze să utilizeze ulterior această abordare în scrierea codurilor de program. În contextul demersurilor didactice moderne, care promovează (pe bună dreptate) educaţia integratoare STEM şi metodele active de învăţare [3, pag. 63], propunem o modalitate non standard de învăţare a structurilor de date menţionate. Eficienţa acesteia a fost validată prin experiment pedagogicorganizat de autori şi se axează pe instruirea în bază de problemă (problem based learning) [3, pag. 63], mai exact pe căutarea soluţiei mai bune a unei probleme matematice. Idea principală a metodei constă în următoarele: problema poate fi rezolvată prin utilizarea oricăror date (simple sau structurate), dar calitatea soluţiei creşte atunci când se aplică o structură nouă de date (în ordinea propusă de curriculă). Întrebările de genul Soluţia poate fi îmbunătăţită? trebuie să aibă un caracter provocator, astfel încât să menţină entuziasmul şi motivaţia elevului. Exemplu de problemă. Să se scrie un program care va realiza înmulţirea a două numere mari. Ultimul cuvânt din enunţ este important, pentru că el va lăsa întotdeauna loc pentru optimizarea soluţiei [4, 5].

Bibliography

[1] ***, *Curriculum pentru învăţământul gimnazial (clasele VII-IX)*. Chişinău, 2010 (`http://ctice.md/ctice2013/wp-content/uploads/2013/02/Informatica_cl_07-09_-Curriculum.pdf`)

[2] ***, *Curriculum pentru învăţământul liceal(clasele X-XII)*. Chişinău, 2010 (`http://ctice.md/ctice2013/wp-content/uploads/2013/02/7548_md_informatica_-roman.pdf`)

[3] Segiu Corlat, Goran Karlsson, Andrei Braicov, ...[et al.], *Metodologia utilizării tehnologiilor informaţionale şi de comunicaţie în învăţământul superior*. Chişinău, UST, 2011, 204 p. ISBN 978-9975-76-070-6

[4] Andrei Braicov, *Informatică. Turbo Pascal. Culegere de probleme*. Chişinău. Editura Prut Internaţional, 2005, 232 p. ISBN 978-9975-69-788-7

[5] Angela Globa, *Aspecte metodice privind elaborarea algoritmilor de operare cu numere mari*, The 20 Conference on Applied and Industrial Mathematics dedicated to academician Mitrofan M. Cioban. Chişinău, August 22-25, 2012. p.161-172.

Despre asimptotele unor funcţii

Cerbu Olga, Ţurcanu Alina

State University of Moldova, Chişinău, Republic of Moldova, Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: olga.cerbu@gmail.com, turcan.alina@civis.md

Definiţie *Se numeşte asimptotă, dreapta verticală, orizontală sau oblică faţă de care graficul funcţiei se apropie oricât de mult.*
O curbă poate avea numai o asimptotă la stânga sau la dreapta, însă pot exista orice număr de asimptote verticale, ca în cazul funcţiei, $f(x) = \operatorname{tg} x$.
Vom examina asimptotele unor funcţii şi momentele la care dorim să atragem atenţia cum ar fi:
- graficul funcţiei şi asimptota se pot intersecta de o infinitate de ori
- unele funcţii cu o infinitate de puncte de discontinuitate de asemenea pot avea asimptote.

Exemplu $f(x) = 2x+3+(-1)^{[x]}\dfrac{\{x\}}{x}$ unde $[x]$ este partea întreagă a lui x iar $\{x\}$ partea fracţionară.

Se vede imediat că $x = k$ este punct de discontinuitate pentru orice $k \in \mathbb{Z}$ iar dreapta $y = 2x + 3$ este asimptotă oblică.
În lucrarea sunt prezentate multe alte exemple cu scopul de a ilustra notiunea de asimptotă.

Utilizarea proprietăţilor modulului la rezolvarea inecuaţiilor ce conţin simbolul lui

Andrei Corlat[1], Ion Jardan[1,2]

[1] *Institute of Mathematics and Computer Science,*
[2] *Technical University of Moldova, Chişinău, Republic of Moldova*
e-mail: an.corlat@gmail.com, ion.jardan@mate.utm.md

În această comunicare sunt analizate metode de rezolvare a inecuaţiilor algebrice ce conţin simbolul modulului, bazate pe utilizarea proprietăţilor lui, ceea ce permite rezolvări succinte. De regulă, inecuaţiile ce conţin simbolul modulului, se rezolvă cu ajutorul metodei intervalelor: domeniul valorilor admisibile al inecuaţiei se împarte astfel, încât în fiecare interval expresiile de sub simbolul modulului să-şi păstreze semnul; în fiecare din aceste intervale inecuaţia se scrie fără simbolul modulului, se rezolvă, iar soluţia se obţine ca reuniunea soluţiilor pe intervale.

În unele cazuri, cunoaşterea proprietăţilor modulului conduce la rezolvări succinte şi optimale ([1]-[3]). Enumerăm în continuare unele proprietăţi ale modulului, ce vor fi utilizate la rezolvarea inecuaţiilor.

1. $|a| \geq 0$; 2. $|a| \geq a$; 3. $|a| \geq -a$; 4. $|a| < b, b > 0 \Leftrightarrow -b < a < b$; 5. $|a| > b, b > 0 \Leftrightarrow \left[\begin{array}{l} a > b \\ a < -b \end{array}\right.$; 6. $|a| > |b| \Leftrightarrow (a-b)(a+b) > 0$;
7. $|a| \leq |b| \Leftrightarrow (a-b)(a+b) \leq 0$.

În continuare vom analiza câteva exerciţii, rezolvarea cărora se bazează pe utilizarea proprietăţilor modulului.

Exemplul I. Să se rezolve inecuaţiile:

a) $\left|\dfrac{x^2 - x - 6}{x^2 + 3x - 10}\right| > -2$; b) $\left|x^2 - x - 2\right| \geq x^2 - x - 2$;

c) $\left|x^2 - x - 2\right| > 2 + x - x^2$; d) $\left|\dfrac{x^2 - 7}{x^2 - 1}\right| < 1$;

e) $\left|x^2 - 5x\right| > 6$; f) $|||x| - 1| - 4| \leq 3$.

Rezolvare. a) Se aplică proprietatea 1 şi se deduce, că mulţimea soluţiilor inecuaţiei enunţate coincide cu domeniul valorilor admisibile, prin urmare $S = R \setminus \{-5; 2\}$.

b) Se aplică proprietatea 2 şi se obţine $S = R$.

c) Inecuaţia se scrie $\left|x^2 - x - 2\right| > -\left(x^2 - x - 2\right)$, apoi în baza proprietăţii 3, conchidem că mulţimea soluţiilor ei se obţine prin excluderea zerourilor funcţiei $f : R \to R$, $f(x) = x^2 - x - 2$. Astfel $S = R \setminus \{-1; 2\}$.

d) Se aplică proprietatea 4 şi se obţine:

$$\left|\frac{x^2 - 7}{x^2 - 1}\right| < 1 \Leftrightarrow -1 < \frac{x^2 - 7}{x^2 - 1} < 1 \Leftrightarrow \begin{cases} \dfrac{x^2 - 7}{x^2 - 1} - 1 < 0 \\ \dfrac{x^2 - 7}{x^2 - 1} + 1 > 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \dfrac{-6}{x^2-1} < 0 \\ \dfrac{2(x^2-4)}{x^2-1} > 0 \end{cases} \Leftrightarrow \begin{cases} x^2-1>0 \\ x^2-4>0 \end{cases} \Leftrightarrow x^2-4>0 \Leftrightarrow$$

$\Leftrightarrow x \in (-\infty;-2) \cup (2;+\infty)$.

e) Se aplică proprietatea 5 şi se obţine:

$$\left|x^2-5x\right| > 6 \Leftrightarrow \left[\begin{matrix} x^2-5x>6 \\ x^2-5x<-6 \end{matrix}\right. \Leftrightarrow \left[\begin{matrix} x^2-5x-6>0 \\ x^2-5x+6<0 \end{matrix}\right. \Leftrightarrow \left[\begin{matrix} \left[\begin{matrix} x<-1 \\ x>6 \end{matrix}\right. \\ 2<x<3 \end{matrix}\right.,$$

de unde $S=(-\infty;-1)\cup(2;3)\cup(6;+\infty)$.

f) Se aplică proprietăţile 4 şi 5 şi se obţine:

$$|||x|-1|-4| \le 3 \Leftrightarrow -3 \le ||x|-1|-4 \le 3 \Leftrightarrow \begin{cases} ||x|-1| \le 7 \\ ||x|-1| \ge 1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -7 \le |x|-1 \le 7 \\ \left[\begin{matrix} |x|-1 \ge 1 \\ |x|-1 \le -1 \end{matrix}\right. \end{cases} \Leftrightarrow \begin{cases} \begin{cases} |x| \le 8 \\ |x| \ge -6 \end{cases} \\ \left[\begin{matrix} |x| \ge 2 \\ |x| \le 0 \end{matrix}\right. \end{cases} \Leftrightarrow \begin{cases} \begin{cases} -8 \le x \le 8 \\ x \in R \end{cases} \\ \left[\begin{matrix} \left[\begin{matrix} x \ge 2 \\ x \le -2 \end{matrix}\right. \\ x=0 \end{matrix}\right. \end{cases},$$

de unde $S=[-8;-2]\cup\{0\}\cup[2;8]$.

În continuare vom aduce câteva afirmaţii referitor la echivalenţa inecuaţiilor ce conţin simbolul modulului.

Afirmaţia 1. Inecuaţia $|f(x)|<g(x)$ este echivalentă cu sistemul de inecuaţii $\begin{cases} f(x)<g(x) \\ f(x)>-g(x) \\ g(x)>0 \end{cases}$.

Observaţie. Dacă $g(x) \le 0$ pentru orice $x \in DVA$ al inecuaţiei, atunci mulţimea soluţiilor inecuaţiei este vidă.

Afirmaţia 2. Inecuaţia $|f(x)|>g(x)$ este echivalentă cu totalitatea de inecuaţii $\left[\begin{matrix} f(x)>g(x) \\ f(x)<-g(x) \end{matrix}\right.$.

Afirmaţia 3. Inecuaţia $|f(x)|<|g(x)|$ este echivalentă inecuaţia $(f(x)-g(x))(f(x)+g(x))<0$.

Afirmaţia 4. Inecuaţia $|f(x)+g(x)|<|f(x)|+|g(x)|$ este echivalentă inecuaţia $f(x)\cdot g(x)<0$.

Vom ilustra prin exemple utilizarea afirmaţiilor 1-4.

Exemplul II. Să se rezolve inecuaţiile:

a) $|x-6|<x^2-2x+6$; b) $\left|x^2-x-2\right|>x+1$;

c) $\left|x^4-x^3+6x^2-5x-16\right|>\left|x^4+x^3-6x^2+5x-16\right|$;

d) $|3x-5|<|x-4|+|2x-1|$.

Rezolvare. a) Se aplică afirmaţia 1 şi se obţine:

$$|x-6|<x^2-2x+6 \Leftrightarrow \begin{cases} x-6<x^2-2x+6 \\ x-6>-x^2+2x-6 \\ x^2-2x+6>0 \end{cases} \Leftrightarrow \begin{cases} x^2-3x+12>0 \\ x^2-x>0 \\ x \in R \end{cases} \Leftrightarrow \Leftrightarrow \begin{cases} x \in R \\ \left[\begin{matrix} x<0 \\ x>1 \end{matrix}\right. \\ x \in R \end{cases},$$ de unde

$S=(-\infty;0)\cup(1;+\infty)$.

b) Se aplică afirmaţia 2 şi se obţine:

$$\left|x^2-x-2\right|>x+1 \Leftrightarrow \begin{cases} x^2-x-2>x+1 \\ x^2-x-2<-x-1 \end{cases} \Leftrightarrow \begin{cases} x^2-2x-3>0 \\ x^2-1<0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \begin{cases} x<-1 \\ x>3 \end{cases} \\ x^2-x-2<-x-1 \end{cases}, \text{ de unde}$$

$S=(-\infty;-1)\cup(-1;1)\cup(3;+\infty)$.

c) Aplicând afirmaţia 3 şi metoda intervalelor de rezolvare a inecuaţiilor se obţine:
$\left(x^4-x^3+6x^2-5x-16+x^4+x^3-6x^2+5x-16\right)\left(x^4-x^3+6x^2-5x-\right.$
$\left.-16-x^4-x^3+6x^2-5x+16\right)>0 \Leftrightarrow 2\left(x^4-16\right)(-2x)\left(x^2-6x+5\right)>0 \Leftrightarrow$
$x(x-2)(x+2)(x-1)(x-5)<0 \Leftrightarrow x\in(-\infty;-2)\cup(0;2)\cup(1;5)$.

d) Se observă, că $3x-5=(x-4)+(2x-1)$ şi se aplică afirmaţia 4:
$|3x-5|<|x-4|+|2x-1| \Leftrightarrow (x-4)(2x-1)<0$, de unde $S=\left(\frac{1}{2};4\right)$.

Bibliography

[1] Cojuhari P., Corlat A. *Ecuaţii şi inecuaţii algebrice.* Chişinău, Editura Centrală, 1995, Repub. Moldova.

[2] Cojuhari P. *Ecuaţii şi inecuaţii. Teorie şi practică.* Chişinău, Editura Universitas, 1993, Repub. Moldova.

[3] ***, https://www.math.md/school

Elements of teaching mathematics

Viorica-Cornelia Hoffmann-Bronţ

Liceul Tehnologic Special nr.1, Oradea,Romania
e-mail: vigyorika@yahoo.com

Methodology, irrespective of the subject matter, defines the special didactic discipline regarding the methods and procedures appropriate to the acquisition of a subject by the pupils. After Professor Şt. Barsanescu "method determines the place of the respective subject within the educational plan, the programming of the content of the forms and means of learning (regarding the pupil) and the teaching (regarding the teacher), the relationship between the student and the subject to be taught. So we can say that "methodologies are considered special theories of the educational process". Mathematics is the discipline that assures the development of logical thinking and outlines the practical dimensions of knowledge, that is, its study is based on knowledge and skills that contribute to multilateral intellectual development. The learning outcomes of mathematical discipline are determined by its cognitive character. The teacher should use the creative-creative potential of the subject, its ability to structure thinking, develop its flexibility, form skills and attitudes according to the content of ideas.

Interpretarea geometrică a rezolvării sistemului de ecuaţii de forma $\begin{cases} ax + by + c = 0 \\ a_1x^2 + b_1x + c_1 - y = 0 \end{cases}$ cu $a_1 \neq 0$

Viorica-Cornelia, Hoffmann-Bronţ

Liceul Tehnologic Special nr.1, Oradea,Romania
e-mail: vigyorika@yahoo.com

Rezolvarea unor sisteme de două ecuaţii de forma

$$\begin{cases} ax + by + c = 0, \\ y - (a_1x^2 + b_1x + c_1) = 0, \end{cases}$$

cu $a_1 \neq 0$ conduc la aflarea intersecţiei dintre o parabolă şi o dreaptă.
Dacă dreapta intersectează parabola în două puncte distincte sistemul are două soluţii distincte.Dreapta este secantă parabolei. Dacă dreapta intersectează parabola într-un singur punct ,sistemul are soluţie unică..Dreapta este tangentă parabolei. Dacă dreapta nu intersectează parbola sistemul de ecuaţii nu are soluţii.Dreapta este exterioară parabolei.

O Nouă paradigmă a ingineriei: Educaţia ŞTIM: Ştiinţă–Tehnologie-Inginerie-Matematică

Dulgheru Valeriu

Universitatea Tehnică a Moldovei
e-mail: valeriu.dulgheru@bpm.utm.md

Motto. *Matematicienii de regulă elaborează metodele matematice dar nu le pot aplica fiindcă unii nu cunosc bine domeniile de aplicare. Inginerii cunosc bine domeniile de aplicare ingunereşti dar nu cunosc metodele matematice.*
Ingineria se confruntă cu noi provocări precum: evoluţia spectaculoasă a tehnologiilor comunicaţiei şi procesării informaţiilor; evoluţia sistemelor de fabricaţie spre sisteme inteligente → procese şi produse inteligente → întreprinderi inteligente; explozia noilor domenii: nanotehnologii, biotehnologii şi cyber – infrastructuri; transformări tehnologice şi societale foarte rapide. Toate acestea cer pentru inginerul viitorului o pregătirea fundamentală solidă cu abilităţi practice consistente: matematica, ştiinţele (fizică, chimia, biologia), tehnologia şi ingineria – elemente esenţiale pentru pregătirea viitorului inginer (STEM). Aceasta se poate realiza doar prin restructurarea întregului sistem de educatie inginerească în concordanţă cu cerinţele actuale dar mai ales cu cele de viitor.

Impactul utilizării platformei de e-învăţare MOODLE în procesul de predare-învăţare-evaluare la unitatea de curs Tehnologii informaţionale şi comunicaţionale

Radames Evdochimov

Facultatea de Ştiinţe Reale, Economice şi ale Mediului, Universitatea de Stat "Alecu Russo" din Bălţi
e-mail: evdochimov.radames@gmail.com

În condiţiile societăţii informaţionale în dezvoltare tehnologiile informaţionale au atins un nivel de performanţă sporit, pătrunzând, practic, în toate domeniile activităţii societăţii, inclusiv, în învăţământ [3]. În acest sens, rolul lecţiilor de informatică în pregătirea unui absolvent competitiv al universităţii, care ştie să se orienteze şi să se realizeze în condiţiile sociale moderne, este în creştere. Prin urmare, a apărut problema căutării mijloacelor cele mai optime de organizare a procesului de predare – învăţare –evaluare a unităţii de curs *Tehnologii informaţionale şi comunicaţionale*. În acest context în calitate de soluţie a fost implementat un curs electronic pe MOODLE.
În rezultatul utilizării cursului au fost evidenţiate următoarele cerinţe faţă de un curs pe platforma MOODLE, care ar permite creşterea eficacităţii acestuia :

1. Cursul trebuie să fie flexibil, oferând celui care învaţă posibilitatea de a alege traseul propriu de învăţare;
2. Cursul trebuie să ghideze procesul de învăţare a celui ce învaţă (prin evaluări sistematice, imposibilitatea trecerii la altă temă fără asimilarea suficientă a materialului curent etc.)[2];
3. Materialul informativ utilizat în cadrul cursului trebuie să fie prezentat în diverse forme pentru a acoperi toate nevoile şi specificul de asimilare a informaţiei de către studenţi [2];
4. Materialul cursului trebuie să fie motivant, adică unele evaluări pot fi realizate în forma de jocuri interactive, pentru crearea cărora pot fi utilizate atât instrumentele platformei MOODLE, cât şi tehnologiile Web 2.0, care permit utilizatorului să descarce şi să editeze el însuşi conţinutul resursei, să folosească materialele didactice, create de alţi profesori, sau şabloane de materiale didactice, adaptându-le la cerinţe proprii, materialele astfel elaborate pot fi utilizate direct din cursul electronic printr-un link la acestea [1, 4, 5];
5. Nu se recomandă plasarea tuturor materialelor utilizate în curs direct pe server MOODLE, este bine ca cursul electronic să conţină multe link-uri la alte resurse Web 2.0, care oferă servicii de stocare gratis a diferitor informaţii, jocuri, sau altor materiale didactice, în acest caz se va economisi spaţiul pe server local a instituţiei de învăţământ.

Drept consecinţe ale utilizării cursului electronic au fost observate creşterea interesului faţă de procesul de studiere a unităţii de curs TIC, creşterea reuşitei studenţilor, dar şi succese mai bune la Olimpiada universitară TIC, organizată anual în luna mai.

Bibliography

[1] ***, Documente Google – Google Play Ajutor – Google Support, https://support.google.com/googleplay/answer/7394452?hl=ro.

[2] Evdochimov R., *Utilizarea diverselor resurse pentru elaborarea materialelor interactive, folosite în predare – învăţare – evaluare prin intermediul tablei interactive. Bunele practici de instruire e-learning– CRUNT*, Chişinău, 24 - 27 Septembrie 2014, pp. 120-123.

[3] ***, *Hotărâre a Guvernului Republicii Moldova cu privire la Strategia naţională de dezvoltare a societăţii informaţionale, "Moldova Digitală 2020", nr. 857 din 31.10.2013*, Monitorul Oficial nr.252-257/963 din 08.11.2013.

[4] ***, *Learning Apps.*, `https://learningapps.org`

[5] Avramescu Ana Nicoleta, *Platforma educaţională Moodle, un succes în e-learning*, `http://www.elearning.ro/platforma-educationala-moodle-un-succes-in-e-learning`

Complex Mathematical Analysis Applied on Potential Theory and Dynamical Systems in Engineering Problems

Mircea Lupu[1,2], Olivia Florea[1]

[1] *Transilvania University of Brasov, Faculty of Mathematics and Informatics, Romania;*
[2] *Member of A. O. Ş. R.*
e-mail: `m.lupu@unitbv.ro, olivia.florea@unitbv.ro`

In this paper we will present the efficiency of the methods and the theoretical support which are applied at different courses and subject matters, showing the best learning way and the interaction of mathematics with new domains of research. These topics are indirect integrating in the post - university, post - graduate and doctoral courses - aligned to the European reform education, which we are adhering. The themes which are presenting are reefers to:

1. Using the complex functions in the determining of the plane potential functions with singularities, these fields are encountered in hydrodynamics, electromagnetism, heat, symmetry theorems and potentials. The dynamical fields came in contact with screens, obstacles and the field lines modify. We will give a method for the determination of the new fields and the influence of the non-homogeneous environments.

2. Using the complex functions in elasticity are obtained new methods for the thermic potential

3. The determination of the potential spatial fields with axial symmetry knowing the potential plane fields which are generated with the complex functions.

4. Using the complex functions in the discrete dynamical systems to determine the stability criterion (the difference equations) very useful in biology, automatics, robotics, telecommunications . In this situation the dynamic of evolution occurs in the discreet time (the second, the hour, the day, the month, the year)

5. The inverse problems and the integral equations method in the optimization

Bibliography

[1] M. Lupu, *L'Etude de l'equation integrale dans la probleme inverse*, The Memories od Acad. Rom. IX, 1996.

[2] M. Lupu, E. Scheiber, *Optimization method in Aerodynamics Integral Nonlinear operator*, J. Comput. And Appl. Math., Elsevier (2002).

[3] C. Iacob, *Matematici clasice si moderne*, Vol. II, Ed. Tehnica, 1979, Bucuresti.

[4] D. Homentkovski *Complex functions with the applications*, Ed. Tehnica, 1989, Bucuresti.

[5] C. Iacob, *Sur la resolution du probleme biarmonique foundamentale pour le cercle*, Rev. Roum. Math. Pures Appl. 9 (1964), pp 925-928.

[6] M. Lupu, *The Dirichlet problem for biharmonics equation of the half plane*, J. Demonstratio Mathematica 32 (1999) pp 41-53, Poland.

[7] M. Lupu, O. Florea, *Exact Solutions And Analytical Methods For Special Limit Problems Applied In Continuous Environments Mechanics*, Transilvania Univ. of Brasov Printing House, ISBN (10) 973-635-827-5, (13) 978-973-635-827-2, pp. 65 – 74.

Interactive Environment GeoGebra Application for Teaching Students the Fields of Mathematics

Lilia Solovei

State University of Moldova, Chişinău, Republic of Moldova
e-mail: soloveililia@gmail.com

It has recently become topical the usage of various packages of computer mathematics systems to improve the efficiency of the educational process. In our presentation we discuss the role of visualization when studying higher mathematics, also the application possibilities of the GeoGebra interactive geometry environment for making a second-order curve image. The GeoGebra interactive geometry environment application in studying second-order curves makes it possible to ensure studied visualization of mathematics objects. Using GeoGebra expediently when studying the areas of mathematics, where it is necessary to make not only calculation but also the geometrical construction, that contributes to better understanding of the studied material, to the development of spatial thinking, facilitates and accelerates the process of problem solving [1]. For instance, multimedia support created with the help of GeoGebra is used at the lecture on *Multiple integrals* for calculating the volume of solid with limited surface area, for making the right choice of a picture plane, and for determining the limits of integration. Colored pictures will help the students form the general view of these surfaces, so they could construct similar shapes in future. GeoGebra can export some images in graphic form (function graphs, figure constructions etc.) and they can be easily placed into the text document, so it allows to make good-looking sketches for lectures, tests and tasks.

GeoGebra is an instrument of electronic digital resources that ensures the most important educational principle – principle of visibility. This program is a very good instrument for promoting the quality of teaching higher mathematics at universities and colleges [2]. The application of interactive teaching means along with the classical education will make it possible to create an up-to-date higher mathematics course.

Bibliography

[1] Hohenwarter, J., Hohenwarter, M., *Introducing Dynamic Mathematics Software to Secondary School Teachers: The Case of GeoGebra*, Journal of Computers in Mathematics & Science Teaching: Association for the Advancement of Computing in Education, 2009.

[2] Petrovici, A., Rădulescu, Z., *Platforma Moodle, softurile GeoGebra şi Wiris - vectori de continuitate a instruirii matematice între liceu şi facultate*, XXI-a Conferinţă Anuală Societăţii de Ştiinţe Matematice din Romˆania, 2017.

Etapa de Pregătire a unui experiment pedagogic la unitatea de curs Tehnologii informaţionale şi comunicaţionale

Lidia Popov

Universitatea de Stat "Alecu Russo" din Bălţi,
Universitatea de Stat din Tiraspol, Republica Moldova
e-mail: popov.lidia@usarb.md

În lucrare se descrie etapa de pregătire a unui experiment pedagogic la unitatea de curs Tehnologii informaţionale şi comunicaţionale (TIC) realizată de autor, sunt prezente instrumentele şi metodele de lucru. În calitate de model serveşte un experiment real, în care au fost implicaţi 343 de studenţi, în doi ani de studii, 2014 - 2015 şi 2015 - 2016 din cadrul Universităţii de Stat "Alecu Russo" din Bălţi, Facultatea de Drept şi Ştiinţe Sociale, specialităţile: Drept, Administraţie publică şi Asistenţă socială, atât de la învăţământul cu frecvenţă (F), cât şi de la învăţământul cu frecvenţă redusă (FR).

Pentru veridicitatea rezultatelor experimentului pedagogic [1,2] este necesar de selectat corect grupele de control şi cele experimentale, criteriile de bază fiind numărul aproximativ egal de studenţi şi nivelul de pregătire comparabil în grupele de experiment şi de control corespunzătoare.

Grupele au fost împărţite pe subgrupe: subgrupa experimentală şi subgrupa de control. De asemenea, a fost verificat dacă între fiecare din cele două subgrupe create nu există diferenţe semnificative între medii. În acest scop s-a folosit testul t - *Student* pentru două subgrupe independente.

Calculele au fost efectuate utilizând aplicaţia SPSS (Statistical Package for the Social Sciences) ca fiind unul dintre cele mai aplicate programe statistice pentru analiza datelor în domeniul Ştiinţe ale Educaţiei [3].

Subgrupa experimentală şi de control sunt independente deoarece, studentul A este inclus într-o singură subgrupă.

În continuare formulăm următoarele două ipoteze de cercetare în vederea repartizării studenţilor pe subgrupe experimentale şi de control:

1. **H0**: $m_1 = m_2$ - nu există diferenţe semnificative între media subgrupei experimentale şi media subgrupei de control;
2. **H1**: $m_1 \neq m_2$ - există diferenţe semnificative între media subgrupei experimentale şi media subgrupei de control.

Aplicăm testul t - *Student* pentru a verifica dacă există diferenţe semnificative între mediile obţinute la testarea iniţială de către subgrupele experimentale şi de control.

Condiţia egalităţii varianţei este testată cu teste specifice, de exemplu testul LEVENE, iar în funcţie de rezultat se calculează t pe două căi. Dacă cele două subgrupe au aceleaşi varianţe, compararea mediilor se poate face conform formulei:

$$t = \frac{m_1 - m_2}{\sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

unde n_1 şi n_2 - numărul de subiecţi din fiecare eşantion; m_1 şi m_2 - mediile celor două subgrupe experimentală şi de control; s_1 şi s_2 - abaterile (deviaţia) standard pentru fiecare subgrupă (eşantion). Formula (1) se aplică în cazul în care abaterile standard s_1 şi s_2 sunt egale. În cazul în care acestea sunt diferite, se aplică formula (2).

$$t = \frac{m_1 - m_2}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$$

Rezultatul p al testului t, furnizat ca un număr între 1 şi 0, reprezintă probabilitatea de a face o eroare dacă respingem ipoteza **H0** (ipoteză de nul). Dacă p este mai mic decât pragul de semnificaţie $\alpha = 0.05$, atunci respingem ipoteza **H0** şi admitem ca adevărată ipoteza **H1**, în caz contrar considerăm adevărată ipoteza **H0**.
În urma calculelor efectuate cu ajutorul aplicaţiei SPSS s-au obţinut indicatorii statistici de bază pentru fiecare subgrupă implicată în experimentul pedagogic [1].
Analizând aceşti indicatori, conchidem că nu există diferenţe semnificative între medii, din acest motiv se respinge ipoteza **H1** şi se adevereşte ipoteza **H0**, adică nu există diferenţe semnificative între media subgrupelor experimentale şi media subgrupelor de control. În aşa fel, considerăm că repartizarea pe subgrupe experimentale şi de control, în anii de studii 2014- 2015 şi 2015 - 2016 s-a efectuat corect, ceea ce permite efectuarea experimentului pedagogic.

Bibliography

[1] Aniţei, M., *Psihologie experimentala*, Iaşi: Editura Polirom, 2007. 400 p.

[2] Dumitriu, C., *Introducere în cercetarea psihopedagogică*, Bucureşti: Editura didactică şi pedagogică, R.A., 2004. 230 p.

[3] Labăr, A., SPSS *pentru Ştiinţele Educaţiei*, Iaşi: Editura Polirom, 2008. 347 p.

The Design of Metacognitive Strategies for Training Abilities to Solve Combinatorics Problems

Marcel Teleucă, Larisa Sali

Universitatea de Stat din Tiraspol
e-mail: mteleuca@gmail.com, salilarisa@gmail.com

In our research, we started from the thesis that teachers' metacognitive actions positively impact student performance on state and national tests as well as on measures of higher-order thinking more typically associated with metacognition. It will be necessary to examine more closely the way in which teachers adapt. That is, how do they decide to make the adaptations they do? What knowledge do they access? What mental process do they engage in? Obviously, we must learn more about this process if we are to teach other teachers to engage in metacognitive actions. [1, p. 249]
Polya G. considered that a teacher of mathematics has a great opportunity to challenge the curiosity of his students by assigning them problems proportionate to their knowledge. The teacher who wishes to develop his students' ability to solve problems must instill some interest in problems into their minds and give them plenty of opportunity for imitation and practice. [2, p.5] Researchers in longitudinal studies aimed to provide the students with mathematical problems for which they did not know the algorithms, and which would provide them opportunities to find patterns, be systematic, and generalise findings. Combinatorics problems were well suited for these goals. We will consider ideas that are elicited by the tasks that were used in these longitudinal studies. [3, p.18] In many solvable problems using combinatorial elements, various real situations structured on the same mathematical model are described. The statement of the problem hides the common structure, and the role of the solver is to reveal the relationships between the dimensions that appear therein. One of G. Polya's recommendations given to solvers is "If you cannot solve the proposed problem, solve first a suitable related problem!" [4, p. 2]
In this presentation, we provide a procedure for the design of metacognitive strategies for the

development of problem solving abilities based on the examination of a cascade of suitable combinatorial problems, which allows the identification of these problems' solutions in a retrospective manner.

The process can be illustrated using the following set of problems, which are solved by creating combinatorial series:

Problem 1. How many different ways are there to spell out "abracadabra", always going from one letter to an adjacent letter? (The statement of the problem contains the image of the letters "abcdr" positioned within a square with a vertical diagonal. [4, p. 2])

Problem 2. In a network of streets of a city all blocks are the same size. How many ways are there of getting from the northern corner to the southern corner in the minimum number (10) of blocks? (That 10 is the minimum can be seen from the fact that each block, in addition to taking us either east or west, takes us southward one-tenth the total southward distance between the two corners. [4, p. 2])

Problem 3. A town in form of a rectangle is given with vertexes: A (south-west), B (north-west), C (north-east), D (south-east). The streets are situated parallel to AB or parallel to BC. Let n be length of AB, m length of BC. The tourist travels from A to C, passing the streets of the town either in the northern or eastern direction. How many ways are there for the tourist to manage that?

Problem 4. (Moivre problem) How many positive integer solutions does the equation $x_1 + x_2 + x_3 + \cdots + x_n = k$ have?

Problem 5. (Tube problem) A tube is given. It is filled with blue and red balls of the same size (in particular, radius of the bottom equals radius of the balls, so that balls can be placed in the tube one by one in vertical trajectory) in the following way: first, k_1 blue balls are placed, then one red one is added; after that k_2 blue balls are added and then one red ball is added and so on, finally, k_n blue balls are added and the last red ball is added. So, n is the number of red balls, $k_1 + k_2 + k_3 + \cdots + k_n = m$ - the number of blue balls. How many ways are there to place m blue balls in tube?

Bibliography

[1] ***, *Handbook of metacognition in education*, eds. Douglas J. Hacker, John Dunlosky, Arthur C. Graesser, in *The educational psychology series*, Routledge, 2009, 462 p.

[2] Polya G, *Hoe to Solve It. A New Aspect of Mathematical Method*, Second Edition, Princeton University Press, Princeton, New Jersey, 1975.

[3] Maher CA, Powell AB, Uptegrove E, *Combinatorics and reasoning: representing, justifying and building isomorphisms*, Springer, New York, 2011.

[4] Woods Donald R, *Notes on introductory combinatorics*, Stanford University, California, 1979.

Asupra unei ecuaţii Diofant

Boris Ţarălungă[1], Tatiana Druc[2]

[1] *UPS "Ion Creangă", Chişinău, Republica Molodova;*
[2] *Instituţia Publică Liceul Teoretic, Varniţa, Republica Molodova*
e-mail: Borisstar@mail.ru, dructa@gmail.com

Un subiect important în teoria numerelor este studiul ecuaţiilor Diofant, ecuaţii pentru care sunt permise numai soluţii întregi. În [1] sunt prezentate soluţiile ecuaţiilor

$$x^2 + y^2 + z^2 + t^2 = w^2, x^2 + y^2 + z^2 = w^2, x^2 + y^2 = w^2.$$

În lucrarea dată se cercetează ecuaţia Diofant de forma

$$x^2 + y^2 + z^2 + t^2 + v^2 = w^2 \tag{1}$$

Soluţia $(x_0, y_0, z_0, v_0, w_0)$, a ecuaţei (1) se numeşte soluţie primitivă, dacă c.m.d.c$(x_0, y_0, z_0, t_0, v_0, w_0) = 1$. Se arată,că soluţiile primitive ale ecuaţiei (1) sunt date de formulele:

$$\begin{aligned}
&x = m^2 - n^2 - p^2 + q^2 + r^2 + s^2,\\
&y = 2mn + 2pq,\\
&z = 2mp - 2nq,\\
&t = 2nr + 2ps,\\
&v = 2ns - 2pr,\\
&w = m^2 + n^2 + p^2 + q^2 + r^2 + s^2,
\end{aligned}$$

unde m, n, p, q, r, s sunt numere întregi pentru care c.m.d.c$(m, n, p, q, r, s) = 1$, iar toate soluţiile ecuaţiei (1) sunt date de formulele:

$$\begin{aligned}
&x = (m^2 - n^2 - p^2 + q^2 + r^2 + s^2)k,\\
&y = (2mn + 2pq)k,\\
&z = (2mp - 2nq)k,\\
&t = (2nr + 2ps)k,\\
&v = (2ns - 2pr)k,\\
&w = (m^2 + n^2 + p^2 + q^2 + r^2 + s^2)k,
\end{aligned}$$

unde k este un numar întreg

Bibliography

[1] R.D. Carmichael, *Diophantine analysis*, New-York, 1915,Add. John Wiley and Sons.

Implementarea algoritmilor de rezolvare a problemelor din combinatorică

Vitalie Ţîcău

Universitatea de Stat "A. Russo" din Bălţi
e-mail: vitaliesticau@gmail.com

Deseori apar următoarele probleme: de a alege dintr-o mulţime oarecare de obiecte – numite elementele mulţimii – submulţimi de elemente care posedă anumite proprietăţi; de a aranja elementele uneia sau a mai multe mulţimi într-o anumită ordine; de a determina numărul tuturor submulţimilor unei mulţimi, constituite după anumite reguli. Deoarece în astfel de probleme este vorba de anumite combinaţii de obiecte, ele se numesc probleme de combinatorică. Domeniul matematicii care studiază astfel de probleme este combinatorică. Combinatorica are o importanţă considerabilă pentru teoria probabilităţilor, cibernetică, logica matematică, teoria numerelor, precum şi pentru alte ramuri ale ştiinţei şi tehnicii. În lucrare sunt cercetate avantajele aplicării elementelor din combinatorică la rezolvarea unor probleme practice şi este elaborată o aplicaţie de rezolvare a unor astfel de probleme.

Au fost cercetate şi rezolvate un sir de exemple tipice:

- aplicarea binomului Newton;
- generarea de permutări, combinări, aranjamente;

- calculul numărului de variante de generare a numerelor distincte, alcătuite din unele cifre indicate;
- diversificarea componenței delegațiilor;
- formarea echipelor de dansatori;
- aranjarea mesei într-o odaie din căminul studențesc;

Exemple prezentate în aplicație prezintă interes în practică şi permit de a înțelege avantajele aplicării permutărilor, combinărilor, aranjamentelor în practică.

Bibliography

[1] Lupu, I., Poştaru, A., *Metodologia studierii combinatoricii*, Chişinău: Prut internațional, 2007, 108 p.

[2] Achiri, I., Garit, V., Poştaru, A., Prodan, N., *Matematică. Manual pentru clasa a X-a*, Chişinău: Prut Internaţional, 2002.

[3] Goian, I., Grigor, R., Marin, V., Smarandache, F., *Algebra în exerciţii si probleme pentru liceu*, Chişinău: Cartier, 2000.

[4] ***, *Permutări. Combinări. Aranjamente. Binomul lui Newton.*, `https://www.scribd.com/doc/38645013/Permutari-combinari-aranjamente-binomul-lui-Newton`

Resolvarea ecuaţiilor prin metoda iterativă

Ţurcanu Alina[1], Domentean Rita[2]

[1] *Technical University of Moldova, Chişinău, Republic of Moldova;*
[2] *State University of Moldova, Chişinău, Republic of Moldova*

e-mail: `turcan.alina@civis.md, ritaturcanu@yahoo.com`

De obicei, ecuaţiile care apar în practică, au coeficienţii obţinuţi prin anumite măsurări. Cel mai des aceşti coeficienţi nu pot fi calculaţi exact, ci în mod aproximativ. La rândul său, rădăcinile acestor ecuaţii sunt calculate cu o anumită precizie, comparativ egală cu precizia de calcul a coeficienţilor ecuaţiei. Astfel apare necesitatea de a rezolva unele ecuaţii în mod aproximativ. Sunt cunoscute foarte multe metode de rezolvare aproximativă a ecuaţiilor şi sistemelor de ecuaţii, cum ar fi: Metoda bisecţiei (înjumătăţirii intervalului), Metoda falsei poziţii (metoda coardei, metoda secantei, metoda împărţirii intervalului în părţi proporţionale), Metode de tip Newton, Metoda Jacobi, Metoda Gauss-Seidel, Metoda suprarelaxării, Metoda lui Richardson, Metoda gradientului conjugat, etc.
În metodele iterative se construieşte un şir $\{x_n\}_{n\geq 0}$ de aproximaţii succesive al soluţiei ecuţiei date. Pentru o ecuaţie de forma $\phi(x) = x$ se porneşte de la o aproximaţie iniţială $x = x_0$ şi se calculează iterativ $x_{n+1} = \phi(x_n)$. Evident convergenţa şirului depinde de proprietăţile funcţie $\phi(\cdot)$.
Fie de rezolvat ecuaţia $f(x) = g(x)$. Schematic algoritmul de calcul al soluţiei poate fi descris dupa cum urmează.

Algoritmul de calcul

```
1   Se defineşte φ(x) = x + f(x) − g(x)
2   Se definesc: x_0, precizia ϵ, numărul maxim de iteraţii admis, N_ϵ, n = 0
3   Se calculează x_{n+1} = φ(x_n), ξ_n = |x_{n+1} − x_n|
4   Test: dacă ξ_n < ϵ atunci Go to 8
5   Test: dacă n > N_ϵ atunci Go to 9
6   n = n + 1
7   Go to 3
8   Convergenţă, x = x_{n+1}, stop
9   Algoritm divergent, stop.
```

În lucrare sunt analizate mai multe exemple care pun în evidenţă avantajele şi dezvantajele metodei.

Legături interdisciplinare Matematica-Informatica în interogarea bazelor de date: Calcul relaţional şi SQL

Teodora Vascan

Tiraspol State University, Republic of Moldova
e-mail: teodora_vascan@mail.ru

În lucrare este descrisă metodologia aplicării cunoştinţelor matematice în studiul bazelor de date. În particular, este analizat calculul relaţional aplicat la interogarea bazelor de date pe exemplul limbajului SQL.

Elaborarea scenariilor pedagogice la matematică prin învăţarea în bază de cercetare-investigare

Tatiana Veveriţa

Universitatea de Stat din Tiraspol, Chişinău, Republica Moldova
e-mail: veveritat@gmail.com

Noua paradigmă a educaţiei în învăţământul preuniversitar a determinat nivelul sporit de complexitate a finalităţilor de studii prin apariţia necesităţii de dezvoltare echilibrată a tuturor competenţelor cheie ale secolului XXI, inclusiv competenţele de cercetare şi investigare.
Una din direcţiile pe care se axează Strategia Europa 2020 (continuitate a strategiei Lisabona) o constituie creşterea economică bazată pe cunoaştere şi implicit competitivitate, punându-se accent pe activitatea de cercetare-dezvoltare. La baza învăţării prin cercetare-investigare stau cinci principii:

1. Utilizează întrebări care provin de la nivelurile superioare ale taxonomiei lui Bloom.
2. Implică întrebări interesante şi motivante pentru elevi.
3. Utilizează o mare varietate de resurse, astfel încât elevii să poată colecta informaţii.
4. Profesorii joacă un rol nou - ca îndrumător sau facilitator.
5. Produsele *pline de înţeles* provin din învăţarea bazată pe cercetare.

Pentru a realiza un scenariu pedagogic la matematică prin învăţarea în bază de cercetare-investigare trebuie să răspundem la următoarele întrebări:

✓ Cum să abordăm investigaţia?

✓ Cum să dezvoltăm abilităţile de cercetare/investigare?

✓ Cum să introducem la clasă noi concepte ştiinţifice folosind investigaţia?

Într-o învăţare bazată pe cercetare este importantă:

- Autenticitatea sarcinilor şi activităţilor de învăţare;
- Înţelegerea aprofundată a ideilor preconcepute;
- Performanţa înţelegerii;
- Evaluarea;
- Utilizarea corespunzătoare a tehnologiei;
- Legătura cu experţii care oferă sarcini autentice şi atrăgătoare;
- Succesul elevilor.

Cercetătoarea L. Ciascai propune următorul model de predare-învăţare bazat pe investigare [1]:

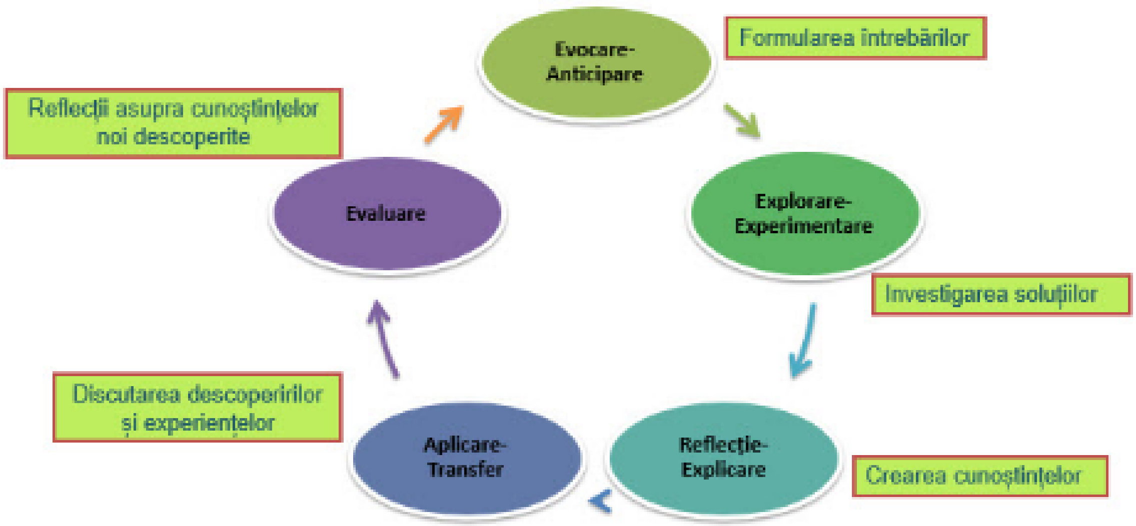


Figure 1. Model de predare-învăţare bazat pe investigaţie

Deoarece învăţarea bazată pe cercetare are la bază punerea întrebărilor, instructorii înşişi trebuie să înveţe arta de a pune întrebări bune. În acest context întrebările bune posedă următoarele caracteristici:

- Întrebările trebuie formulate astfel încât să se poată răspunde la ele;
- Răspunsul nu poate fi un simplu fapt;
- Răspunsul nu poate fi deja cunoscut;
- Întrebările trebuie să aibă la bază anumite obiective pentru un răspuns;

- Întrebările nu pot fi prea personale.

Într-o cercetare întrebările pot fi de cinci tipuri:

1. Întrebări deductive.
2. Întrebări interpretative.
3. Întrebări de transfer.
4. Întrebări referitoare la ipoteze.
5. Întrebări reflexive.

În formularea întrebărilor deductive este important:

- Să cereţi elevilor să depăţească informaţiile disponibile;
- Să cereţi elevilor să caute indici, să-i examineze şi să decidă dacă au un careva rol în problema cercetată.

Conţinutul întrebărilor interpretative trebuie să prezică ce consecinţe pot să apară ca urmare a unui anumit scenariu şi să combine cunoştinţele anterioare despre careva situaţii sau informaţii noi despre anumite fapte.
Întrebările de transfer trebuie să fie focusate pe aplicarea cunoştinţelor în situaţii noi şi pe extinderea gândirii, iar cele referitoare la ipoteze să prezică anumite rezultate şi să conducă la descoperirea noilor cunoştinţe.
La crearea întrebărilor reflexive este important să cereţi elevilor să compare convingerile pe care le au cu dovezile reale şi să conduceţi elevii înapoi la investigaţie.
Prin acest model de elaborare a scenariilor pedagogice autorul valorizează învăţarea prin investigaţie care presupune implicare şi dezvoltare a abilităţilor de gândire la nivel superior.

Bibliography

[1] Cascai.L. *Model ciclic de predare-învăţare bazat pe investigaţie*, Presa universitară clujeană. 2016. 57p.

[2] Adavidoaiei M., *Consolidarea motivaţiei elevilor prin metoda învăţării bazate pe cercetare - investigare : Ghid pentru profesori, învăţători şi educatori*, Iaşi : PIM, 2015.

[3] Ali Abdi, *The Effect of Inquiry-based Learning Method on Students' Academic Achievement in Science Course*, Universal Journal of Educational Research 2(1): 37-41, 2014.

[4] Cleverly D, *Implementing inquiry-based learning in nursing* London, NY: Routlege, Taylor&Francis e-Lybrary.

[5] Masha Smallhorn, Jeanne Young, Narelle Hunter and Karen Burke da Silva, *Inquiry-based learning to improve student engagement in a large first year topic*, Student Success ISSN: 2205-0795 Volume 6, Issue 2, pp. 65-71 August 2015.

The beginning development of pedagogical skills of future teachers using the means of informational and communicational technologies

Liubov Zastînceanu

"Alecu Russo" State University of Bălţi, Republic of Moldova
e-mail: liubaz@mail.ru

Modern university studies imply an obligatory use of opportunities offered by informational and communicational technologies. The MOODLE platform connection and access possibility to different digital resources directly from the platform: information depositors, interactive testing systems, specialized educational software, URLs and the opportunity to create well planned interactive activities allows the creation of a course that is electronic, flexible, non linear and adaptive. The students from the Educational Science domain, during the university studies always face a dual situation: on one hand, they are the ones that are trained, on the other any of the methods, strategies, applied technologies are involuntarily appreciated by them as pedagogical efficiency and potential use in future professional activity. In this article we will present the impact of using the opportunities offered by the information and communicative technologies on the development of pedagogical skills trening.

Index

Index