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Plenary Talks

Bloch-Iserles Hamiltonian system

Vasile Brînzănescu

Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Romania
e-mail: Vasile.Brinzanescu@imar.ro

In a paper in 2009, Bloch, Brînzănescu, Iserles, Marsden and Ratiu studied the Hamiltonian system introduced by Bloch and Iserles in 2006 and showed that this system is completely integrable. In 2015, Brînzănescu and Ratiu proved that this Hamiltonian system is algebraically complete integrable. We shall present the ideas of the proofs.

General concept of multiplied fixed points of mappings

Mitrofan M. Choban

Tiraspol State University, Republic of Moldova
e-mail: mmchoban@gmail.com

In 1987, D. Guo and V. Lakshmikantham [5] introduced and initiated the study of the coupled fixed point. In 2010, Samet and Vetro presented one concept of a fixed point of m-order as an extension of the coupled fixed point. In 2004 A.C.M. Ran and M.C.B. Reurings and in 2006 T. Gnana Bhaskar and V. Lakshmikantham are proved interesting fixed point theorems in partially ordered metric spaces. Later, V. Berinde and M. Borcut introduced the concept of tripled fixed point and proved tripled fixed-point theorems using mixed monotone mappings. After that a substantial number of articles was dedicated on tripled fixed point and quadruple fixed point theory (see [3]). In [6, 7] were introduced a new concept of a fixed point of m-order.

Our main aim is to introduced a general concept of a multidimensional fixed point in the ordered spaces with distance [4].

Fix $m \in \mathbb{N} = \{1, 2, ...\}$. Denote by $\lambda = (\lambda_1, ..., \lambda_m)$ a collection of mappings $\{\lambda_i : \{1, 2, ..., m\} \longrightarrow \{1, 2, ..., m\} : i \leq m\}$.

Let X be a space and $F : X^m \longrightarrow X$ be an operator (mapping). The operator F and the mappings λ generate the operator $\lambda F : X^m \longrightarrow X^m$, where $\lambda F(x_1, ..., x_m) = (y_1, ..., y_m)$ and $y_i = F(x_{\lambda_i(1)}, ..., x_{\lambda_i(m)})$ for each point $(x_1, ..., x_m) \in X^m$ and any index $i \leq m$.

A point $a = (a_1, ..., a_m) \in X^m$ is a λ-multiplied point of the operator F if $a = \lambda F(a)$, i. e. $a_i = F(a_{\lambda_i(1)}, ..., a_{\lambda_i(m)})$ for any index $i \leq m$. This concept of multiplied point is developed in context with methods from [1, 2, 4].

Let (X, d) be a distance space, $m \in \mathbb{N} = \{1, 2, ...\}$. On X^m consider the distance

– $d^m((x_1, ..., x_m), (y_1, ..., y_m)) = sup\{d(x_i, y_i) : i \leq m\}$,

– $\bar{d^m}((x_1, ..., x_m), (y_1, ..., y_m)) = \Sigma\{d(x_i, y_i) : i \leq m\}$.

We say that the operator F:

- is a λ-*contraction* if there exist a number $k \in [0, 1)$ such that

$$d(F(x_1, ..., x_m), F(y_1, ..., y_m)) \leq ksup\{d(x_i, y_i) : i \leq m\}$$

for all $(x_1, ..., x_m), (y_1, ..., y_m) \in X^m$;

- is a $\bar{\lambda}$-*contraction* if there exist a number $k \in [0, 1)$ such that

$$d(F(x_1, ..., x_m), F(y_1, ..., y_m)) \leq k/m\Sigma\{d(x_i, y_i) : i \leq m\}$$

for all $(x_1, ..., x_m), (y_1, ..., y_m) \in X^m$.

We analyze some results of the following kind:

Theorem 1. *Let (X, d) be a complete s-distance symmetric space.*

1. If F is a λ-contraction, then any Picard sequence of the operator λF is a convergent Cauchy sequence and F has a unique multidimensional fixed point.

2. If F is a $\bar{\lambda}$-contraction and for any $i \leq m$ the mapping λ_i is a surjection or, more general, $| \cup \{\lambda_i^{-1}(j) : j \leq n\}| = m$ for each $i \leq m$, then any Picard sequence of the operator λF is a convergent Cauchy sequence and F has a unique multidimensional fixed point.

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The inverse problem and the coalitional rationality for values of cooperative TU games

Irinel Dragan

University of Texas, Mathematics, Arlington, Texas, USA
e-mail: dragan@uta.edu

In this paper, we discuss two algorithmic procedures for solving what we call The Inverse problem, with coalitional rationality constraints, for cooperative TU games. First, for a TU game with the Shapley Value L find out a game, with the same Shapley value L, that belongs to the Core of the game. Second, for a TU game with the Semivalue M, defined by a weighted vector p subject to some normalization condition, find out a game, with the same Semivalue M, that belongs to its Power Core of the game. The main tools are the potential bases of the vector space of TU games, using the Hart/Mas-Colell potentials in the first case, and our potentials in the second case. A few examples illustrate the procedures.

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Approximation of the controls for the beam equation

Sorin Micu

Department of Mathematics, University of Craiova, Romania
e-mail: sd_micu@yahoo.com

We consider the following equation

$$\begin{cases} \ddot{u}(x,t) + \partial_x^4 u(x,t) = 0, & (x,t) \in (0,1)\times(0,T) \\ u(x,0) = u_0(x), \quad \dot{u}(x,0) = u_1(x), & x \in (0,1), \end{cases} \tag{1}$$

modelling the vibration of an elastic beam, where $\dot{u}$ denotes the derivative of u with respect to time. To (1) we may attach different boundary conditions. For instance, if we consider that

$$\begin{cases} u(0,t) = u(1,t) = 0, & t \in (0,T) \\ \partial_x^2 u(0,t) = 0, \quad \partial_x^2 u(1,t) = v(t), & t \in (0,T), \end{cases} \tag{2}$$

then we are dealing with a controlled hinged beam, whereas

$$\begin{cases} u(0,t) = u(1,t) = 0, & t \in (0,T) \\ \partial_x u(0,t) = 0, \quad \partial_x u(1,t) = v(t), & t \in (0,T), \end{cases} \tag{3}$$

models a controlled clamped beam. In both cases the function v is a control acting at the extremity $x = 1$ of the beam. It should be chosen in such a way to ensure that the beam reaches the equilibrium $u(\cdot,T) = \dot{u}(\cdot,T) = 0$ at some time $T > 0$.

Our aim is to study the approximation of the boundary controls v of (1) by using a finite difference space semi-discrete scheme. More precisely, for each $N \in \mathbb{N}^*$, we consider the equidistant points, $x_j = jh$, $-1 \le j \le N+2$, where $h = \frac{1}{N+1}$ represents the mesh-size. A finite differences space semi–discretization of (1) is given by the following system

$$\begin{cases} u_j''(t) + \frac{u_{j+2}(t) - 4u_{j+1}(t) + 6u_j(t) - 4u_{j-1}(t) + u_{j-2}(t)}{h^4} = 0 & 1 \le j \le N, \ t \in (0,T) \\ u_j(0) = u_j^0(x), \quad u_j'(0) = u_j^1(x) & 1 \le j \le N. \end{cases} \tag{4}$$

To this we have to add the discrete boundary conditions

$$\begin{cases} u_0(t) = 0, \quad u_{N+1}(t) = 0 & t \in (0,T) \\ u_{-1}(t) = -u_1(t), \quad u_{N+2}(t) = h^2 v_h(t) - u_N(t) & t \in (0,T), \end{cases} \tag{5}$$

or

$$\begin{cases} u_0(t) = 0, \quad u_{N+1}(t) = 0 & t \in (0,T) \\ u_{-1}(t) = u_1(t), \quad u_{N+2}(t) = 2hv_h(t) + u_N(t) & t \in (0,T), \end{cases} \tag{6}$$

corresponding to (2) and (3), respectively.

The resulting system consists of N linear equations with N unknowns $u_1, u_2, \ldots, u_N$. For each $j \in \{0, 1, ..., N+1\}$, the quantity $u_j(t)$ approximates $u(t, x_j)$, the solution of (1) at time t and in the point x_j, provided that $\left(u_j^0, u_j^1\right)_{1\le j\le N}$ is an approximation of the initial data $\left(u^0, u^1\right)$ of (1).

Due to the high frequency numerical spurious oscillations, the corresponding semi-discrete model is not uniformly controllable with respect to the mesh-size and the convergence of the approximate controls v_h corresponding to initial data in the finite energy space cannot be guaranteed. In the hinged beam case (4)-(5), we analyze how do the initial data to be controlled and their discretization affect the result of the approximation process. We prove that the convergence of the scheme is ensured if the continuous initial data are sufficiently regular or if the highest frequencies of their discretization have been filtered out. In the clamped beam case (4)-(6) we show that we can uniformly control a projection of the discrete solution over a space of finite dimension, which tends to infinity as the mesh size goes to zero.

Well-posedness and numerical analysis for a nonlinear reaction-diffusion equation

Costică Moroşanu

Faculty of Mathematics, "Al. I. Cuza" University, Iaşi, Romania
e-mail: `costica.morosanu@uaic.ro`

The paper concerns with the existence, uniqueness, regularity and the approximation of solutions to the reaction-diffusion equation endowed with a cubic nonlinearity and Neumann boundary conditions, relevant in a wide class of physical phenomena, including phase separation and transition. The convergence and error analysis for an iterative scheme of fractional steps type, are also established. We prove L^∞ stability by maximum principle arguments and derive error estimates using energy methods for two implicit-explicit approaches, a linearized scheme and a fractional step method. A numerical experiment validates the theoretical results, comparing the accuracy of the methods.

MSC: 35Bxx; 35K45; 35K55; 35K57; 35K60; 35Qxx; 74A15; 80Axx.
Keywords: Qualitative properties of solutions; nonlinear initial-boundary value problems for nonlinear parabolic systems; reaction-diffusion equations; thermodynamics, phase-changes.

Open problems in the general efficiency

Vasile Postolică

Romanian Academy of Scientists
"Vasile Alecsandri" University of Bacău, Department of Mathematics, Informatics and Educational Sciences, Romania
e-mail: `vpostolica@ambra.ro`

This research work is devoted to significant open problems in the general Efficiency presented inside the best appropriate environment of the infinite dimensional Ordered Vector Spaces, following our refined recent results, especially using the largest class of the Convex Cones discovered till now in separated Locally Convex Spaces, named by us "Isac's Cones", and ensuring the existence together with important properties for the efficient points under completeness instead of compactness. These Open problems are also generated by the new links between the General Efficiency, the Vector (Strong) Optimization and the Choquet Boundaries, since the Efficiency is strongly related to the General Optimization, the Potential Theory and conversely, with projections in the new recent and future fields of Scientific Research: Theory and Applications of the Generalized Dynamical Systems, Fixed points Theory, the Best Approximation Theory, the study of the Conically Bounded Sets, the study of the Nuclearity for the Topological Vector Spaces and being based on the main conclusions given in [1] - [8].

Mathematics Subject Classification 2010 : 90C48; 90B50, 90B70.

Keywords : ordered vector space, general efficiency, multifunction, Pareto optimality, locally convex space, Isac's (nuclear, supernormal) cone (set), Choquet boundary, best approximation, spline function.

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A Hamiltonian approach to implicit systems and applications in optimization

Dan Tiba

"Simion Stoilov" Institute of Mathematics of Romanian Academy, Bucharest, Romania
e-mail: Dan.Tiba@imar.ro dtiba@imar.ro

We discuss general implicit systems in arbitrary dimension via a new approach and in the absence of independence type assumptions. The applications concern shape optimization and nonlinear programming problems.

Third grade fluids from fundamentals to real applications

Victor Ţigoiu

University of Bucharest, Romania
e-mail:

We start this short presentation by putting into evidence the fundamental aspects concerning the general constitutive law for third grade fluids (and the corresponding constitutive restrictions). We emphasize two distinct models. After that we present some results concerning the existence, uniqueness and stability of the solutions for the initial and boundary value problems (we underline the role played by the constitutive restrictions). We finish our presentation with some "real" applications which put into evidence the importance of the above mentioned constitutive restrictions in the description of the behaviour of these fluids.

Section 1. Real, Complex, Functional and Numerical Analysis

Two–parameter exponentially–fitted Obkreckhoff methods for second-order boundary value problems

Moses Adebowale Akanbi and Ashiribo Senapon Wusu

Department of Mathematics, Lagos State University, Lagos, Nigeria
e-mail: Moses.Akanbi@lasu.edu.ng, akanbima@gmail.com

It is well known that classical methods may not be well suited for problems with oscillatory/periodic behaviour. To overcome this deficiency, classical methods are modified using a technique called exponential fittings. This modification makes it possible to construct new methods suitable for the efficient integration of oscillatory/periodic problems from classical ones. A new family of two–parameter two–step exponentially–fitted Obreckkoff methods for approaching problems that exhibit oscillatory/periodic behaviour is proposed. These methods depend upon two frequencies which can be tuned to solve the problem at hand more accurately. The qualitative properties of the constructed methods are investigated. Numerical experiments on standard problems confirming the theoretical expectations regarding the constructed methods compared with existing standard methods are also presented.

On some properties of Modified Humbert polynomials in several variables

Rabia Aktas

Ankara University, Faculty of Science, Department of Mathematics, Turkey
e-mail: raktas@science.ankara.edu.tr

The aim of this paper is to present a multivariable extension of the generalized Humbert polynomials which include many well-known special polynomials such as Humbert, Louville, Gegenbauer, Legendre, Tchebycheff, Pincherle and Kinney polynomials and to give various properties satisfied by these polynomials. We derive several recurrence relations and expansions in the series of orthogonal polynomials for this family of multivariable polynomials. We also obtain generating functions for some special cases of these polynomials. Furthermore, various families of multilinear and multilateral generating functions are derived and special cases of these generating functions are discussed.

Search of symmetric composition methods of symmetric integrators

Elisabete Alberdi[1], Joseba Makazaga[2], and Ander Murua[2]

[1] *Engineering School of Bilbao, University of the Basque Country (UPV/EHU), Bilbao, Spain*
[2] *Informatika Fakultatea, University of the Basque Country (UPV/EHU), Donostia/San Sebastian, Spain*
e-mail: elisabete.alberdi@ehu.es

Composition methods are useful when solving Ordinary Differential Equations (ODEs) as they increase the order of accuracy of a given basic numerical integration scheme. We will focus on symmetric composition methods involving some basic second order symmetric integrator with different

step sizes [1]. The introduction of symmetries into these methods simplifies the order conditions and reduces the number of unknowns. Several authors have worked in the search of the coefficients of these type of methods: the best method of order 8 has 17 stages [2], methods of order 8 and 15 stages were given in [3, 5, 6], 10-order methods of 31, 33 and 35 stages have been also found [2, 4].

In this work a technique that we have built to obtain 10-order symmetric composition methods of symmetric integrators of $s = 31$ stages (16 order conditions) is explained. Given some starting coefficients that satisfy the simplest five order conditions, the process followed to obtain the coefficients that satisfy the sixteen order conditions is provided.

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Scattered data interpolation using radial basis functions

Alvin Asimi, Adrian Naço, and Lulëzim Hanelli

Department of Mathematic Engineering, Polytechnic University of Tirana, Albania

e-mail: `alvinasimi87@hotmail.com`

In this paper we study the effects of the shape parameter ϵ for the numerical stability and accuracy of the solution to the scattered data interpolation problem (two dimensional data) using Gaussians radial basis functions. For this purpose we report the results of a series of experiments in which we compute Gaussian RBF interpolant with a fixed value of shape parameter ϵ for different data sets. This approach is known as non-stationary interpolation. In the other hand we compute another series of experiments in which we scale the shape parameter ϵ according to the fill distance of the data set. This approach is known as stationary interpolation. Finally, we will compare the results of the two approaches in order to draw conclusions.

On Hamming type distance functions

Ivan Budanaev

Doctoral School of Mathematics and Information Science
Institute of Mathematics and Computer Sciences of ASM
Tiraspol State University, Republic of Moldova

e-mail: ivan.budanaev@gmail.com

By a space we understand a topological T_0-space. In the theory of topological spaces we explore the following problems:

Problem 1. *A space is quasi-metrizable under what conditions?*

Problem 2. *Let X be a subspace of a semigroup Z and let d be a quasimetric on X. What conditions on Z allow for an invariant quasi-metric ρ, which is an extension of d?*

In the present article, we prove the following theorems:

Theorem 1. *On a space X there exists a quasi-metric with natural values if and only if X is an Alexandroff space.*

Theorem 2. *If Z is the free monoid generated by the set X, then for any quasi-metric d on X there exists its invariant extension ρ on Z.*

Theorem 2 will be applied in the study of Hamming, Levenshtein, and other types of distances on strings.

The extension of the Null-Space algorithm to consistent linear systems of linear equations

Doina Carp[1], Constantin Popa[2], and Cristina Şerban[2]

[1] *Constanţa Maritime University, Romania*
[2] *Ovidius University of Constanţa, Romania*
e-mail: doina.carp@gmail.com, cpopa1956@gmail.com, cgherghina@gmail.com

The Null-Space algorithm was introduced by M. Benzi in [1] (see also [2] for a developed version) as a direct method for the numerical solution of square nonsingular linear systems. It uses directional projections onto the hyperplanes defined by the systems equations, with respect to an appropriate set of vectors. In our paper we extend this algorithm to rectangular, but consistent linear systems of equations. The extended algorithm finds a solution of the system, which generally differs from the minimal norm one.

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Fitting exponential data using nonlinear least squares

Catherine Costigan[1], John Coulter[2], and Sabin Tabirca [1]

[1] *University College Cork, Ireland*
[2] *Cork University Maternity Hospital, Ireland*
e-mail: 110325243@umail.ucc.ie

There are many examples in nature of data that decreases exponentially. Most of these decrease to zero but there are also some examples of data that decreases exponentially with a vertical shift ($y(t) = Ae^{-\alpha t} + B$). Data in this form can be difficult to fit a curve to. Techniques such as those used by software like Mathematica can work well for some curves but often, they are fail to converge when fitting a curve of this form to data. It was found that they often fail to converge, in particular, when only a small number of data points are used.

A new method will be presented in this paper that is more accurate and reliable than the Mathematica method. It uses the method of least squares to fit a curve to the data while minimizing the residuals. This method will be tested on both simulated and real data of hCG measurements of women with Gestational Trophoblastic Disease. It has been shown previously that the hCG measurements decrease exponentially according to the curve mentioned.

The new method produces very accurate results on the simulated data. It is then used to fit a curve to the real data in order to make predictions about future hCG measurements. The predictions are not as accurate as other methods produce.

About topologies on some spaces of multifunctions

Anca Croitoru and Gabriela Apreutesei

Faculty of Mathematics, "Alexandru Ioan Cuza" University of Iaşi, Romania
e-mail: croitoru@uaic.ro

The aim of this talk is to present different topologies on some spaces of multifunctions, describing some of their properties and comparative results.

On the control and evaluation of the coverage quality with radio-signals of the celular networks. A Case Study from Tirana, Albania

Shkelqim Kuka[1], Adrian Naço[1], and Robert Kosova[2]

[1] *Faculty of Mathematic Engineering and Physic Engineering, Polytechnic University of Tirana, Albania*
[2]*Department of Mathematics, Faculty of Information Technology, University "Aleksander Moisiu" Durres, Albania*
e-mail: sh.kuka@fimif.edu.al, a.naco@fimif.edu.al, robertko60@yahoo.com

For the most accurate evaluation of the indices that feature a net coverage quality, a very important role do play the disposal of a given most accurate numerical model of the terrain , and also this for the maps to describe the coverage and the land usage. During the implementation of various theoretical models of propagation the signal that they receive and take it into consideration the changes in the Land Cover, of the cellular nets operators in Albania (AMC, Vodafone, Eagle, Plus) it is indispensable that these models have to be confronted with the factual measurements in terrains and various conditions. In this letter are introduced some of the results achieved during the evaluation in order to verify the coverage quality and the accuracy of the afforded models from the modules GRASS-RaPlaT, being based at an investigation made at one of the most populated areas of Tirana. In order to be investigated the nature of differences between the factual signal and the signal calculated via propagation models if they are randomized or they are systematic ones and also this for the sizes of these differences, their dependency as a function of the distance from the relevant source of broadcasting, the correlation of the deflections via urban mediums, etc was realized the so called variographic analysis.

Implementation of all the data and the attaining the results is completed in the modules and technologies "Open Source".

Keywords: Signal valuation, Radio waves, DTM, Cellular network.

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Some Gronwall inequalities and applications

Mihaela Mioara Mirea

Liceul cu Program Sportiv "Petrache Trişcu" Craiova, Romania
e-mail: m_mirea2000@yahoo.com

R. Bellman pointed out in 1953 in his book “Stability Theory of Differential Equations”, McGraw Hill, New York, the Gronwall type integral inequalities of one variable for real functions play a very important role in the Qualitative Theory of Differential Equations.

In this paper are presented some applications of Gronwall inequalities with nonlinear kernels of Lipschitz type to problems of boundedness and convergence to zero at infinity of the solutions of certain Volterra integral equations.

Keywords: integral inequalities, Gronwall.

Poincaré inequalities and geometric properties of metric measure spaces

Marcelina Mocanu

"Vasile Alecsandri" University of Bacău, Romania
e-mail: mmocanu@ub.ro

In the framework of first order calculus on metric measure spaces, Poincaré inequalities provide a way to pass from local to global, from infinitesimal information of the upper gradient to larger scales. In the presence of a doubling measure, Poincaré inequalities guarantee the existence of a richer structure of the metric space, such as a measurable differentiable structure with which Lipschitz functions can be differentiated almost everywhere.

We discuss some geometric implications of Poincaré inequalities based on Banach function spaces, including Orlicz-Poincaré inequalities, generalizing some results known for Poincaré inequalities based on Lebesgue spaces.

Reductional method in perturbation theory of generalized spectral E. Schmidt problem

Davran G. Rakhimov[1], Boris V. Loginov[2], and Anastasiya N. Kuvshinova[2]

[1] *"Branch of the Moscow State University after M.V. Lomonosov in Tashkent, Tashkent, Uzbekistan*
[2] *Ulyanovsk State Technical University,Ulyanovsk, Russia*
e-mail: `davranaka@yandex.com, bvllbv@yandex.ru, erasya@rambler.ru`

In cycle of works at the origin of XX century on linear and nonlinear integral equations E. Schmidt had introduced eigenvalues λ_k, of the operator $B : H \to H$ in a Hilbert space H taking into account their multiplicities and eigenelements $\{u_k\}_1^\infty, \{v_k\}_1^\infty$ satisfying the relations

$$Bu_k = \lambda_k v_k, \;\; B^* v_k = \lambda_k u_k.$$

This allows to extend Hilbert-Schmidt theory on nonsymmetrical completely continuous operators in abstract separable Hilbert spaces (Goursat E., 1933; Mogilevskii, 1958). Some physical applications of E. Schmidt spectral problems are indicated in (Loginov B.V., Pospeev V.E., 1967), in the article (Kuvshinova A. N., Loginov B.V., 2014) it is given the development of two variants Hamilton-Cayley-Arzhanikh theorems (Arzhanikh I.S., 1951; Arzhanikh I.S., Gugnina V.I., 1962) on polynomial dependent by spectral parameter on relevant E. Schmidt spectral problems with development of the corresponding characteristic polynomials by E. Schmidt spectral parameter. In monographs (Ilinsky A.S., Slepyan G.A., 1988; Valovik D.V., Smirnov Yu. G., 2010) nonlinear problems about electromagnetic waves development in waveguides and resonators in nonlinear media are studied, which are essentially spectral E. Schmidt problems in the linearization.

Let E_1, E_2 be Banach spaces $E_1 \subset E_2 \subset H$ with dense embeddings, H be a Hilbert space and $B \in L(E_1, E_2)$ be closed linear operator. Let $\lambda_0 \in R$ - be n-multiple Fredholmian point of the following E. Schmidt unperturbed spectral problem, analytically dependent on E. Schmidt spectral parameter $\lambda \in R$, with relevant E. Schmidt eigenelements $(\varphi_{i0} \;\; \psi_{i0})^T$

$$B\varphi_{i0} = A(\lambda_0)\psi_{i0} \tag{1}$$
$$B^*\psi_{i0} = A^*(\lambda_0)\varphi_{i0} \tag{2}$$

Here it is considered the following perturbed E.Schmidt's spectral problem:

$$B\varphi = A(\lambda, \varepsilon)\psi, \tag{3}$$
$$B^*\psi = A^*(\lambda, \varepsilon)\varphi, \tag{4}$$

where $\mu = \lambda - \lambda_0$, $\varepsilon \in R, |\varepsilon| < \rho_0$ -small parameter, and $A(\lambda, \varepsilon)\psi = \sum\limits_{i+j \geq 0} A_{ij}\mu^i\varepsilon^j\psi$. It is required to determine the perturbed eigenvalues with relevant eigenvectors $(\varphi(\varepsilon) \;\; \psi(\varepsilon))^T$ in the form of series on small parameter ε degrees. In the direct sum $H \oplus H$ our problem can be rewritten in the following matrix form

$$(\mathcal{B}_0 - \mathcal{A}(\lambda_\prime))\Phi \equiv \begin{pmatrix} \mathbf{B} & -\mathbf{A}(\lambda_0) \\ -\mathbf{A}^*(\lambda_0) & \mathbf{B}^* \end{pmatrix} \begin{pmatrix} \varphi(\varepsilon) \\ \psi(\varepsilon) \end{pmatrix} = $$
$$= \sum_{i+j \geq 1} \mu^i \varepsilon^j \begin{pmatrix} 0 & A_{ij} \\ A_{ij}^* & 0 \end{pmatrix} \begin{pmatrix} \varphi(\varepsilon) \\ \psi(\varepsilon) \end{pmatrix} \tag{5}$$

E. Schmidts eigenelements of the adjoint perturbation problem corresponding to the same eigenvalues are determined analogously

$$(\mathcal{B}_0^* - \mathcal{A}^*(\lambda_\prime))\Psi \equiv \begin{pmatrix} \mathbf{B}^* & -\mathbf{A}(\lambda_0) \\ -\mathbf{A}^*(\lambda_0) & \mathbf{B} \end{pmatrix} \begin{pmatrix} \overline{\varphi}(\varepsilon) \\ \overline{\psi}(\varepsilon) \end{pmatrix} = $$
$$= \sum_{i+j \geq 1} \mu^i \varepsilon^j \begin{pmatrix} 0 & A_{ij} \\ A_{ij}^* & 0 \end{pmatrix} \begin{pmatrix} \overline{\varphi}(\varepsilon) \\ \overline{\psi}(\varepsilon) \end{pmatrix} \tag{6}$$

In this article at the usage of the reduction method suggested in the articles (Rakhimov D. G., 2015) the investigation of perturbation of multiple eigenvalues is reduced to the investigation of perturbation of simple ones.

As application of the obtained results the problem about boundary perturbation for the system of two Sturm-Liouville problem with E. Schmidts spectral parameter is considered.

Everywhere below the terminology and notations of the monograph (Vainberg M.M. and Trenogin V.A., 1969) are used.

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On the convergence of some iterative algorithms to approximate fixed points

Balwant Singh Thakur

School of Studies in Mathematics Pt.Ravishankar Shukla University Raipur, India e-mail: balwantst@gmail.com

In this paper we present a new iterative algorithm to approximate fixed points of nonexpansive and pseudocontractive mappings. We establish convergence results. We also discuss numerical examples to illustrate theoretical results. In addition, we compare the convergence of these iterations with existing iterations using the numerical computation.

Vector integrals for multifunctions

Cristina Stamate

"Octav Mayer" Institute of Mathematics of Romanian Academy, Iaşi Branch, Romania
e-mail: cstamate@ymail.com

In this paper we present a general Pettis type integral for vector multifunctions relative to a vector multisubmeasure and present several classic properties.

Some new results on quasi-monotone sequences

Şebnem Yildiz

Ahi Evran University, Art and Science Faculty, Mathematics Department, Kırşehir Turkey
e-mail: sebnemyildiz@ahievran.edu.tr

In this paper, a known theorem on $|\hat{N}, p_n|_k$ summability methods of infinite series have been generalized to $|A, p_n|_k$ summability method of infinite series by using quasi-monotone sequences. And then, in the consequence of this theorem, a new result dealing with summability factors of Fourier series has been obtained.

2010 AMS Subject Classification: 26D15, 40F05, 40G99, 42A24.
KeyWords: Summability factors, absolute matrix summability, Fourier series, infinite series, Hölder inequality, Minkowski inequality.

Maximal and Calderon-Zygmund operators on local Morrey-Lorentz spaces and some applications

Canay Aykol Yuce

Ankara University, Turkey
e-mail: aykol@science.ankara.edu.tr

In this talk the boundedness, including the limiting cases, of the Hardy-Littlewood maximal operator M, the Calderon-Zygmund operators T and the maximal Calderon-Zygmund operators T on the local Morrey- Lorentz spaces $M^{loc}_{p,q;\lambda}$ will be proved. Further some applications of obtained results will be given.

On perpendicular bisectors in normed spaces

Gheorghiță Zbăganu[1,2]

[1] "*Gheorghe Mihoc - Caius Iacob*" *Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania*
[2] *University of Bucharest, Romania*
e-mail: gheorghitazbaganu@yahoo.com

Let $(X, \|.\|)$ be a normed space and $a \in X$. The set $M_a = \{x \in X : \|x\| = \|x - a\|\}$ is called the perpendicular bisector of the segment $[0, a]$. The set $L_a = \{x \in X : \|x\| \leq \|x - a\|\}$ is the Leibnizian halfpace given by a. In a inner-product space the set M_a is a hyperplane. It is known from 1937 that if L_a is a convex set for all $a \in X$ then X is an inner product space, meaning that there exists a scalar product such that $\|x\|^2 =< x, x >$. A byproduct is that if M_a is a hyperplane for all $a \in X$ then X is an inner product space.

We investigate the sets $H_a = \{x \in X : \|x\| > \|x - a\|\}$ and $\overline{H}_a = \{x \in X : \|x\| \geq \|x - a\|\}$. We prove that X is 2-dimensional, then $(H_a, +)$ is a semigroup and if the norm is strictly convex then $\left(\overline{H}_a, +\right)$ is a semigroup, too. We use the following representation of a 2-dimensional norm: $\|x, y\| = \mathrm{E}|xX - y|$ for some integrable random variable X.

We show that in higher dimensions this is not true and conjecture that if $\left(\overline{H}_a, +\right)$ is a semigroup for any $a \in X$ then X has an inner product.

Section 2. PDEs with Applications in Mechanics, Biology, etc.

Existence and nonexistence for nonlinear problems with singular potential

B. Abdellaoui[1], K. Biroud[1], J. Davila[2], and F. Mahmoudi[2]

[1] *Département de Mathématiques, Université Abou Bakr Belkaïd, Tlemcen, Algeria*
[2] *Departamento de Ingenieria Matematica, CMM, Universidad de Chile, Santiago, Chile*
e-mail: boumediene.abdellaoui@uam.es, kh_biroud@yahoo.fr, jdavila@dim.uchile.cl, fmahmoudi@dim.uchile.cl

Let $\Omega \subset \mathbb{R}^N$ be a bounded regular domain of $\mathbb{R}^N$ we consider the following class of elliptic problem

$$\begin{cases} -\Delta u = \dfrac{u^q}{d^2} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $0 < q \leq 2^* - 1$. We investigate the question of existence and nonexistence of positive solutions depending on the range of the exponent q.

2010 Mathematics Subject Classification: 35D05, 35D10, 35J20, 35J25, 35J70.

Keywords: Hardy inequality, Nonlinear elliptic problems, singular weight.

Search of unconventional error representations for time-domain goal-oriented adaptivity

Elisabete Alberdi Celaya[1], Judit Muñoz Matute[2], and David Pardo Zubiaur[1,2,3]

[1] *University of the Basque Country (UPV/EHU), Bilbao, Spain*
[2] *BCAM - Basque Center for Applied Mathematics, Bilbao, Spain*
[3] *IKERBASQUE, Basque Foundation for Science, Bilbao, Spain*
e-mail: elisabete.alberdi@ehu.es

Goal-oriented adaptive algorithms [4] have been widely employed during the last three decades to produce optimal grids in order to solve challenging engineering problems.

In this work, we extend the error representation using unconventional dual problems for goal-oriented adaptivity in the context of frequency-domain wave-propagation problems developed in [2], to the case of time-domain problems. To do that, we express the entire problem in weak form in order to formulate the adjoint problem and apply the goal-oriented adaptivity [3]. We have also chosen specific spaces of trial and test functions that allow us to express a classical Method of Lines in terms of a Galerkin scheme [1]. Some numerical results are provided in 1D which show that the upper bounds of the new error representation are sharper than the classical ones [5] and therefore this new error representation can be used to design goal-oriented adaptive processes.

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Nonlinear fourth-order diffusion-based image restoration scheme

Tudor Barbu

Institute of Computer Science of the Romanian Academy, Romania
e-mail: `tudor.barbu@iit.academiaromana-is.ro`

We consider a nonlinear fourth-order PDE-based image restoration technique in this paper. The proposed denoising approach is based on a novel fourth-order diffusion-based model.

The partial differential equations have been widely used for image denoising and restoration in the last 25 years. They represent a much better smoothing solution than conventional two-dimension image filters which generate the blurring effect that destroys the image details. The nonlinear second-order anisotropic diffusion models provide an effective deblurring but they could also produce the undesired staircase effect, representing creating of flat zones separated by artifact boundaries.

For this reason some nonlinear fourth-order PDE schemes have been introduced for image restoration. While these fourth-order diffusion models inspired by the influential You-Kaveh isotropic scheme remove successfully the Gaussian noise and overcome the blocky effect, they may also produce image blurr and speckle noise. So, we develop here a novel fourth-order nonlinear PDE-based image filtering approach that is totally different from the well-known You-Kaveh method and provide much better results. Unlike other fourth-order diffusion methods, this differential model provides a proper trade-off between noise reduction, feature preservation and overcoming unintended effects. The proposed PDE model is composed of a fourth-order partial differential equation and some boundary conditions. It is based on a positive and monotonically decreasing diffusivity function, which is properly constructed for an effective restoration. A mathematical investigation of the well-posedness of this diffusion model is also considered. Then, a consistent and fast-converging explicit numerical approximation scheme, based on the finite-difference method, is developed for this nonlinear fourth-order PDE.

The successfully performed experiments and method comparison prove the effectiveness of the proposed technique. Our denoising method outperforms not only the classic 2D image filters, but also numerous state of the art nonlinear second and fourth order diffusion models.

Keywords: Image denoising and restoration, Gaussian noise, Nonlinear diffusion, Fourth-order PDE model, Finite difference method, Numerical approximation scheme.

On one nonlocal problem for a semilinear parabolic equation

Olga Danilkina and Marcellina Andrea Mjenda

Department of Mathematics, College of Natural and Mathematical Sciences, University of Dodoma, Tanzania
e-mail: olga.danilkina@gmail.com

We consider a class of nonlocal problems with integral conditions to a semilinear parabolic equation and study the question of existence and uniqueness of a generalized solution. The first approach to nonlocal problems with integral conditions was initiated in [1] and later it has been widely extended by many authors (see [2], [3] and references therein).

In this paper, we begin with the classification of nonlocal problems with integral conditions and the overview of general techniques. Further, we consider one nonlocal problem with the integral condition of the first kind, demonstrate its equivalence to a problem with the integral condition of the second kind and define the notion of a generalized solution. Then we prove the existence result based on Faedo-Galerkin approximations and obtained apriori estimates. Finally, we show uniqueness of the generalized solution.

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Modelling and programming with finite element method (FEM) of partial differential equations with non-linear variable coefficients in an engineering problem

I. G. Demiriz

Yildiz Technical University, Turkey
e-mail: idemiriz@yildiz.edu.tr

In this study, natural and forced vibration of the composite rectangular plates with curvilinear building mathematical formulation of the of non-linear variable coefficients modeling using FEM of partial differential equations and algorithms necessary for the numerical solution of these problems and programs are made.

Computational issues for avascular tumor growth

Gabriel Dimitriu

Department of Mathematics and Informatics, University of Medicine and Pharmacy "Grigore T. Popa" Iasi, Romania
e-mail: dimitriu.gabriel@gmail.com

Tumor growth is a fundamental scientific problem and has received considerable attention by the mathematics community. Mathematical modelling of avascular tumors can be seen as a first step toward building models for fully vascularized tumors in later stages and can play a very important role in cancer research.

This work focusses on Sherratt and Chaplain's avascular tumor growth model, defined in terms of the continuum densities of proliferating, quiescent and necrotic cells, together with the consideration of the generic nutrient supply from underlying tissue.

Numerical simulations visualize the tumor growth process, by including both effects of nutrient supply in the tumor dynamics and random effects in the necrosis equation, depending on different parameter values and functional forms used in the model.

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Minimum traveling wave for the delayed diffusive Nicholson blowflies equation

Adrian Gomez[1] and Nolbert Morales[1]

[1] *Grupo de Sistemas Dinámicos y Aplicaciones(GISDA), Departamento de Matemática, Universidad del Bío-Bío, Concepción, Chile*
e-mail: agomez@ubiobio.cl

In this work we consider the Delayed Nicholson Blowflies Equation with diffusion

$$u_t(t,x) = \Delta u(t,x) - \delta u(t,x) + pu(t-\hat{h},x)e^{-u(t-\hat{h},x)}, \quad u(t,x) \geq 0, \quad x \in \mathbb{R}^m, \tag{1}$$

where $\hat{h} \geq 0$ and the parameters p, δ satisfy $p/\delta \in (1, e]$. Equation (1) is a prototype of monostable delayed reaction-diffusion equations, and it has been extensively studied.

A *Travelling wave solution* to (1) is a special type of positive solution $u(t,x)$ having the form $u(t,x) = \phi(x+ct)$, where $\phi : \mathbb{R} \to \mathbb{R}$ is C^2. The function $\phi(s)$ is called a *wave profile* and it must satisfy $\phi(-\infty) = 0, \phi(+\infty) = \kappa$, where $\kappa = \ln(p/\delta)$ is the positive equilibria of (1). To find traveling waves solutions to (1) is equivalent to found positive heteroclinic solutions to the second-order delayed differential equation

$$\phi''(t) - c\phi'(t) - \phi(t) + \frac{p}{\delta}\phi(t-r)e^{-\phi(t-r)} = 0, \quad r := ch, \tag{2}$$

satisfying $\phi(-\infty) = 0, \quad \phi(+\infty) = \kappa$, and $\phi(t) > 0$ for all $t \in \mathbb{R}$.

The number c is called the wave speed propagation. In our case, it is well known that there exists a minimum $c_0(h)$ such that equation (1) can have a travelling wave solution.

In our present work we show the existence of a traveling wave solution with minimum speed of propagation $c_0(h)$, using the super-sub solution method, and then we can give an explicit approximation of the wave via the iterative procedure developed in [2].

In the case $h = 0$ the wave moving to a minimum speed (minimal wave) is, according to Kolmogorov *et. al.* [1], the most important wave, because the solution of the Cauchy Problem

$$u_t(t,x) = u_{xx}(t,x) + f(u(t,x)), \qquad u(0,x) = \begin{cases} 0, & \text{if } x < a \\ 1, & \text{if } x \geq a \end{cases}, \tag{3}$$

with some $a \in \mathbb{R}$ and f satisfying $f(0) = f(1) = 0$, $f'(0) = \alpha > 0$, $f'(v) < \alpha$, for all $v \in (0,1]$, $f(v) > 0$ for all $v \in [0,1]$, converges to the minimal wave, i.e., in practice, propagations of genes, viruses, or another modelled amounts are at a minimum speed.

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On the global in time existence and uniqueness of solutions to a nonlinear evolution equation in an AL - space

Cecil P. Grünfeld

"Gheorghe Mihoc - Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania
e-mail: grunfeld@spacescience.ro

In this work, we reconsider a previously obtained result on the existence and uniqueness of global in time solutions to the initial value problem for a nonlinear evolution equation in an Abstract Lebesgue (AL) - space, with the purpose to include new applications to the mathematical theory of nonlinear kinetic models.

Analytical and numerical methods for the compressible free surfaces fluid flow in the presence of bodies with axial symmetries

Luminita Grecu[1] and Mircea Lupu[2]

[1] *Department of Applied Mathematics, University of Craiova, Romania*
[2] *Transylvania University, member of the Academy of Romanian Scientists*
e-mail: lumigrecu@hotmail.com, m.lupu@unitbv.ro

In this paper there are presented some direct and inverse methods regarding the compressible fluid flow with free surfaces in the presence of bodies with axial symmetries. Dirichlet and Riemann-Hilbert boundary problems are formulated and singular integral equations for analytical functions

appear. The complex potential of the flow and the complex velocity are obtained using as canonic domain the upper half-plane. There are found analytical solutions for some particular cases, namely for the movement in the presence of conical surfaces or in the presence of the circular disk. Comparisons and discussions about the correspondence with the incompressible case and with other results known from literature are also made.

Approximation of the boundary controls for the wave equation

Pierre Lissy[1] and Ionel Roventa[2]

[1] *CEREMADE, Université Paris-Dauphine, France*
[2] *Department of Mathematics, University of Craiova, Romania*
e-mail: lissy@ceremade.dauphine.fr, ionelroventa@yahoo.com

We consider a finite-differences semi-discrete scheme for the approximation of boundary controls for the one-dimensional wave equation. The high frequency numerical spurious oscillations lead to a loss of the uniform (with respect to the mesh-size) controllability property of the semi-discrete model in the natural setting. We prove that, by filtering the high frequencies of the initial data in an optimal range, we restore the uniform controllability property. Moreover, we obtain a relation between the range of filtration and the minimal time of control needed to ensure the uniform controllability, recovering in many cases the usual minimal time to control the (continuous) wave equation.

On the existence of solutions for a generalized strong vector quasi-equilibrium problem

Monica Patriche

University of Bucharest, Romania
e-mail: monica.patriche@yahoo.com

In this talk, we consider a generalized strong vector quasi equilibrium problem and we prove the existence of its solutions by using some auxiliary results. One of the established theorems is proved by using an approximation method.

On magnetic relaxation equation for anisotropic reacting fluid mixtures

Liliana Restuccia[1], Arcangelo Labianca[2], and Lidia Palese[2]

[1] *University of Messina, Messina, Italy*
[2] *University of Bari, Bari, Italy*
e-mail: lrestuccia@unime.it, arcangelo.labianca@uniba.it, lidiarosaria.palese@uniba.it

In some previous papers [1]-[7] a linear theory for magnetic relaxation phenomena in magnetizable continuous media was developed, that is based on thermodynamics of irreversible processes with internal variables [8]-[13]. Here, we consider reacting fluid mixtures where irreversible microscopic phenomena give rise to magnetic relaxation, and these phenomena are described splitting the total specific magnetization in two irreversible parts and introducing one of these partial specific magnetizations as internal variable in the thermodynamic state vector. The phenomenological

equations for these fluid mixtures are derived and, in the linear case, a generalized Snoek equation for magnetic relaxation phenomena is derived. The obtained results have applications in several fields of applied sciences, as, for instance, in medicine and biology, where complex fluids are taken into consideration, in which different types of molecules, having different magnetic susceptibilities and relaxation times, present magnetic relaxation phenomena and contribute to the total magnetization (as an example such physical situations arise in nuclear magnetic resonance).

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Section 3. ODEs and Dynamical Systems

Modelling the impact of misdiagnosis and treatment on the dynamics of malaria concurrent and co-infection with pneumonia

Okaka C. Akinyi

Department of Mathematics, Masinde Muliro University of Science and Technology, Kakamega, Kenya
e-mail: letticoll@yahoo.com

Incidences of misdiagnosis of diseases with similar symptoms as malaria have increased lately due to the increasing levels of endemicity of these diseases and their co-infection with malaria. In this paper a mathematical model for the impact of misdiagnosis and treatment of pneumonia as malaria has been developed and analyzed. The dynamics of misdiagnosis is compared with that of accurate diagnosis and prompt and correct treatment. We establish the existence of local asymptotic stability of the disease-free equilibrium when $R_{MP} < 1$. The results show that the disease-free equilibrium may not be globally asymptotically stable whenever $R_{MP} < 1$. The existence of backward bifurcation has been shown using the Lyapunov second method. We deduce that in a situation of prompt pneumonia treatment there exists a unique endemic equilibrium which is globally asymptotically stable.

Spectral properties of a selfadjoint matrix quantum difference equation

Yelda Aygar

Ankara University, Faculty of Science, Department of Mathematics, Turkey
e-mail: yaygar@science.ankara.edu.tr

The aim of this work is to get some spectral properties of a self adjoint matrix quantum difference equation. In this way, we find the polynomial type Jost solution of this equation. Using the asymptotic behavior and analytical properties of Jost solution, we introduce the set of eigenvalues of the operator L generated by same q-difference equation. Also, we investigate the spectral properties of eigenvalues and continuous spectrum of L.

First integrals of the family of cubic differential systems with invariant lines of total multiplicity 8

Cristina Bujac and Nicolae Vulpe

Institute of Mathematics and Computer Science, Academy of Science of Moldova
e-mail: cristina@bujac.eu, nvulpe@gmail.com

Polynomial differential systems on the plane are systems of the form

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where $P, Q \in \mathbb{R}[x, y]$, i.e. P and Q are the polynomials over $\mathbb{R}$. We call *cubic* system a cubic polynomial differential system (1) with degree $n = \max\{\deg P, \deg Q\} = 3$.

There are several open problems on polynomial differential systems, especially on the class of all cubic systems (1). In this paper we are concerned with the *algebraic integrability in the sense of Darboux* of cubic systems (1) which possess invariant straight lines of total multiplicity eight (including the line at infinity with its own multiplicity). We denote such class of cubic systems by $\mathbb{CSL}_8$.

We remark that an interesting survey on the problem of Darboux integrability of systems (1) possessing invariant algebraic curves could be find in [1]. According to [1] if a cubic system has a sufficient number of invariant straight lines considered with their multiplicities, then this system has the first integral of Darboux type.

In paper [2] the classification of systems in $\mathbb{CSL}_8$ with respect to configurations of invariant straight lines has been done in the case of the existence of four distinct infinite singularities (real or/and complex). It was proved that such a cubic system could possess only one of the 17 possible configurations. We mention that applying the notion of *parallel multiplicity* of invariant straight lines (see for details [3]) the classification of the whole class of cubic systems with invariant affine lines of total parallel multiplicity 7 was done in the article [3]. As a result the authors have obtained exactly the same 17 configurations of invariant straight lines. Moreover in [3] there were determined all the first integrals as well as the integrating factors for the constructed canonical systems possessing such configurations.

We note that here we also have constructed the first integrals as well as the integrating factors for our canonical forms which differ from the ones from [3]. However in spite of [3] we have also determined necessary and sufficient conditions, in terms of affine invariant polynomials, for a cubic system to be brought via an affine transformation and time rescaling to one of the corresponding canonical forms.

We point out that the problem of the integration of a cubic system in $\mathbb{CSL}_8$ possessing less than 4 distinct infinite singularities was completed in this paper. First we mention that in the article(s) [4] (respectively [5] and [6]; [7]) the classification of systems in $\mathbb{CSL}_8$ with respect to the configurations of their invariant straight lines has been done in the case of the existence of three (respectively two; one) distinct infinite singularities obtaining 5 (respectively 25; 4) distinct configurations. As a result we have proved that cubic systems in $\mathbb{CSL}_8$ possess a total of 51 distinct configurations. Using the corresponding constructed canonical systems we have determined all the first integrals as well as the integrating factors. We also remark that the first integrals are constructed using the invariant lines, their multiplicities and the method of Darboux.

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Some existence results for solutions of a fractional differential inclusion with ”maxima”

Aurelian Cernea

Faculty of Mathematics and Computer Science, University of Bucharest, Romania
e-mail: acernea@fmi.unibuc.ro

We study a the following boundary value problem associated to a fractional differential inclusion with ”maxima”.

$$D_c^q x(t) \in F(t, x(t), \max_{s\in[t-h_1,t]} x(s), \max_{s\in[t,t+h_2]} x(s)) \quad a.e. \ ([0,1]),$$

with ”boundary conditions” of mixed type

$$x(t) = \alpha(t), \quad t \in [-h_1, 0], \quad x(t) = \beta(t), \quad t \in [1, 1 + h_2],$$

where $q \in (1, 2]$, D_c^q is the Caputo fractional derivative, $h_1, h_2 > 0$ are given, $\alpha(.) : [h_1, 0] \to \mathbf{R}$, $\beta(.) : [1, 1 + h_2] \to \mathbf{R}$ are continuous mappings and $F : [0, 1] \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \to \mathcal{P}(\mathbf{R})$ is a set-valued map.

Several existence results are obtained by using suitable fixed point theorems when the right hand side has convex or non convex values. Our results are essentially based on a nonlinear alternative of Leray-Schauder type, on Bressan-Colombo selection theorem for lower semicontinuous set-valued maps with decomposable values and on Covitz and Nadler set-valued contraction principle.

Center conditions for a cubic differential system with an invariant quartic curve

Dumitru Cozma and Vadim Repesco

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: dcozma@gmail.com

By using a nondegenerate transformation of variables and a time rescaling, a cubic system having a singular point with pure imaginary eigenvalues and two parallel invariant straight lines can be brought to the form [1]

$$\begin{aligned} \dot{x} &= y(1 + cx + mx^2), \\ \dot{y} &= -(x + gx^2 + dxy + by^2 + sx^3 + qx^2y + nxy^2 + ly^3), \end{aligned} \tag{1}$$

where the variables and coefficients in (1) are assumed to be real. Then the origin $O(0, 0)$ is a singular point of a center or a focus type for (1) and the invariant straight lines are $2 + (c \pm \sqrt{c^2 - 4m}\,)x = 0$, $m(c^2 - 4m) \neq 0$. We pay attention to the problem of distinguishing between a center and a focus.

The problem of the center for cubic system (1) with: at least three invariant straight lines was solved in [1]; two invariant straight lines and one invariant conic was solved in [2]; two invariant straight lines and one invariant cubic was solved in [3]. Sufficient center conditions for cubic system (1) were obtained in [4]. The problem of the center was solved for cubic system (1) when it can be reduced to a Liénard type system [5].

In this paper we obtain the center conditions for cubic system (1) with one invariant quartic curve by using the method of Darboux integrability.

Theorem 1. The cubic system (1) with a center and one invariant quartic curve is always Darboux integrable.

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Accurate spectral solutions to non-standard eigenvalue problems with boundary conditions containing the eigenparameter

Călin-Ioan Gheorghiu

"T. Popoviciu" Institute of Numerical Analysis, Cluj-Napoca, Romania
e-mail: ghcalin@ictp.acad.ro

In this paper the issue of the numerical treatment of the eigenparameter dependent boundary conditions, i.e., of some unusual boundary conditions, is addressed using the so called Petterson-König's rod eigenvalue problem [6] and the Charney baroclinic stability problem [1].

One boundary condition in these problems depends on the eigenparameter and additionally on a physical parameter. The Chebyshev collocation based on both, square [3] as well as rectangular differentiation ([2]) and a Chebyshev tau method are used in order to discretize the first problem.

All these schemes cast the problem in singular algebraic generalized eigenvalue ones ([4]). These problems are solved comparatively by QZ algorithm as well as by some target oriented Jacobi Davidson methods. Thus the spurious eigenvalues are completely eliminated.

A particular attention is paid to the way in which boundary conditions are imposed in collocation methods ([5]).

However, the numerical experiments carried out show that the critical eigenvalues found out by both collocation methods are practically undistinguishable. They are confirmed by tau method, which is robust with respect to the issue of boundary conditions, as well as by an analytical study. The dependence of the critical values of the eigenparameter on the involved physical parameter is displayed.

For the second problem, defined on the positive real semi-axe, we make use of the Laguerre Gauss Radau collocation. Again, the dependence of the eigenparameter on an involved physical parameter confirms the previous published results.

Thus, the issue of imposing parameter dependent boundary condition seems to be solved but the dispute over the superiority of rectangular or square differentiation remains an open question.

Key words: collocation; Chebyshev; Laguerre; square differentiation; rectangular differentiation; tau method; singular eigenproblems; eigenparameter boundary conditions; singular algebraic generalized eigenproblems.

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Solving some linear systems of two ordinary differential equations with variable coefficients

Raluca Mihaela Georgescu

University of Piteşti, Romania
e-mail: gemiral@yahoo.com

An approach using Maple environment of teaching and learning aspects of analytic geometry aspects concerning the points and the vectors in the three-dimensional Euclidean space is presented. First, the problems are resolved in a classical mode, following the application of the formulas presented in the course. Then, the same problems are resolved with help of Maple environment. Finally, the graphics representation of the solution is shown.

Keywords: points, vectors.

Differential transformations for linearizing the morphogenesis mathematical model

Adela Ionescu[1] and Evantia Coles[2]

[1] *University of Craiova, Romania*
[2] *University of Medicine and Farmacy Craiova, Romania*
e-mail: adela0404@yahoo.com, fruev@yahoo.com

This paper aims to study the influence of center manifold method on linearising the activator-inhibitor dyanamical system associated to the morphogenesis mechanisms. It is considered the activator-inhibitor mechanism with different sources, for the simplicity of calculus.

The flow on the center manifold contains all the necessary information for analyzing the asymptotic behavior of the small solutions of the entire system. The parameter influence is important in pointing out the variation of the equilibrium states on the center manifold.

Numerical implementation of IVPs with ABC algorithm

Iclal Gor and Korhan Gunel

Adnan Menderes University, Faculty of Arts and Sciences, Department of Mathematics, Turkey
e-mail: iclal@adu.edu.tr

Global optimization algorithms are utilized for the numerical solution of differential equations. Artificial bee colony (ABC) algorithm is an optimization method based on the behaviour of honey bee swarm intelligence.

In this work, we solve the Initial Value Problems (IVPs) with ABC algorithm and and compared them with the exact solution. The approximated results show that the ability of ABC solution of IVPs.

Nonlinear dynamics in the study of a hybrid system of Rayleigh - Van der Pol type

Mircea Lupu[1] and Gheorghe Radu[2]

[1] *Transylvania University, member of the Academy of Romanian Scientists*
[2] *"Henri Coanda" Air Force Academy of Brasov, Romania*
e-mail: m.lupu@unitbv.ro, gh.radu@gmail.com

In this paper, we study the mathematical model for nonlinear dynamical systems with distributed parameters given by a generalized Rayleigh-Van der Pol equation. In the autonomous case, as well as in the non-autonomous case, conditions for stability, bifurcations, and self-oscillations are studied using some criteria of Lyapunov, Bendixon, Hopf [12], [15], [16]. Asymptotic and numerical methods are often used [4]. The equation has the form

$$\ddot{x} + \omega^2 x = \left(\alpha - \beta \cdot x^2 - \gamma \cdot \dot{x}^2\right) \cdot \dot{x} + f(t), \quad \alpha \in \mathbb{R};\ \beta > 0;\ \gamma > 0;\ \omega > 0.$$

where resonance and limit cycles can be remarked [1], [15]. Note that for $\alpha \neq 0, \beta = 0, \gamma \neq 0$ we have the Rayleigh equation [1], while for $\alpha \neq 0, \beta \neq 0, \gamma = 0$ we have the Van der Pol equation [2], [5], [15], [16]. Besides the theoretical study, the applications to techniques are very important: dynamical systems in the mechanics of vibration, oscillations in electromagnetism and transistorized circuits [13], aerodynamics of the flutter with two degrees of freedom (which are presented in the second part of this paper, see also [10]), are modeled by this hybrid equation that we propose (see [9]).

Keywords: nonlinear dynamical systems, self-oscillations, stability, bifurcation, limit cycle, resonance.
2010 Mathematics Subject Classification: 34C15, 37B25, 37B55, 37C75, 37G15.

Approximate method for an initial value problem for difference equations with non-instantaneous impulses

Ekaterina Madamlieva, Snezhana Hristova, Svetoslav Enkov

Plovdiv University, Plovdiv, Bulgaria
e-mail: ekaterinaa.b.m@gmail.com, snehri@gmail.com, enkov@yahoo.com

An algorithm for constructing two monotone sequences of upper and lower solutions of the initial value problem for nonlinear difference equations with non-instantaneous impulses is given. The impulses start abruptly at some points and their action continue on given finite intervals.

It is proved both functional sequences are convergent and their limits are minimal and maximal solutions of the considered problem. An example is given to illustrate the theoretical results.

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Invariant conditions for the stability of the unperturbed motion for the four-dimensional nonlinear polynomial differential systems

Victor Orlov[1,2] and Mihail Popa[2]

[1] *Technical University of Moldova,*
[2] *Institute of Mathematics and Computer Science of ASM, Chişinău, Republic of Moldova*
e-mail: orlovictor@gmail.com, mihailpomd@gmail.com

Let us consider the four-dimensional nonlinear polynomial differential system

$$\frac{dx^j}{dt} = a^j_\alpha\, x^\alpha + \sum_{i=1}^{l} a^j_{\alpha_1\alpha_2\ldots\alpha_{m_i}}\, x^{\alpha_1}x^{\alpha_2}\ldots x^{\alpha_{m_i}}\ (j,\alpha,\alpha_1,\alpha_2,...,\alpha_{m_i}=\overline{1,4};\ l<\infty), \tag{1}$$

where the coefficient tensor $a^j_{\alpha_1\alpha_2\ldots\alpha_{m_i}}$ is symmetrical in lower indices in which the complete convolution holds, and $\{m_1,m_2,...,m_l\}$ $(m_i\ge 2)$ is a finite set of different natural numbers. In [1] it was shown that the expressions

$$I_{1,4}=a^\alpha_\alpha,\quad I_{2,4}=a^\alpha_\beta a^\beta_\alpha,\quad I_{3,4}=a^\alpha_\gamma a^\beta_\alpha a^\gamma_\beta,\quad I_{4,4}=a^\alpha_\delta a^\beta_\alpha a^\gamma_\beta a^\delta_\gamma, \tag{2}$$

are the center-affine invariants relative to the group $GL(4,\mathbb{R})$ of the system (1). The characteristic equation of the system (1) as well as the system of first approximation $\frac{dx^j}{dt}=a^j_\alpha\, x^\alpha$ $(j,\alpha=\overline{1,4})$ can be written as

$$\rho^4+L_{1,4}\,\rho^3+L_{2,4}\,\rho^2+L_{3,4}\,\rho+L_{4,4}=0, \tag{3}$$

where

$$L_{1,4}=-I_{1,4},\quad L_{2,4}=\frac{1}{2}\,(I^2_{1,4}-I_{2,4}),\quad L_{3,4}=\frac{1}{6}\,(3I_{1,4}I_{2,4}-2I_{3,4}-I^3_{1,4}),$$
$$L_{4,4}=\frac{1}{24}\,(8I_{1,4}I_{3,4}-6I_{4,4}-6I^2_{1,4}I_{2,4}+3I^2_{2,4}+I^4_{1,4}), \tag{4}$$

and $I_{i,4}$ $(i=\overline{1,4})$ are from (2).

Using the Lyapunov's theorems for stability in the first approximation and the Hurwitz's theorem on the signs of roots of equation (3) (see for example [2]) we have proved the following results:

Theorem 1. *If the center-affine invariants of the system* (1) *satisfy the inequalities*

$$L_{i,4}>0,\ (i=\overline{1,4}),\quad L_{1,4}L_{2,4}L_{3,4}-L^2_{3,4}-L^2_{1,4}L_{4,4}>0, \tag{5}$$

then unperturbed motion of this system is asymptotically stable regardless of the member higher than the first order of smallness, where $L_{i,4}$ $(i=\overline{1,4})$ *are from* (4), (2).

Theorem 2. *If among the center-affine invariant expressions* (5) *of the system* (1) *there is at least one with a negative sign, then unperturbed motion of this system is unstable regardless of the member higher than the first order of smallness.*

We have obtained the center-affine invariant conditions for the existence of periodic solutions for the four-dimensional Lyapunov-Darboux differential systems with quadratic nonlinearities [3].

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The common Hilbert series of the differential system $s(1,3,7)$ and the Krull dimensions of Sibirsky algebras

Victor Pricop

Institute of Mathematics and Computer Science,
Chişinău, Republic of Moldova
e-mail: pricopvv@gmail.com

Let G be a linearly reductive group over an algebraically closed field K and V an n-dimensional rational representation. Through $H(K[V]^G,t)$ is denoted the Hilbert series of invariants ring $K[V]^G$ [1].

Theorem 1. [1]

$$H(K[V]^G,t)=\frac{1}{2\pi i}\int\limits_{S^1}\frac{1}{det(I-t_{\rho_V}(z))}\frac{dz}{z} \tag{1}$$

where $S^1\subset\mathbb{C}$ is the unit circle $\{z:|z|=1\}$.

Using the formula (1) and corresponding generating function [2] it was obtained a common Hilbert series for the Sibirsky graded algebras of comitants $S_{1,3,7}$ and invariants $SI_{1,3,7}$ of the differential system $s(1,3,7)$.

Theorem 2. *For the differential system $s(1,3,7)$ the following common Hilbert series for Sibirsky graded algebras of comitants and invariants was obtained*

$$H_{S_{1,3,7}}(t)=\frac{1}{(1-t^2)^5(1+c)^3(1-t^4)^5(1-t^3)^8(1-t^5)^5(1-t^9)(1-t^7)^3}(1+4c+8t^2+20t^3+$$
$$+119t^4+630t^5+2704t^6+10022t^7+33698t^8+104818t^9+304181t^{10}+826655t^{11}+2112616t^{12}$$
$$+5098405t^{13}+11666106t^{14}+25400587t^{15}+52790206t^{16}+105011044t^{17}+200416900t^{18}$$
$$+367773321t^{19}+650140950t^{20}+1109089748t^{21}+1828673257t^{22}+2918286116t^{23}+$$
$$+4513317434t^{24}+6772373326t^{25}+9869976204t^{26}+13983988556t^{27}+19277729149t^{28}+$$
$$+25877612329t^{29}+33848259389t^{30}+43167949995t^{31}+53708076135t^{32}+65220413010t^{33}+$$
$$+77335714909t^{34}+89575940034t^{35}+101380841773t^{36}+112147463549t^{37}+121279087722t^{38}+$$
$$+128238286339t^{39}+132597788686t^{40}+134082589969t^{41}+132597788686t^{42}+128238286339t^{43}+$$
$$+121279087722t^{44}+112147463549t^{45}+101380841773t^{46}+89575940034t^{47}+77335714909t^{48}+$$
$$+65220413010t^{49}+53708076135t^{50}+43167949995t^{51}+33848259389t^{52}+25877612329t^{53}+$$
$$+19277729149t^{54}+13983988556t^{55}+9869976204t^{56}+6772373326t^{57}+4513317434t^{58}+$$
$$+2918286116t^{59}+1828673257t^{60}+1109089748t^{61}+650140950t^{62}+367773321t^{63}+$$
$$+200416900t^{64}+105011044t^{65}+52790206t^{66}+25400587t^{67}+11666106t^{68}+5098405t^{69}$$

$$+2112616t^{70}+826655t^{71}+304181t^{72}+104818t^{73}+33698t^{74}+10022t^{75}+$$
$$+2704t^{76}+630t^{77}+119t^{78}+20t^{79}+8t^{80}+4t^{81}+t^{82}),$$

$$H_{SI_{1,3,7}}(t)=\frac{1}{(1-t^2)^4(1+t)^3(1-t^4)^6(1-t^3)^8(1-t^5)^5(1-t^7)^2}(1+4t+9t^2+22t^3+114t^4+$$
$$+576t^5+2433t^6+8812t^7+28787t^8+86580t^9+242349t^{10}+633691t^{11}+1554313t^{12}+$$
$$+3589873t^{13}+7838767t^{14}+16239174t^{15}+32018338t^{16}+60242752t^{17}+108417618t^{18}+$$
$$+187010583t^{19}+309738539t^{20}+493386952t^{21}+756961044t^{22}+1119980967t^{23}+$$
$$+1599914185t^{24}+2208870842t^{25}+2949986298t^{26}+3814040685t^{27}+4777086279t^{28}+$$
$$+5799732655t^{29}+6828681083t^{30}+7800621224t^{31}+8648294432t^{32}+9307907390t^{33}+$$
$$+9726879111t^{34}+9870564527t^{35}+9726879111t^{36}+9307907390t^{37}+8648294432t^{38}+$$
$$+7800621224t^{39}+6828681083t^{40}+5799732655t^{41}+4777086279t^{42}+3814040685t^{43}+$$
$$+2949986298t^{44}+2208870842t^{45}+1599914185t^{46}+1119980967t^{47}+756961044t^{48}+$$
$$+493386952t^{49}+309738539t^{50}+187010583t^{51}+108417618t^{52}+60242752t^{53}+32018338t^{54}$$
$$+16239174t^{55}+7838767t^{56}+3589873t^{57}+1554313t^{58}+633691t^{59}+242349t^{60}$$
$$+86580t^{61}+28787t^{62}+8812t^{63}+2433t^{64}+576t^{65}+114t^{66}+22t^{67}+9t^{68}+4t^{69}+t^{70}).$$

From this theorem result that the Krull dimension [2] of Sibirsky graded algebra $S_{1,3,7}$ (respectively $SI_{1,3,7}$) is equal to 27 (respectively 25).

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Integrability of the cubic differential systems with six real invariant straight lines along at least five directions

Vitalie Puţuntică

Chişinău, Republic of Moldova
e-mail: vitputuntica@mail.ru

We consider the cubic differential system $\dot{x}=P(x,y)$, $\dot{y}=Q(x,y)$, where P, $Q\in\mathbb{R}[x,y]$, $max\{deg(P),deg(Q)\}=3$ and $GCD(P,Q)=1$.

The straight line $Ax+By+C=0$ is said to be invariant for this system if there exists a polynomial $K(x,y)$ such that $A\cdot P+B\cdot Q\equiv(Ax+By+C)\cdot K$ holds. The polynomial $K(x,y)$ is called the cofactor of the invariant straight line. This system is said to be Darboux integrable if it has a first integral of the form $\prod_{j=1}^{m}(A_jx+B_jy+C_j)^{\alpha_j}$, where $A_jx+B_jy+C_j=0$, $j=\overline{1,m}$ are invariant straight lines and $\alpha_j\in\mathbb{C}$.

A classification of cubic systems with exactly seven real invariant straight line was carried out in [1] and [2], and for cubic systems with exactly six real invariant straight line along two and tree directions it is investigated in [3] and [4]. We give here a similar classification for cubic differential systems with exactly six real invariant straight lines along five and six directions.

Theorem. *Any cubic differential system with six real invariant straight lines along at least five directions via affine transformation and time rescaling can be brought one to one of the following forms:*

$$(1)\ \begin{cases} \dot{x} = x(x+1)(1+ax-2y), \\ \dot{y} = y(1+2x-y+ax^2-2xy), \quad a(a-1)(a-2) \neq 0, \end{cases}$$

$$(2)\ \begin{cases} \dot{x} = x(a+(a+b)x+bx^2+y^2), \\ \dot{y} = y(-a+(a-1)y+bx^2+y^2), \ ab(a+1)(b+1)(a-b)(a^2+b) \neq 0. \end{cases}$$

The invariant straight lines of system (1) are: $x = 0$, $x+1 = 0$, $y = 0$, $y-x-1 = 0$, $y+(1-a)x = 0$, $y - ax - 1 = 0$ and for system (2): $x = 0$, $y = 0$, $y - x - 1 = 0$, $y + a(x + 1) = 0$, $y + bx + a = 0$, $ay - bx - a = 0$.

Systems (1) and (2) have a Darboux first integral of the form respectively

$$F(x, y) = \frac{x(x+1)}{y(y-ax-1)}, \quad F(x, y) = \frac{xy}{(y-x-1)(y+bx+a)}.$$

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Approximations of homoclinic orbits for double zero bifurcation with symmetry of order two

Carmen Rocşoreanu and Mihaela Sterpu

University of Craiova, Romania
e-mail: rocsoreanu@yahoo.com, msterpu@yahoo.com

Consider the two-dimensional system of differential equations corresponding to the normal form of the double zero bifurcation with symmetry of order two. This is a degenerated Bogdanov-Takens bifurcation of order two. The associated family of dynamical systems exhibits, among others, a homoclinic bifurcation [1]. For parameters situated on the homoclinic bifurcation curve, we obtain second-order approximations for the homoclinic solutions.

To perform this task, we reduce first the normal form to a perturbed Hamiltonian system, using a blow-up technique.

Then, by means of a perturbation method, we determine explicit first and second order approximations of the homoclinic orbits.

Finally, the solutions deduced theoretically are compared with those obtained numerical for several values of the parameters.

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Lipschitz stability of differential equations with non-instantaneous impulses

Radoslava Terzieva and Snezhana Hristova

Plovdiv University, Plovdiv, Bulgaria
e-mail: radoslavaterzieva@abv.bg, snehri@gmail.com

In this paper we consider differential equation with noninstantaneous impulses. In these equation we have impulses which start abruptly at some points and their action continue on given finite intervals. We pursue the study of Lipschitz stability using the techniques of Liapunov functions. We established sufficient conditions for Lipschitz stability. Examples are given to illustrate the results.

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Cubic differential systems with two parallel complex invariant straight lines of multiplicity $m(2,2;3)$

Alexandru Şubă and Olga Vacaraş

Institute of Mathematics and Computer Science of ASM, Chişinău, Republic of Moldova
e-mail: suba@math.md, vacaras_olga@mail.md

We consider the real cubic system of differential equations

$$\begin{gathered}\dot{x}=\sum_{j=0}^{3} P_j(x,y)\equiv P(x,y),\ \dot{x}=\sum_{j=0}^{3} Q_j(x,y)\equiv Q(x,y),\\ \gcd(P,Q)=1,\ yP_3(x,y)-xQ_3(x,y)\not\equiv 0.\end{gathered}\tag{1}$$

The straight line $\alpha x+\beta y+\gamma=0,\ \alpha,\beta,\gamma\in\mathbb{C},\ (\alpha,\beta)\neq(0,0)$ is called invariant for (1) if there exists a polynomial $K(x,y)$ such that the identity $\alpha P(x,y)+\beta Q(x,y)\equiv(\alpha x+\beta y+\gamma)K(x,y)$ holds. The invariant straight line $f=0$ for a cubic vector field $\mathbb{X}=P\partial/\partial x+Q\partial/\partial y$ has multiplicity m if there exists a sequence of cubic vector fields $\mathbb{X}_k$ converging to $\mathbb{X}$, such that each $\mathbb{X}_k$ has m distinct invariant straight lines $f_{1,k}=0,...,f_{m,k}=0$, converging to $f=0$ as $k\to\infty$, and this does not occur for $m+1$. We note that this definition of multiplicity can be applied to the infinite line $Z=0$ in the case when this line is not full of singular points, i.e. $yP_3(x,y)-xQ_3(x,y)\not\equiv 0$.

Theorem. *Every cubic system having two parallel complex invariant straight lines of multiplicity $m=2$ and the line at infinity of the multiplicity $m_\infty=3$ via affine transformations and time rescaling can be brought to the form*

$$\dot{x}=1+x^2,\qquad \dot{y}=a+2xy+x^3.\tag{2}$$

The system (2) has only the affine invariant straight lines $f_{1,2}\equiv x\pm i=0$, and the first integral: $\mathbb{F}=(1+x^2)\exp((1+ax-2y)/(1+x^2)+a\cdot arctg(x))$. The perturbed cubic system

$$\begin{aligned}\dot{x}=(x^2+1)(\epsilon x+1),\quad &\dot{y}=(4x^3+8xy+4a-3\epsilon+8ax\epsilon-\\ &-12x^2\epsilon+4y\epsilon+24x^2y\epsilon+4y^2\epsilon-4a\epsilon^2+6x\epsilon^2+8ay\epsilon^2-\\ &-24xy\epsilon^2+24xy^2\epsilon^2-\epsilon^3+6y\epsilon^3-12y^2\epsilon^3+8y^3\epsilon^3)/4\end{aligned}\tag{3}$$

has six affine distinct invariant straight lines: $f_1 = 0$, $f_2 = 0$, $f_{3,4} \equiv x \pm i\sqrt{1+a\epsilon} + \epsilon y - \epsilon/2 = 0$, $f_5 \equiv x + 1/\epsilon = 0$, $f_6 \equiv 2x + 2\epsilon y - \epsilon + 1/\epsilon = 0$. If $\epsilon \to 0$, then (3)$\to$ (2), $f_3 \to f_1$, $f_4 \to f_2$ and $f_5, f_6 \to \infty$.

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Daboux integrability of Lotka-Volterra cubic differential systems with 1:-2 resonant singularity and six straight lines of three directions

Silvia Turuta

Institute of Mathematics and Computer Science of ASM, Chişinău, Republic of Moldova
e-mail: poderioghin_silvia@yahoo.com

We consider the real Lotka-Volterra cubic system of differential equations with 1:-2 singularity

$$\begin{aligned} \dot{x} &= x(a_{30}x^2 + a_{21}xy + a_{12}y^2 + a_{20}x + a_{11}y + 1) \equiv P(x,y), \\ \dot{y} &= y(b_{21}x^2 + b_{12}xy + b_{03}y^2 + b_{11}x + b_{02}y - 2) \equiv Q(x,y), \end{aligned} \tag{1}$$

where $gcd(P,Q) = 1$.

A straight line $\alpha x + \beta y + \gamma = 0, \alpha, \beta, \gamma \in C$, $(\alpha, \beta) \neq (0,0)$ is invariant for (1) if there exists a polynomial $K(x,y)$ such that the following identity $\alpha P(x,y) + \beta Q(x,y) \equiv (\alpha x + \beta y + \gamma)K(x,y)$ holds. We say that an invariant straight line l has algebraic multiplicity m if m is the greatest natural number that l^m divide $P(PQ'_x + QQ'_y) - Q(PP'_x + QP'_y)$.

In this paper we give a classification and integrability of systems (1) with six invariant straight lines of three directions.

Theorem. *The system (1) has invariant straight lines of three directions of total algebraic multiplicity six if and only if modulus affine transformations and time rescaling it has one of the following twelve forms :*

1) $\dot{x} = -x(x-1), \quad \dot{y} = y(y-1)(2-x-y)$ **;**
2) $\dot{x} = x(1 - axy + (1+d)y^2 + ax), \quad \dot{y} = y(y-1)(dy+2)$**;**
3) $\dot{x} = x(x-1)(ax-1), \quad \dot{y} = y(-2 + 3x + dy + (a-2)x^2 - dxy)$**;**
4) $\dot{x} = \frac{1}{4}x(4 + f^2x^2 + 4fxy + 4(1+c)y^2 - 4fx), \quad \dot{y} = y(y-1)(2+cy)$ **;**
5) $\dot{x} = \frac{1}{4c^2}x(4f^2x^2 + 4c^2fxy + c^3(4+c)y^2 + 8cfx + 4c^2), \quad \dot{y} = y(y-1)(cy+2)$**;**
6) $\dot{x} = x(x-1)(ax-1), \quad \dot{y} = -\frac{1}{8a^2}(16a^2 - 24a^3x - 8a^3(1+2a)x^2 + 8afy - 8a^2fxy + f^2y^2)$ **;**
7) $\dot{x} = x(x-1)(ax-1), \quad \dot{y} = -\frac{1}{8}y(f^2y^2 - 8fxy + 8(2-a)x^2 - 24x + 8fy + 16)$ **;**
8) $\dot{x} = \frac{1}{4}x(f^2x^2 + 4fxy - 4y^2 - 4fx + 4), \quad \dot{y} = -2y(y-1)^2$**;**
9) $\dot{x} = x(x-1)^2, \quad \dot{y} = -\frac{1}{b^2}y(2b^2 - 3b^2x + b^2x^2 - 4by + 4bxy + 2y^2)$**;**
10) $\dot{x} = \frac{1}{2}x(x-1)((2+c)x-2), \quad \dot{y} = \frac{1}{2}y(y-1)(4-(4+c)x+2cy)$**;**
11) $\dot{x} = \frac{1}{2+c}x(x-1)(-2+2x-c), \quad \dot{y} = \frac{1}{2+c}y(y-1)(4+2c-(4+c)x+2cy+c^2y)$**;**
12) $\dot{x} = x(x-1)(ax-1), \quad \dot{y} = -\frac{1}{9a}y(18a - 9a(1+a)x + 6f(1+a)y - 9afxy + 2f^2y^2)$**;**

where $abcdf(a-1)(c+2)(d+2) \neq 0$ *and* $a,b,c,d,f \in \mathbb{R}$*. The systems 1)-12) are Darboux integrable and have the following first integral or integrating factor, respectively:*

1) $\mu(x,y) = x(x-1)^{-1}(y-2)^{-2}(x+y-1)^{-1}$**;**
2) $F(x,y) = x^2y(dy+2)^{-\frac{2+d}{d}}(y-ax-1)^{-2}$**;**
3) $F(x,y) = x^2y(ax-1)^{\frac{2-2a}{a}}(dy+2x-2)$**;**
4) $\mu(x,y) = (xy(cy+2))^{-1}(y-1)(2y+fx-2)^{-2}$**;**
5) $\mu(x,y) = (xy(y-1))^{-1}(cy+2)(c^2y+2fx+2c)^{-2}$**;**
6) $\mu(x,y) = (xy(x-1))^{-1}(ax-1)(fy-4a^2x+4a)^{-2}$**;**

7) $\mu(x,y) = (xy(ax-1))^{-1}(x-1)(fy-4x+4)^{-2}$**;**
8) $\mu(x,y) = (xy)^{-1}(fx+2y-2)^{-2}$**;**
9) $\mu(x,y) = (xy)^{-1}(bx+y-b)^{-2}$**;**
10) $\mu(x,y) = x^{-2}y^{-\frac{3}{2}}(x-1)^{-\frac{1}{2}}(y-1)^{-\frac{1+c}{2+c}}(2+(2+c)x)^{\frac{1}{2+c}}\ ((2+c)x-cy-2)^{-\frac{4+c}{4+2c}}$**;**
11) $\mu(x,y) = x^{-2}y^{-\frac{3}{2}}(x-1)^{\frac{1}{2+c}}(y-1)^{-\frac{1+c}{2+c}}(2x-2-c)^{-\frac{1}{2}}(2x-cy-2)^{-\frac{4+c}{4+2c}}$**;**
12) $\mu(x,y) = x^{-2}y^{-\frac{3}{2}}(x-1)^{\frac{a}{2a-2}}(ax-1)^{\frac{1}{2-2a}}(3ax-fy-3)^{\frac{2-a}{2a-2}}\ (3ax-fy-3a)^{\frac{1-2a}{2a-2}}$**.**

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The study of evolution price of electricity in households with a dynamic model

Laura Ungureanu

Spiru Haret University, Craiova
e-mail: `laura.ungureanu@spiruharet.ro`

This scientific approach is starting from the premise that in the near future the Romanian Electricity Market will be complete liberalized in order to be integrated in the Single Market Electricity (31.12.2017). These profound changes will have a strong impact on household behaviour. In this regard, we study for electricity market a dynamic model based on the principle of supply and demand equilibrium in which it is presented the reaction that the electricity price p produces on the amount consumed q. Also, by means of extrapolation methods we are deduced the expressions of demand and supply functions for electricity price. These expressions are entered in a demand-supply model given by a dynamical system. We study the equilibrium and the evolution of this system based on Eurostat data sets from 1999 to 2015 in order to determinate the electricity price evolution and to establish competitive advantages for a sustainable consumption.

The developed model is nonlinear and it is studied by the means of bifurcation theory which allows the pointing in a specific manner of negative phenomena such as lower production and especially those characterized by positive modernization of production capacities which can trigger both increased demand, and determining consumption.

Periodic solutions to indefinite singular equations

Manuel Zamora

Departamento de Matemáticas, Grupo de Investigación en Sistemas Dinámicos y Aplicaciones (GISDA), Universidad del Bío-Bío, Concepción, Chile
e-mail: `mzamora@ubiobio.cl`

The efficient conditions guaranteeing the existence of a $T-$periodic solution to the second order differential equation

$$u'' = h(t)g(u)$$

will be established in this talk. Here, g is a positive and decreasing function with singularity at the origin, and the weight $h \in L(\mathbb{R}/T\mathbb{Z})$ is a sign-changing function. The approach is based on Leray-Schauder continuation degree theory.

Section 4. Probability Theory, Mathematical Statistics, Operations Research

Reliability for semi-Markov systems: modelling and estimation

Vlad Stefan Barbu [1]

Université de Rouen, Laboratoire de Mathématiques Raphaël Salem, France
e-mail: barbu@univ-rouen.fr

Semi-Markov processes and Markov renewal processes represent a class of stochastic processes that generalize Markov and renewal processes. As it is well known, for a discrete-time (respectively continuous-time) Markov process, the sojourn time in each state is geometrically (respectively exponentially) distributed. In the semi-Markov case, the sojourn time distribution can be any distribution on $\mathbb{N}$ (respectively on $\mathbb{R}_+$). This is the reason why the semi-Markov approach is much more suitable for applications than the Markov one.

The purpose of our talk is: to make a general introduction to semi-Markov processes; to investigate some reliability and survival analysis problems for this type of system and to address some statistical topics.

We start by briefly introducing the semi-Markov framework and by giving some basic definitions and results. These results are applied in order to obtain closed forms for some survival or reliability indicators, like survival/reliability function, availability, mean hitting times, etc. The last part of our talk is devoted to the estimation of the main characteristics of a semi-Markov system (semi-Markov kernel, semi-Markov transition probabilities, etc.) and to the asymptotic properties of these estimators. Statistical issues for the reliability indicators are also presented.

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Parallel SMM-DMM algorithm for determining Bayes solutions of the bimatrix incomplete information game

Boris Hâncu

Moldova State University, Moldavia
e-mail: boris.hancu@gmail.com

To determine Bayes-Nash equilibria profiles of the bimatrix incomplete information game $\widetilde{\mathbf{\Gamma}} = \left\langle \{1,2\}, I, J, \left\{ AB(\alpha,\beta) = \left\| \left(a_{ij}^{\alpha\beta}, b_{ij}^{\alpha\beta} \right) \right\|_{i\in I}^{j\in J} \right\}_{\alpha=\overline{1,\kappa_1}}^{\beta=\overline{1,\kappa_2}} \right\rangle$ we have to follow the next steps.

1) Root MPI process broadcasts to all $\varkappa_1 \cdot \varkappa_2$ MPI processes the initial matrices $A = ||a_{ij}||_{i\in I}^{j\in J}$, and $B = ||b_{ij}||_{i\in I}^{j\in J}$ of the bimatrix game $\Gamma = \langle A, B \rangle$.

[1] parts of this talk are joint works with Alex Karagrigoriou, University of the Aegean, Samos, Greece; N. Limnios, Université de Technologie de Compiègne, France; Andreas Makrides, University of Cyprus, Nicosia, Cyprus

2) Using the MPI and open source library ScaLAPACK-BLACS, initialize the processes grid $\{(\alpha,\beta)\}_{\alpha=\overline{1,\varkappa_1}}^{\beta=\overline{1,\varkappa_2}}$. All fixed MPI processes (α,β) using the OpenMP directives and combinatorial algorithm construct the sets $\widetilde{\mathbf{I}}(\alpha), \widetilde{\mathbf{J}}(\beta)$.

4) MPI process with rank α, for all $\alpha = \overline{1,\kappa_1}$ constructs payoff matrix $A(\alpha)$ and MPI process with rank β, for all $\beta = \overline{1,\varkappa_2}$ constructs payoff matrix $B(\beta)$ and using open source library ScaLAPACK-BLACS broadcasts to all $\varkappa_1 \cdot \varkappa_2$ the matrix $A(\alpha)$ the matrix $B(\beta)$.

6) All fixed MPI processes (α,β) using the OpenMP functions eliminate from matrix $A(\alpha)$ and from matrix $B(\beta)$ the lines that are strictly dominated in matrix $A(\alpha)$ and columns that are strictly dominated in matrix $B(\beta)$. Finally we obtain the matrices $\left(\widehat{\mathbf{A}}(\alpha), \widehat{\mathbf{B}}(\beta)\right)$, where $\widehat{\mathbf{A}}(\alpha) = \left\|\mathbf{a}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}}\right\|_{\widetilde{\mathbf{i}}\in\widetilde{\mathbf{I}}'(\alpha)}^{\widetilde{\mathbf{j}}\in\widetilde{\mathbf{J}}'(\beta)}$ and $\widehat{\mathbf{B}}(\beta) = \left\|\mathbf{b}_{\widetilde{\mathbf{i}}\widetilde{\mathbf{j}}}\right\|_{\widetilde{\mathbf{i}}\in\widetilde{\mathbf{I}}'(\alpha)}^{\widetilde{\mathbf{j}}\in\widetilde{\mathbf{J}}'(\beta)}$ and cardinals $\left|\widetilde{\mathbf{I}}'(\alpha)\right| \leq \left|\widetilde{\mathbf{I}}(\alpha)\right|, \left|\widetilde{\mathbf{J}}'(\beta)\right| \leq \left|\widetilde{\mathbf{J}}(\beta)\right|$.

7) All fixed MPI processes (α,β) using the OpenMP functions and ScaLAPACK routines and existing algorithm, determine all Nash equilibrium profiles in the bimatrix game with matrices $\left(\widehat{\mathbf{A}}(\alpha), \widehat{\mathbf{B}}(\beta)\right)$.

8) Using ScaLAPACK-BLACS routines, the root MPI process gather from processes grid $\{(\alpha,\beta)\}_{\alpha=\overline{1,\varkappa_1}}^{\beta=\overline{1,\varkappa_2}}$ the set of Nash equilibrium profiles in the bimatrix game with matrices $(\mathbf{A}(\alpha), \mathbf{B}(\beta))$.

Applications of the Rosenthal- type inequality for $\tilde{\rho}$-mixing random variables

Zahra Shokooh Ghazani

Department of Mathematics, Islamic Azad university, Central Tehran Branch, Tehran, Iran
e-mail: zah.shokooh_ghazani@iauctb.ac.ir

In this paper, a sequence of $\tilde{\rho}$-mixing random variables that are stochastically dominated to random variable ξ is considered. We will prove the complete convergence of the weighted sums of these random variables by presenting new assumptions as well as using Rosenthal- type inequality.

An overview of nonparametric methods

Kulwant Singh Kapoor

Department of Biostatistics, All India Institute of Medical Sciences, New Delhi, India
e-mail: kulwantsinghus@yahoo.com

In medical research sample observations are used to generalize about the population from which the observations have been drawn. Using appropriate statistical tools does this. These statistical tools or methods can be broadly classified into two categories: Parametric and Nonparametric methods. Parametric methods of statistical inference require the assumption that the data has come from some underlying distribution whose general form is known, such as normal, binomial, poisson, etc. Statistical methods for estimation and hypothesis testing are then based on these assumptions. The focus is on estimation and testing hypothesis about them. When the stringent assumptions about the distribution of parent populations cannot be met, parametric tests are no longer applicable, and some alternative tests called 'non-parametric' or 'distribution-free' tests are employed instead. Strictly speaking, only those procedures that are not statements about the population parameters are classified as non-parametric, while those that make no assumption about the sampled population are called distribution free procedures. Despite this distinction, it is customary to use the terms non-parametric and distribution free interchangeably and to discuss the various procedures of both types under the heading of non-parametric statistics. Non-parametric methods have its advantages and disadvantages also.

Probabilistic study of some topological indices of random trees

Ramin Kazemi

Imam Khomeini International University, Qazvin, Iran
e-mail: r.kazemi@sci.ikiu.ac.ir

Topological indices are numerical parameters of a graph which characterize its topology and are usually graph invariant. If G is a connected graph with vertex set V, then the eccentric connectivity index of G, $\xi^c(G)$, is defined as $\sum_{v\in V(G)} d(v)ecc(v)$ where $d(v)$ is the degree of a vertex v and $ecc(v)$ is its eccentricity. The first and second Zagreb index of G are defined as $M_1(G) = \sum_{v\in V(G)}(d(v))^2$ and $M_2(G) = \sum_{uv\in E(G)} d(u)d(v)$, where $V(G)$ and $E(G)$ are the vertex and edge sets of a graph G, respectively. A size-m bucket recursive tree T_m with variable bucket capacities and maximal bucket size b starts with the root labeled by 1. The tree grows by progressive attraction of increasing integer labels: when inserting label $j+1$ into an existing bucket recursive tree T_j, except the labels in the non-leaf nodes with capacity $< b$ all labels in the tree (containing label 1) compete to attract the label $j+1$. For the root node and nodes with capacity b, we always produce a new node $j+1$. But for a leaf with capacity $c < b$, either the label $j+1$ is attached to this leaf as a new bucket containing only the label $j+1$ or is added to that leaf and make a node with capacity $c+1$. This process ends with inserting the label m (i.e., the largest label) in the tree.

In this paper, we give the mean and variance of the first and second Zagreb index, and the eccentric connectivity index. Also, we show an asymptotic normality based on these indices. With the same manner, we can extend our results to another random trees.

Optimal cooperative strategies for stochastic games with final sequence of states

Alexandru Lazari

State University of Moldova, Chişinău, Republic of Moldova
e-mail: lazarialexandru@mail.md

Let L be a stochastic system with finite set of states V, where $|V| = \omega$. The system L starts its evolution from the state v with the probability $p^*(v)$, $\forall v \in V$. Also, the transition from one state $u \in V$ to another state $v \in V$ is performed in one unit of time according to the probability $p(u,v)$. Additionally we assume that a sequence of states $X = (x_1, x_2, \ldots, x_m) \in V^m$ is given and the stochastic system stops transitions as soon as the states $x_1, x_2, \ldots, x_m$ are reached consecutively in given order.

The stochastic system L represents a Markov process with final sequence of states X. Various problems related to such systems were studied in [3].

Next, the following game is considered. Initially, each player $\mathcal{P}_\ell$ defines his strategy (transition matrix) $(p^{(\ell)}(u,v))_{u,v\in V}$, $\ell = \overline{0, r-1}$. The initial distribution of the states is established according to the given distribution $(p^*(v))_{v\in V}$. The game is started by first player $\mathcal{P}_0$ and at every moment of time, the stochastic system passes consecutively to the next state according to the transition matrix given by current player. After the last player $\mathcal{P}_{r-1}$, the first player $\mathcal{P}_0$ acts on the system evolution and the game continues in this way until the given final sequence of states X is achieved. In [2] it was proved that the distribution of the game duration T is a homogeneous linear $m\omega r$-recurrence and its main probabilistic characteristics were determined.

In this paper we consider that all the players have the same goal – minimization of the game duration. We apply the geometric and signomial programming approaches ([4]) for determining the optimal stationary strategies of the players that minimize the duration of the game.

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Stochastic optimal control of a mixing flow model

Mario Lefebvre

Polytechnique Montréal, Canada
e-mail: mlefebvre@polymtl.ca

We consider the following two-dimensional controlled stochastic mixing flow model:

$$\begin{aligned} \dot{x}_1(t) &= Gx_2(t), \\ \dot{x}_2(t) &= KGx_1(t) + b_0x_2(t)u(t) + m_0x_2(t) + \sigma_0x_2(t)\dot{B}(t), \end{aligned}$$

where G (> 0), $-1 < K < 1$, b_0 $(\neq 0)$, m_0 and σ_0 (> 0) are constants, and $\{B(t), t \geq 0\}$ is a standard Brownian motion. Optimal control problems involving a random final time are considered for this model, and explicit solutions are obtained by making use of the method of similarity solutions.

Informational extended games and applications

Ludmila Novac

Moldova State University, Moldavia
e-mail: novac-ludmila@yandex.com

The informational extension concept for games has on its basis the assumptions that the participants of the game have possibility to send and to receive, (as well as to guess) some information about the chosen strategies of other participants and about their behavior [1]. Our aim is to show the importance of the information' possession for all decision-making problems and for conflict problem solving [3].

We analyze a static informational extended game with n players, who choose their actions simultaneously. The game will assume that players' payoff functions are common knowledge.

In this informational extended game we will consider, that each player is informed about the strategies that will be chosen by other players. In this case the set of strategies for each player will be a set of functions defined on the product of strategies' sets of all other players [1].

An informational extended game can be described in the normal form by the triplet: ${}_n\Gamma = \left\langle I; \overline{X}_i, i = \overline{1,n}; \overline{H}_i, i = \overline{1,n} \right\rangle$, *where* $I = \{1, 2, ..., n\}$ *is the set of players, and the sets of strategies for players are defined by:* $\overline{X}_i = \left\{ \varphi_i : \prod_{j \in I, j \neq i} X_j \to X_i \right\}, i = \overline{1,n}$, *where* $X_i, \left(i = \overline{1,n}\right)$ *is the set of strategies without information for the i-th player. The payoff functions are defined on the product of the extended strategies' sets:* $\overline{H}_i : \prod_{i \in I} \overline{X}_i \to R, \left(i = \overline{1,n}\right)$.

In this case we analyze the informational extended game in which we consider that all players know the chosen strategies of all other players, and each player $i \in I$ chooses his strategy from the set $\overline{X}_i$.

If some players $j \in I$ don't know the strategies of other players, that will be chosen, then they will choose their strategies from their initial sets X_j without information.

Thus, we can define some different informational extended games in which the outcome will consist of strategies $x_j \in X_j, j \in J$ and $\varphi_k \in \overline{X}_k, k \in I \backslash J$, where J is the set of players which don't have information about chosen strategies of other players [3].

We can consider some games with continuous sets of strategies, or various cases when the sets of strategies are discrete and finite[1-3]. For the informational extended games, in the both cases (games with continuous or discrete sets of strategies), we can determine the solutions (Nash equilibria), using the best-response sets [1-3].

One of the most common interpretations of the "informational extended set of strategies" can be described by next example.

Let's to consider an informational extended game with two players. A trivial example is a decision about entering a new market. Suppose the company, which acts as a monopoly in any market, and another outsider company which have to decide on joining or not joining the market. If in a static game, the outsider company would have some information about the reaction of the company, which acts as a monopolist, then it could chose a better strategy in order to avoid some unexpected situation and not to make his partner to act aggressively.

These models can be used in the several situations for decision-making problems in various social domains, inclusively in economy, management and others.

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On statistical tests for parametric and semiparametric regression models

George Andrei Padureanu

University of Bucharest, Doctoral School in Mathematics, Romania
e-mail: george.padureanu@gmail.com

A subject that is least addressed in the literature is Testing of Statistical Hypotheses on Semiparametric Regression Methods. A very important place would occupy comparing two regression models, one parametric versus one semiparametric.

The paper uses an entropic measure called divergence of conditional distributions corresponding to two regression models. Regression models that are studied are parametric and semi-parametric regression models with binary response. Divergences are defined as of type

$$D(F_1(Y|(x,u))||F_2(Y|(x,u))),$$

both Renyi and Shannon versions.

A simulation study allows the identification empirical distributions of these divergences and on this basis; the paper proposes statistical tests to verify the hypothesis $H : F(Y|(x,u)) = F_2(Y|(x,u))$ with alternative $H_a : F(Y|(x,u)) = F_2(Y|(x,u))$.

Monetary policy indicators analysis based on regression function

Anamaria Popescu

Faculty of Science, University of Petrosani, Romania
e-mail: am.popescu@yahoo.com

This article highlights the effective possibilities for the use of linear regression model to analyze the values of interest rates the standing facilities. In this context, I consider these indicators as dependent variables whose variation is significantly determined by the evolution of the monetary policy interest rate representing the interest rates used for the principal money market operations of the BNR. To emphasize the practical aspects related to the use of linear regression in analyzing the instruments of monetary policy, we have developed a practical study in which we defined as independent variable rate interest rate monetary policy, in the period 2010- 2014. The objectives of this analysis is to determine the function that best describes the relationship of the three indicators, observing the relationship that is established between them and estimating an valid and statistically significant econometric model.

Model for the waiting time of the first conception Bayesian approach to probability

Chitaranjan Sharma

Government girls college Dhar M.P, India
e-mail: drcsharma3@gmail.com

Work on models of human reproductive process comprised of both discrete and continuous models postulating both homogeneity and heterogeneity of the population. A historical review of the models reveals that the models differ only in-treatment of time as Discrete" or "Continuous". In many of the models, the probability of conception is held constant and further assumption is made in regard to all conceptions which lead to live births and which are associated with a fixed non-susceptible period covering the gestation period and the period of post-partum amenorrhea. However, introducing further the notion of time when conception is recorded has also evolved new type of more flexible and realistic probability models. Uniform prior density, the Baye's estimate the parameter $'\theta'$ has been calculated which is equal to the maximum likelihood estimate. If the prior information exists, then by the corresponding values of α, β, and λ can be substitute in the above expressions to obtain exact posterior estimates of the desired functions can be obtained and the complete Bayesian analysis of the model can be performed.

A note on profile-likelihood-based confidence intervals for the mean of inverse Gaussian distribution

Patchanok Srisuradetchaia and Nantapath Trakultraipruk

Department of Mathematics and Statistics, Faculty of Science, Thammasat University, Thailand
e-mail: spatchan@tu.ac.th

The inverse Gaussian distribution is applied in a wide range of applications from physics, engineering, medicine to even business. In this research, a confidence interval of the mean is constructed for the inverse Gaussian distribution with an unknown shape parameter. The profile likelihood is employed as a method to eliminate a shape parameter. Since the large-sample theory is used to calibrate the likelihood, a sufficient sample size is required. Thus the optimal sample size is determined by the Monte-Carlo simulation method. In this work, we also prove that the profile likelihood function does not converge to zero as the mean approaches infinity - in both known and unknown shape parameters - but to a certain quantity depending on sample. Moreover, the profile-likelihood-based interval is derived to obtain a useful form of analytical solution.
Keywords: inverse Gaussian distribution, profile likelihood, likelihood- based confidence intervals, Monte-Carlo simulation.

Comparative analysis of estimation methods for CES production function

Nadia Elena Stoicuta and Olimpiu Stoicuta

University of Petroşani, Romania
e-mail: stoicuta_nadia@yahoo.com, stoicuta_olimpiu@yahoo.com

This article describes the analysis of the stationary and dynamic case of the Kmenta method for estimating the CES production function. The data series which occur in the analyzed models, are given by the real gross value added, regarded as output variable, and the tangible assets, respectively the average number of employees, regarded as input variables. The parameters of the models, are determined using the least squares method (LSM), using the software package Eviews.

Keywords: CES production function, least squares method (LSM), Kmenta approximation.

The fractional multi-objective transportation problem with fuzzy time criterion

Alexandra Tkacenko

State University of Moldova, Chişinău, Republic of Moldova
e-mail: alexandratkacenko@gmail.com

In this paper we propose to study a new approach to solving the multi-objective fractional transportation problem with the same denominators, which are functions of time. Additionally, we include in model separate this function as a time criterion of "bottleneck" type. We develop

the case when it is of not deterministic, but of fuzzy type [1], [2]. The mathematical model of the proposed problem is the follows:

$$\min Z^k = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}}{\max_{ij} \tilde{t}_{ij} \,|x_{ij} > 0\}} \tag{1}$$

$$\min Z^{k+1} = \max_{ij} \tilde{t}_{ij} \,|x_{ij} > 0\,\} \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = a_i,\, i = 1, 2, \ldots, m; \quad \sum_{i=1}^{n} x_{ij} = a_i, j = 1, 2, \ldots, n; \tag{3}$$

$$x_{ij} \geq 0,\, i = 1, 2, \ldots, m,\, j = 1, 2, \ldots, n,\, k = 1, 2, \ldots, r. \tag{4}$$

In order to solve the model (1)-(4) we proposed an iterative algorithm. It generates the crowds efficient model solutions for different types of approaches to the time required for transport from optimistic to pessimistic, using for this purpose the possible ranges of variation thereof. The algorithm was tested on several examples and was found to be quite effective.

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Robust estimates for the single index model

Aida Toma[1,2]

[1] *Bucharest Academy of Economic Studies*

[2] *"Gheorghe Mihoc - Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania*

e-mail: aida_toma@yahoo.com

For portfolios with a large number of assets, the single index model allows to express the large number of covariances between individual assets through a significantly smaller number of parameters. This avoid the constraint of having very large samples for parameters estimation, which would be unrealistic given the dynamic of market conditions. The traditional way to estimate the regression parameters in the single index model is the least square method. Although the least square estimators have desirable properties when the model is exactly satisfied, they may give completely erroneous results when outliers are present in the data set. In this paper we define and study minimum pseudodistance estimators for the parameters of the single index model and using them we construct new optimal robust portfolios.

Network games

Valeriu Ungureanu

Moldova State University, Moldavia
e-mail: v.a.ungureanu@gmail.com

We expose results of investigating games that appear in real situations when several companies manage the activity of a big network. Decision-making subjects may have antagonistic interests. In such circumstances, well-known extremal network/digraph problems and problems of constructing various structures on networks/digraphs become mono or multi criteria strategic network game problems. Systems of human, information, hardware or other types, controlled by different subjects, involve their interactions. As a consequence, many traditional network problems have to be treated from the perspective of game theory, including problems of routing, load balancing, facility location, network design, etc.

A series of related problems have been investigated and described in scientific literature in the context of cyclic games solving. That approach used a special type of strategy definition.

This work is based on previous author's results [1] which introduced some types of games on digraphs by defining originally the notions of pure strategies, outcome and payoff functions.

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Section 5. Algebra, Logic, Geometry (with applications)

The ideal structure of Lattice-ordered groups and their Crossed products

Mamoon A. Ahmed

Princess Sumaya University for Technology, Amman, Jordan
e-mail: m.ahmed@psut.edu.jo

Let (G, G_+) be a quasi-lattice-ordered group with positive cone G_+ and H_+ a hereditary subsemigroup of G_+. In [2] we showed that the ideal $I_{H_+} \times_\alpha G_+$ is generated by $\{i_{B_{G_+}}(1 - 1_u) : u \in H_+\}$ and is therefore contained in the commutator ideal $\mathcal{C}_G$ of the C^*-algebra $B_{G_+} \times_\alpha G_+$. Adji and Raeburn have shown a result about the structure of the primitive ideal space of the C^*-algebra $B_{G_+} \times_\alpha G_+$ for a totally ordered abelian group [1], Theorem 3.1. In this paper we extend their result to this more general setting. In particular we show that if $\sum(G)$ is the set of subgroups $H := H_+ - H_+$ partially ordered by inclusion, then there exists a well-defined map F from the disjoint union $\bigsqcup\{\widehat{H} : H \in \sum(G)\}$ to the primitive ideals of the Toeplitz algebra $B_{G_+} \times_\alpha G_+$. This allows us to deduce information about the irreducible representations of the C^*-crossed product $B_{G_+} \times_\alpha G_+$.

Keywords: Lattice-ordered group, C^*-algebra, crossed product, short exact sequence, primitive ideal, irreducible representation.

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The collection of operators with the b- property

Baris Akay[1] and Omer Gok[2]

[1] *Istanbul University, Faculty of Sciences, Turkey*
[2] *Yildiz Technical University, Faculty of Arts and Sciences, Istanbul, Turkey*
e-mail: baris.akay@istanbul.edu.tr, gok@yildiz.edu.tr

In this talk, we are interested in the operators on Archimedean vector lattices(Riesz spaces) with the b-property.

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On the Slyvester-Padovan-Jacobsthal-type sequences

Yeşim Aküzüm and Ömür Deveci

Department of Mathematics, Faculty of Science and Letters, Kafkas University, Turkey e-mail: yesim_036@hotmail.com, odeveci36@hotmail.com

Let f and g be polynomials of degrees (k) and (m), respectively, and let these polynomials be given by

$$f = a_k x^k + a_{k-1}x^{k-1} + \cdots + a_1 x + a_0$$

$$g = b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0.$$

The Sylvester matrix $S_{f,g} = [S_{ij}]_{(k+m)\times(k+m)}$ associated with the polynomial f and g is defined as follows:

$$\begin{bmatrix} a_k & a_{k-1} & \cdots & a_2 & a_1 & a_0 & 0 & 0 & \cdots & 0 \\ 0 & a_k & a_{k-1} & \cdots & a_2 & a_1 & a_0 & 0 & \cdots & 0 \\ 0 & 0 & a_k & a_{k-1} & \cdots & a_2 & a_1 & a_0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \cdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & a_k & a_{k-1} & \cdots & a_2 & a_1 & a_0 \\ b_m & b_{m-1} & \cdots & b_2 & b_1 & b_0 & 0 & 0 & \cdots & 0 \\ 0 & b_m & b_{m-1} & \cdots & b_2 & b_1 & b_0 & 0 & \cdots & 0 \\ 0 & 0 & b_m & b_{m-1} & \cdots & b_2 & b_1 & b_0 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \cdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & b_m & b_{m-1} & \cdots & b_2 & b_1 & b_0 \end{bmatrix}.$$

In this work, we define the Slyvester-Padovan-Jacobsthal-type sequences of the first and second kind via the Slyvester matrices which are obtained from the characteristic polynomials of the Padovan and Jacobsthal sequences and we obtain the cyclic groups and the semigroups from the generating matrices of the Slyvester-Padovan-Jacobsthal-type sequences of the first and second kind when read modulo m. Also, we study the sequences modulo m and then, we derive the relationships among the orders of the cyclic groups and the periods of the sequences.

2010 Mathematics Subject Classification: 11B37, 11B50, 05A15, 20D60, 11C20.
*Keywords:*The Sylvester matrix,Sequence, Group, Period.

A problem connected with Terao conjecture

Cristian Anghel

"Simion Stoilov" Institute of Mathematics of Romanian Academy, Bucharest, Romania
e-mail: Cristian.Anghel@imar.ro

I shall speak about Terao's conjecture and some connections with the notion of stable extendability of vector bundles on projective spaces.

Geometrical aspects in the generalized (almost) Lie algebras/algebroids category

Constantin M. Arcuș and Esmaeil Peyghan

Secondary School CORNELIUS RADU, Romania e-mail: constantin_arcus@yahoo.ro

We build an important example of generalized almost Lie algebra. Using this space, we present an algebraic point of view over nonlinear connections theory. Also, the distinguished linear connections are presented in this general framework. Using the torsion and curvature of a distinguished linear connection we present formulas of Ricci type and identities of Cartan and Bianchi type. A detailed study about (pseudo)metrizability and a theory dedicated to algebraic (generalized) Lagrange spaces are presented. The Einstein equations are presented in this general framework.

The Vandermonde-type sequences in groups

Gizem Artun and Ömür Deveci

Department of Mathematics, Faculty of Science and Letters, Kafkas University, Turkey
e-mail: gizmartn91@hotmail.com, odeveci36@hotmail.com

In this work, we define the Vandermonde-type sequence and then we obtain the relations among the elements of the sequence and generating matrix of the sequence. Also, we study the Vandermonde-type sequence modulo m and we obtain the cyclic groups from the generating matrix of the sequence when read modulo m. Then we derive the relationships among the orders of the obtained cyclic groups and the periods of the Vandermonde-type sequence modulo m. Finally, we redefine the adjacency-type sequences by means of the elements of the groups which have two or more generators and then we obtain the periods of the Vandermonde-type sequence in the polyhedral groups $(n, 2, 2)$, $(2, n, 2)$ and $(2, 2, n)$ for $n \geq 3$ as applications of the results produced.

2010 Mathematics Subject Classification: 11B50, 20F05, 15A36, 20D60.
Keywords: The Vandermonde-Type Sequence, Group, Perio.

On some almost contact metric structures on hypersurfaces in Kählerian manifolds

Mihail B. Banaru and Galina A. Banaru

Smolensk State University, Russian Federation
e-mail: mihail.banaru@yahoo.com

1. One of the most important examples of almost contact metric structures is the structure induced on an oriented hypersurface in an almost Hermitian manifold [5]. We recall that an almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi, \xi, \eta, g\}$ on this manifold, where ξ is a vector, η is a covector, Φ is a tensor of the type $(1,\ 1)$ and $g = \langle \cdot,\ \cdot \rangle$ is the Riemannian metric. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta \circ \Phi = 0,\ \Phi^2 = -id + \xi \otimes \eta,$$

$$\langle \Phi X, \Phi Y \rangle = \langle \Phi X, \Phi Y \rangle - \eta\,(X)\,\eta\,(Y) \quad X, Y \in \aleph(N)$$

where $\aleph(N)$ is the module of smooth vector fields on N [6], [8].
The class of Kählerian manifolds is the most important and best studied Gray–Hervella class of al-most Hermitian manifolds. It belongs to each of sixteen Gray–Hervella classes [7]. That is why every result on the geometry of almost Hermitian manifolds of any class is also relevant to Kählerian manifolds. Note that Kählerian geometry is currently an intensively developing area of

modern differential geometry. This topic has undoubtedly a rich inner content and close ties with other parts of geometry as well as various areas of modern theoretical physics [8].

2. As it is known, an almost Hermitian manifold is a $2n$-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J. Moreover, the following condition must hold

$$\langle JX,\ JY\rangle = \langle X,\ Y\rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n} [8].

We recall that the fundamental (or Kählerian) form of an almost Hermitian manifold is determined by the relation

$$F(X,\ Y) = \langle X,\ JY\rangle\,, \quad X, Y \in \aleph(M^{2n}).$$

An almost Hermitian manifold is called Hermitian, if its structure is integrable. A Kählerian structure must comply with the condition $\nabla F = 0$, where ∇ is the Levi–Civita connection of the metric $g = \langle \cdot, \cdot \rangle$ [7], [8].

3. The main results are the following:

1) The Cartan structural equations of the general type almost contact metric structure on an oriented hypersurface in a Kählerian manifold are obtained;

2) The Cartan structural equations of some important kinds of almost contact metric structures (cosymplectic, Sasaki, Kenmotsu etc.) on an oriented hypersurface in a Kählerian manifold are selected;

3) The Cartan structural equations of almost contact metric structures on oriented totally umbilical and totally geodesic hypersurfaces in a Kählerian manifold are also selected;

4) The Cartan structural equations of almost contact metric structures on oriented 1-hypersurfaces (i.e. on hypersurfaces with type number 1) in a Kählerian manifold are also selected;

5) A characterization in terms of the type number of hypersurfaces in Kählerian manifolds with some important kinds of almost contact metric structures (cosymplectic, nearly cosymplectic, Sasaki, Kenmotsu etc.) is obtained.

We remark that the present work is a continuation of researches of the authors in the theory of the hypersurfaces in Kählerian manifolds (see, for example, [1], [2], [3], [4] and others).

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Endomorphisms of AG-quasigroups

Natalia Bobeica and Liubomir Chiriac

Universitatea de Stat din Tiraspol, Moldavia
e-mail: nbobeica1978@gmail.com, llchiriac@gmail.com

A groupoid $(G, \cdot)$ is called a *groupoid Abel-Grassmann* or *AG-groupoid* if it satisfies the left invertive law $(a \cdot b) \cdot c = (c \cdot b) \cdot a$ for all $a, b, c \in G$.

A quasigroup is an algebra $(Q; *, \backslash, /)$ of type (2,2,2) such that $x\backslash(x * y) = y$, $(x * y)/y) = x$, $x * (x\backslash y) = y$, and $(x/y) * y) = x$, for all $x, y \in G$ [1].

Denote by $End(Q)$ the sets of all endomorphisms of a quasigroup Q. It is interesting to study the following problem.

Problem. *Let Q and $Q^{'}$ be the quasigroups. If their endomorphisms $End(Q)$ and $End(Q^{'})$ are isomorphic, in which conditions Q and $Q^{'}$ are isomorphic too?*

In [2] were studied the endomorphisms of idempotent medial quasigroups and determinability of some classes of medial quasigroups by their endomorphisms. The authors were introduced the concept of the endomorphism algebra of idempotent medial quasigroup.

We study the above *Problem* in the case of *AG*-quasigroup.

We prove that if the endomorphism algebras $End(Q)$ and $End(Q^{'})$ of idempotent *AG*-quasigroups Q and $Q^{'}$ are isomorphic, then the quasigroups Q and $Q^{'}$ are isomorphic too.

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The reflector functor and the lattice $\mathbb{L}(\mathcal{R})$

Dumitru Botnaru and Olga Cerbu

Tiraspol State University, Chişinău, Republic of Moldova
State University of Moldova, Chişinău, Republic of Moldova
e-mail: dumitru.botnaru@gmail.com, olga.cerbu@gmail.com

In the category $\mathcal{C}_2\mathcal{V}$ of locally convex topological vector spaces [4] we examine a class of factorization structures for which the reflector functor transforms the class of projections or the class of injections, or both classes into themselves.

In this paper, for any reflector functor $r \colon \mathcal{C}_2\mathcal{V} \to \mathcal{R}$ constructs a lattice $\mathbb{L}_u(\mathcal{R})$ of factorization structure, so that r preserves the respective classes (Theorem 10) for each element of this lattice.

Concerning to the category of locally convex spaces (see [4]), referring to factorization structures (see [1]).

The factorization structure $(\mathcal{E}_p, \mathcal{M}_u)$ = (the class of exact epimorphisms, the class of universal monomorphisms) was described in the lattice [see 3]. In the category $\mathcal{C}_2\mathcal{V}$, a monomorphism $m \colon X \to Y$ is universal iff any continuous functional defined on X, extends through m.

Definition 1. *Let $\mathcal{A}$ and $\mathcal{B}$ be two classes of morphisms of category $\mathcal{C}$. The class $\mathcal{A}$ is called $\mathcal{B}$-hereditary if $f \cdot g \in \mathcal{A}$ and $f \in \mathcal{B}$ implies $g \in \mathcal{A}$.*

A dual notion: the $\mathcal{B}$-cohereditary class.

Definition 2. *Let $\mathcal{A}$ and $\mathcal{B}$ be two classes of morphisms of the category $\mathcal{C}$. The class*
$\mathcal{A} \circ \mathcal{B} = \{a \cdot b \mid a \in \mathcal{A}, b \in \mathcal{B}$ *and the composition ab exists*$\}$

In the class $\mathbb{R}$ of locally convex topological vector spaces $\mathcal{C}_2\mathcal{V}$ (see [4]) we consider the following order: $\mathcal{R}_1 \leq \mathcal{R}_2$ if $\mathcal{R}_1 \subset \mathcal{R}_2$. On the class of right factorization structures we consider the order $(\mathcal{P}_1, \mathcal{I}_1) \leq (\mathcal{P}_2, \mathcal{I}_2)$ if $\mathcal{P}_1 \subset \mathcal{P}_2$.

Let Π be the subcategory of the complete spaces with weak topology and $\pi\colon \mathcal{C}_2\mathcal{V} \to \Pi$ - the reflector functor. The subcategory Π is the smallest element in the lattice $\mathbb{R}$. Let $\mathcal{R} \in \mathbb{R}$. For each object X of the category $\mathcal{C}_2\mathcal{V}$, let $r^X : X \to rX$ and $\pi^X : X \to \pi X$ the $\mathcal{R}$ and Π-repliques. Since $\Pi \subset \mathcal{R}$, we have $\pi^X = v^X \cdot r^X$, for a morphism v^X. We denote by $\mathcal{U} = \mathcal{U}(\mathcal{R}) = \{r^X \mid X \in |\mathcal{C}_2\mathcal{V}|\}$, $\mathcal{V} = \mathcal{V}(\mathcal{R}) = \{v^X \mid X \in |\mathcal{C}_2\mathcal{V}|\}$. We have the following factorization structures $(\mathcal{P}'', \mathcal{I}'') = (\mathcal{P}''(\mathcal{R}), \mathcal{I}''(\mathcal{R})) = (\mathcal{V}^{\urcorner}, \mathcal{V}^{\urcorner}{}_{\llcorner})$, $(\mathcal{P}', \mathcal{I}') = (\mathcal{P}'(\mathcal{R}), \mathcal{I}'(\mathcal{R})) = (\mathcal{U}^{\llcorner\urcorner}, \mathcal{U}^{\llcorner})$ (see [2]).

For $\mathcal{R} \in \mathbb{R}$ we denote by $\mathbb{L}(\mathcal{R})$ the class of factorization structures $(\mathcal{E}, \mathcal{M})$ for which $\mathcal{P}'(\mathcal{R}) \subset \mathcal{E} \subset \mathcal{P}''(\mathcal{R})$ and $\mathbb{L}_u(\mathcal{R}) = \{(\mathcal{E}, \mathcal{M}) \in \mathbb{L}(\mathcal{R}) \mid \mathcal{M} \subset \mathcal{M}_u\}$, where $\mathcal{M}_u$ is the class of the universal monomorphisms (see [2]).

Let $\mathcal{I}'_u = \mathcal{I}'_u(\mathcal{R}) = (\mathcal{E}_p \cup \mathcal{U}(\mathcal{R}))^{\llcorner}$, $\mathcal{P}'_u = \mathcal{P}'_u(\mathcal{R}) = (\mathcal{I}'_u)^{\urcorner}$.

It is well known (see [2], [1]) that $(\mathcal{P}'_u, \mathcal{I}'_u)$ is a factorization structure in the category $\mathcal{C}_2\mathcal{V}$.

Theorem 3 [2]. *1. For any non-zero reflective subcategory $\mathcal{R}$ of the category $\mathcal{C}_2\mathcal{V}$, the factorization structure $(\mathcal{P}''(\mathcal{R}), \mathcal{I}''(\mathcal{R}))$ has the $\mathcal{M}_u$-hereditary projections class.*
2. The map $\mathcal{R} \mapsto (\mathcal{P}''(\mathcal{R}), \mathcal{I}''(\mathcal{R}))$ establish a one-to-one correspondence between lattice $\mathbb{R}$ of non-zero reflective subcategories $\mathcal{C}_2\mathcal{V}$ in the lattice $\mathbb{B}_u$ of the factorization structures $(\mathcal{P}, \mathcal{I})$ with the properties:

a) $\mathcal{I} \subset \mathcal{M}_u$;

b) the class $\mathcal{P}$ is $\mathcal{M}_u$-hereditary.

3. Let $f\colon X \to Y \in \mathcal{E}pi$ and $f = m \cdot e$ is $(\mathcal{E}_p, \mathcal{M}_u)$ – the factorization of this morphism. The epimorphism $f \in \mathcal{P}''(\mathcal{R})$ iff the reflector functor $r\colon \mathcal{C}_2\mathcal{V} \to \mathcal{R}$ transforms the morphism m into an isomorphism: $r(m) \in \mathcal{I}so$. In other words, $\mathcal{P}''(\mathcal{R}) = (\varepsilon\mathcal{R}) \circ \mathcal{E}_p$, where $\varepsilon\mathcal{R} = \{e \in \mathcal{E}pi \mid r(e) \in \mathcal{I}so\}$.
4. Let $f\colon X \to Y \in \mathcal{M}_u$. $f \in \mathcal{I}''(\mathcal{R})$ iff the square

$$r(f) \cdot r^X = r^Y \cdot f$$

is a copullback.

Corollary 4. 1. The reflector functor $\pi\colon \mathcal{C}_2\mathcal{V} \to \Pi$ is exact.

2. The reflector functor $\pi\colon \mathcal{C}_2\mathcal{V} \to \Pi$ in composition with the inclusion functor $i\colon \Pi \to \mathcal{C}_2\mathcal{V}$ is exact.

Problem 5. To describe the factorization structures $(\mathcal{P}'(\mathcal{L}), \mathcal{I}'(\mathcal{L}))$.

Remark 6. The factorization structure $(\mathcal{P}'(\Pi), \mathcal{I}'(\Pi))$ was described in the [2] paper.

Definition 7. *Let $r\colon \mathcal{C} \to \mathcal{C}$ be a covariant functor, and $(\mathcal{P}, \mathcal{I})$ – a factorization structure (the left or right factorization structure). We say that this functor r is:*

1. $\mathcal{P}$-functor, if $r(\mathcal{P}) \subset \mathcal{P}$.

2. $\mathcal{I}$-functor, if $r(\mathcal{I}) \subset \mathcal{I}$.

3. $(\mathcal{P}, \mathcal{I})$-functor, if $r(\mathcal{P}) \subset \mathcal{P}$ and $r(\mathcal{I}) \subset \mathcal{I}$.

We will examine the case when r is a reflector functor: $r\colon \mathcal{C} \to \mathcal{R}$, but we will examine as a functor from the category $\mathcal{C}$ into category $\mathcal{C}$: $r\colon \mathcal{C} \to \mathcal{C}$, it is in composition with embedding functor $\mathcal{R} \longmapsto \mathcal{C}$. We will use the same notation r, specifying if it will necessary, field values: $r\colon \mathcal{C} \to \mathcal{R}$ or $r\colon \mathcal{C} \to \mathcal{C}$.

Proposition 8. *1. $\mathbb{L}_u(\mathcal{R})$ is a complete lattice with the smallest element $(\mathcal{P}'_u, \mathcal{I}'_u)$ and the bigger element $(\mathcal{P}'', \mathcal{I}'')$.*
2. $\mathbb{L}_u(\mathcal{R})$ is the class of the factorization structures $(\mathcal{E}, \mathcal{M})$, for which $\mathcal{I}'_u \subset \mathcal{M} \subset \mathcal{I}''$.

Lemma 9. *Let $m\colon X \to Y$ be an universal monomorphism. Then $\pi(m)$ is a sectional morphism.*

Theorem 10. *1. Let $(\mathcal{E}, \mathcal{M}) \in \mathbb{L}_u(\mathcal{R})$. We examine the following conditions:*

a) the morphism $f : X \longrightarrow Y$ belongs of the class $\mathcal{E}$;

b) the morphism $r(f) : rX \longrightarrow rY$ belongs of the class $\mathcal{E}$.

Then a $\Longrightarrow$ b. If the class $\mathcal{E}$ is $\mathcal{M}_u$-hereditary, then a $\Longleftrightarrow$ b. In particularity $f \in \mathcal{P}''(\mathcal{R}) \Longleftrightarrow r(f) \in \mathcal{P}''(\mathcal{R})$.

2. *Let* $(\mathcal{E},\mathcal{M}) \in \mathbb{L}_u(\mathcal{R})$. *Then* $r : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{R}$ *is a* $(\mathcal{E},\mathcal{M})$*-functor:* $r(\mathcal{E}) \subset \mathcal{E}$ *and* $r(\mathcal{M}) \subset \mathcal{M}$. *Therefore, if* $f = m \cdot e$ *is the* $(\mathcal{E},\mathcal{M})$*-factorization of the morphism* $f \in \mathcal{C}_2\mathcal{V}$*, then* $r(f) = r(m) \cdot r(e)$ *is the* $(\mathcal{E},\mathcal{M})$*-factorization of the morphism* $r(f) : rX \longrightarrow rY$.

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Normal residuated lattices

Dumitru Bușneag, Dana Piciu, Luisa-Maria Nițu

Faculty of Sciences, Department of Mathematics, University of Craiova, Romania
e-mail: busneag@central.ucv.ro, piciudanamarina@yahoo.com, nituluisamaria@yahoo.com

The aim of this paper is to extend some results from normal lattices to the case of i-normal residuated lattices defined in [1], [2].

A bounded distributive lattice L is said to be *normal* if every prime ideal in L contains a unique prime ideal. While studyng extensively normal lattices, Cornish (see [3]) has given several characterizations of normal lattices.

A residuated lattice L is called *i-normal* if every prime i-filter P of L is contained in a unique maximal i-filter M_P.

BL-algebras are examples of *i-normal residuated lattices* (see [5], [7], [8]).

If L is a Stonean residuated lattice and normal lattice, then L is i-normal residuated lattice, see Proposition 8.

In [4] the i-normal residuated lattices are called *residuated lattices with the Gelfand property* (or, in brief, *Gelfand residuated lattices*).

If L is a residuated lattice and $a, b \in L$, then we consider the set $\langle a, b\rangle = \{x \in L : a^n \odot x \leq b$ for some $n \geq 1\}$. $\langle a, b\rangle$ is called annihilator of a relative to b (see [6]). For every $a, b \in L, \langle a, b\rangle \in Id(L)$, see Lemma 5.

If L is a residuated lattice, we prove that L is i-normal iff the lattice $\mathcal{F}_{ip}(L)$ is co-normal iff the lattice $\mathcal{F}_i(L)$ is co-normal, see Theorem 5. Also, if L is a residuated lattice, then L is i-normal iff $a, b \in L$ and $a \odot b = 0$, then $\langle a, b\rangle \vee_{id} \langle b, a\rangle = L$ iff for every $P \in Spec_i(L)$ and $a, b \in L$ such that $a \odot b = 0$, then there are $x \in P$, $p, q, r \geq 1$ such that $a^p \odot x^q$ and $b \odot x^r$ are comparable, see Theorem 7.

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Refined multiplicty one for ℓ-adic representations

Liubomir Chiriac

University of Massachusetts-Amherst, USA
e-mail: LiubomirC@gmail.com

An important feature of the Atkin-Lehner theory about newforms is the fact that each eigenspace for the Hecke operators is one-dimensional. This phenomenon is commonly referred to as the multiplicity one property for newforms. Therefore, if two normalized newforms have the same Hecke eigenvalues then they must coincide. At the same time, one can attach ℓ-adic Galois representations to newforms, such that at almost all primes the trace of the image of the Frobenius element agrees with the corresponding Hecke eigenvalue. In this context, the natural question arises as to what extent do Frobenius traces determine a Galois representation.

With the aforementioned number-theoretic motivation in mind, we study this question in a slightly more general setting: given an ℓ-adic representation ρ of a compact group G, the traces of what proportion of elements in G determine ρ?

The main ingredient in our analysis is a "unitary trick", first used by Serre in [3] and later on expanded in [4], which describes an elegant way of computing the Haar measure of a subset defined by algebraic relations in terms of the connected components in an algebraic group. It turns out (cf. [4], Theorem 5.15) that if two continuous representations

$$\rho_1 : G \to GL_n(\mathbb{Q}_\ell) \text{ and } \rho_1 : G \to GL_m(\mathbb{Q}_\ell),\ n \geq m,$$

of a compact group G have equal traces on a set of density greater than $1 - \frac{1}{n(n+m)}$ then they are isomorphic (see also [2]). As a consequence, one can infer that for any n-dimensional representation the set of elements with nonzero traces has density at least $\frac{1}{n^2}$.

Better bounds can be obtained for irreducible Galois representations of Artin-type whose adjoint action is also irreducible. As shown in [1], the traces are nonzero for at least half of the primes in that case.

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Classes of filters in residuated lattices

Florentina Chirteș, Christina Dan, Dana Piciu

Faculty of Sciences, Department of Mathematics, University of Craiova, Romania

e-mail: florentinachirtes@yahoo.com, christi_the_d@yahoo.com, piciudanamarina@yahoo.com

Residuation is a fundamental concept of ordered structures and categories. The theory of residuated lattices was used to develop algebraic counterparts of fuzzy logics and substructural logics. Apart from their logical interest, residuated lattices have interesting algebraic properties, and include two important classes of algebras : *BL-algebras* (introduced by Hajek as the algebraic counterpart of his Basic Logic) and *MV-algebras* (introduced by Chang to prove the completeness theorem for Łukasiewicz calculus).

An important role in the theory of lattices is played by the concept of *filter*. Afterwards, the notion of filter was defined on various algebraic structures. *Deductive systems* correspond to subsets closed with respect to Modus Ponens and they are called sometime *(implicative) filters*.

The aim of our work is to put in evidence new types of filters in a residuated lattice: *semi-G-filters, Stonean filters, divisible filters, BL-filters, regular filters, MTL-filters*, and to establish new characterizations and connections between these.

In the last ten years, in the mathematical literature have been studied many types of filters in residuated lattices or in special classes of residuated lattices (BL-algebras, MTL-algebras, etc.) and consequently many names have been proposed for these, such that: *Boolean filters, Heyting filters, implicative filters, positive implicative filters, fantastic filters, easy filters, obstinate filters, fold filters*, etc. This diversity of names for the filters makes them very difficult to study and to see the connections between them, many times different names being use to mean the same notion. To avoid this inconvenience, we propose a new approach for the study of these filters in residuated lattices, in order to be easily accessible to the readers (thus, we propose that the main names of filters F of a residuated lattice L to be given by the name of the class of algebras which contains the quotient residuated lattice L/F).

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About topological Ward quasigroup

Mitrofan Choban and Liubomir Chiriac

Tiraspol State University, Moldavia
e-mail: mmchoban@gmail.com, llchiriac@gmail.com

A groupoid $(Q, \cdot)$ is called a *quasigroup* if for every $a, b \in Q$ the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions.

Alternatively, a quasigroup is an algebra $(Q; *, \backslash, /)$ of type (2,2,2) such that $x\backslash(x * y) = y$, $(x * y)/y) = x$, $x * (x\backslash y) = y$, and $(x/y) * y) = x$, for all $x, y \in G$.

Given a group $(Q; \circ, ^{-1}, e)$, it is possible to construct a quasigroup by defining $x * y = x \circ y^{-1}$. This quasigroup satisfies the identity $(x * y) * (y * z) = x * z$. Quasigroups satisfying the above identity are known as Ward quasigroup [1].

If a operation $*$ in a quasigroup $(Q; *, \backslash, /)$ with a topology is continuous, then Q is called a paratopological quasigroup or a topological groupoid. If in a paratopological quasigroup Q the operations $\backslash$ and $/$ are continuous, then Q is called a topological quasigroup. If in a quasigroup Q the translations $x \to a * x$ and $x \to x * b$ are continuous for any $a, b \in Q$, then Q is called a semitopological quasigroup. The following results are related to $[1-2]$.

Theorem 1. *Any paratopological quasigroup Ward is a topological quasigroup.*
Theorem 2. *Let P be a topological property. If any semitopological group with property P is a topological group, then any semitopological quasigroup Ward with property P is a topological quasigroup.*
Theorem 3. *Let P_1 and P_2 be topological properties. If any topological group with property P_1 is a topological group with property P_2, then any topological quasigroup Ward with property P_1 is a topological quasigroup Ward with property P_2.*
Corollary. *Let $(Q, *)$ be a semitopological quasigroup Ward. If Q is a locally compact space, then $(Q, *)$ is a topological quasigroup with a Haar measure.*

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K-groups and "good" group structures on some plane cubic curves with singularities over arbitrary fields

Adrian Constantinescu[1], Constantin Udriște[2], and Steluța Pricopie[2]

[1] *"Simion Stoilov" Institute of Mathematics of Romanian Academy, Bucharest, Romania*
[2] *"Politehnica" University of Bucharest, Romania*
e-mail: adrian.constantinescu@imar.ro, udriste@mathem.pub.ro

The notion of K-group, with K an arbitrary field, has been introduced in [3].II.

In [3].I, have been described all algebraic K-groups (resp. Lie C-groups) on the smooth locus of the projective Descartes Folium for K an algebraically closed field with $char.K \neq 3$ (resp. $K = C$).

In this talk we present an extension of these results for the general case of an arbitrary (not necessarily algebraically closed) field K, with $char.K \neq 3$, obtained by using K-groups. Some details of this approach will be indicated.

Other involvements of K-groups in some group structures, for $K = R, C$, are possible by using some smooth algebraic C-variety structures on Descartes Folium over C, obtained by modifications of its canonic algebraic C-structure in its unique non-smooth point (involving in particular modifications of its canonic analytic C-space structure, as its complex topology in that point (similarly, as in [4])).

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On completeness relative to implicit reducibility in the chain super-intuitionistic logics

Ion Cucu

State University of Moldova, Chişinău, Republic of Moldova
e-mail: cucuion2012@gmail.com

We examine logics that are intermediary between classical logic and intutionistic one. They are constructed on finite or infinite chains (i.e. linear ordered set) of the values. It is known that the logic is called a chain if the formula $((p \supset q) \vee (q \supset p))$ is true in it.

The function f of the algebra A is called parametrically expressed by means of a system of functions Σ of A if there exist functions $g_1, h_1, \ldots, g_r, h_r$ which are expressed explicitly via Σ using superpositions, such that the predicate $f(x_1, x_2, \ldots, x_n) = x_{n+1}$ is equivalent to $\exists t_1 \exists t_2 \ldots \exists t_l \big((g_1 = h_1)\& \, \& \ldots \& (g_r = h_r)\big)$ on A. If $t_1, t_2, \ldots, t_l$ are absent, it is called implicit expressibility. The function f is called implicitly reducible to system Σ if there exists such sequence of functions $f_1, f_2, \ldots, f_m$ that $f_m = f$ and f_i is implicitly expressible via $\Sigma \cup \{f_1, f_2, \ldots, f_{i-1}\}$, for every $i = 1, 2, \ldots, m$.

Let us consider the pseudo-Boolean algebra

$$Z_m = \big\langle \{0, \tau_1, \tau_2, \ldots, \tau_{m-2}, 1\}; \Omega \big\rangle,$$

where $0 < \tau_1 < \tau_2 < \ldots < \tau_{m-2} < 1$, $\Omega = \{\&, \vee, \supset, \neg\}$; and LZ_m denotes the set of valid formulas, i.e. the logic of Z_m. Let $\varphi(0) = 0, \varphi(\tau_1) = \varphi(\tau_2) = = \tau_2$, $\varphi(1) = 1$ and $\psi(0) = 0, \psi(\tau_1) = 1, \psi(\tau_2) = \tau_2, \psi(1) = 1$ on Z_4.

The system Σ of formulas is called complete relative to implicit reducibility in logic LZ_m if each formula is implicitly reducible in LZ_m to Σ.

The criterion of completeness relative to implicit reducibility in LZ_3 has been obtained earlier by the author.

Theorem. *The system of formulas Σ is complete relative to implicit reducibility in LZ_m, iff Σ is complete in this sense in LZ_3 and for every relation $\varphi(x) = y$, $\psi(x) = y$ there is a formula of Σ that does not preserve them on Z_4.*

The recurrence sequences via the adjacency matrix of the dihedral group

Ömür Deveci

Department of Mathematics, Faculty of Science and Letters, Kafkas University, Turkey
e-mail: odeveci36@hotmail.com

This work develops properties of the recurrence sequences defined by using the characteristic polynomial of the adjacency matrix of the Cayley diagram of the dihedral group. The study of this sequence modulo yields cyclic groups and semigroups from generating matrix. Also, we extend the sequences defined to groups and then we examine these sequences in finite groups. Finally, we obtain the periods of of the extended sequences in the dihedral group D_{2n} as applications of the results obtained.

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On an algebraically reflexive representation of the second dual of $C(K)$

Omer Gok

Yildiz Technical University, Faculty of Arts and Sciences, Department of Mathematics, Istanbul, Turkey
e-mail: gok@yildiz.edu.tr

Let X be a Banach space and let X' be its norm dual. Suppose $m : C(K) \to L(X)$ is a bounded algebra homomorphism such that m is unital. In this talk we show that if $m * (C(K))'')$ has a finite separating set in X', then $m * (C(K))'')$ is algebraically reflexive.

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On the groups of regular mappings in isostrophic loops

Ion Grecu

Moldova State University, Moldavia
e-mail: iongrecu21@gmail.com

A grupoid $(Q, \cdot)$ is called a quasigroup if the ecuations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions, for $\forall a, b \in Q$. A loop is a quasigroup with a neutral element. If $(Q, \cdot)$ is a quasigrup then we will denote: $LM(Q, \cdot) =< L_x^{(\cdot)} | x \in Q >$, $RM(Q, \cdot) =< R_x^{(\cdot)} | x \in Q >$, where $L_a^{(\cdot)}(x) = a \cdot x$, $R_a^{(\cdot)}(x) = x \cdot a$, $\forall x, y \in Q$. A mapping $\varphi \in M(Q, \cdot)$ is called a left (resp. right) regular mapping if $\varphi(x \cdot y) = \varphi(x) \cdot y$ (resp. $\varphi(x \cdot y) = x \cdot \varphi(y)$), $x, y \in Q$, and the mapping $\varphi \in M(Q, \cdot)$ is called a middle (resp. generalized left, generalized right) regular mapping if there exists its adjoint $\varphi^* \in M(Q, \cdot)$ such that $\varphi(x) \cdot y = x \cdot \varphi^*(y)$(resp. $\varphi(x \cdot y) = \varphi^*(x) \cdot y$, $\varphi(x \cdot y) = x \cdot \varphi^*(y)$), $\forall x, y \in Q$. We will denote by $\mathcal{L}_{(\cdot)}$ (resp. $\mathcal{R}_{(\cdot)}$, $\overline{\mathcal{L}}_{(\cdot)}$, $\overline{\mathcal{R}}_{(\cdot)}$, $\overline{\mathcal{L}}^*_{(\cdot)}$, $\overline{\mathcal{R}}^*_{(\cdot)}$) the group of all left (resp. right, left generalized, right generalized, adjoint left generalized, adjoint right generalized) regular mappings of the quasigroup $(Q, \cdot)$. In loops the notions "regular mapping" and "generalized regular mapping" coincide. If $(Q, \cdot)$ is a group and $H \subseteq Q$ then $C_{(Q,\cdot)}(H) = \{g \in Q | g \cdot h = h \cdot g, \forall h \in H\}$, is called the centralizer of H in $(Q, \cdot)$. Recall that a loop $(Q, \cdot)$ is middle Bol if every its loop isotope satisfies the identity $(x \cdot y)^{-1} = y^{-1} \cdot x^{-1}$ [1]. Middle Bol loops are isostrophes of left (right) Bol loops [2, 3].

The actions of multiplication groups have been considered for isotopic loops in [7] and was partially studied for isostrophic loops in [4, 5, 6]. Connections between the centralizer of multiplication groups and the groups of regular mappings are established in the present work for quasigroups and their isotrophic loops.

Theorem. *Let $(Q, \cdot)$ be a quasigroup which isostrophe $(Q, \circ)$ is a loop and let $\alpha, \beta \in S_Q$. The following statements hold:*
1. If $x \circ y = \beta(y) \backslash \alpha(x)$, then $C_{S_Q}(RM(Q, \circ)) = \overline{\mathcal{R}}^*_{(\cdot)}$ and $C_{S_Q}(LM(Q, \circ)) \subseteq \mathcal{F}^*_{(\cdot)}$;
2. If $x \circ y = \alpha(x)/\beta(y)$, then $C_{S_Q}(RM(Q, \circ)) = \overline{\mathcal{L}}^*_{(\cdot)}$ and $C_{S_Q}(LM(Q, \circ)) \subseteq \mathcal{F}_{(\cdot)}$;
3. If $x \circ y = \alpha(x) \backslash \beta(y)$, then $C_{S_Q}(LM(Q, \circ)) = \overline{\mathcal{R}}^*_{(\cdot)}$ and $C_{S_Q}(RM(Q, \circ)) \subseteq \mathcal{F}^*_{(\cdot)}$;
4. If $x \circ y = \beta(y)/\alpha(x)$, then $C_{S_Q}(LM(Q, \circ)) = \overline{\mathcal{L}}^*_{(\cdot)}$ and $C_{S_Q}(RM(Q, \circ)) \subseteq \mathcal{F}_{(\cdot)}$;
5. If $x \circ y = \beta(y) \cdot \alpha(x)$, then $C_{S_Q}(RM(Q, \circ)) = \overline{\mathcal{R}}_{(\cdot)}$ and $C_{S_Q}(LM(Q, \circ)) = \overline{\mathcal{L}}_{(\cdot)}$.

Corollary 1. *If $(Q, \cdot)$ and $(Q, \circ)$ are two isostrophic loops and $\alpha, \beta \in S_Q$ then:*
1. If $x \circ y = \beta(y) \backslash \alpha(x)$, then $C_{S_Q}(RM(Q, \circ)) = \mathcal{R}_{(\cdot)}$ and $C_{S_Q}(LM(Q, \circ)) \subseteq \mathcal{F}^*_{(\cdot)}$;
2. If $x \circ y = \alpha(x)/\beta(y)$, then $C_{S_Q}(RM(Q, \circ)) = \mathcal{L}_{(\cdot)}$ and $C_{S_Q}(LM(Q, \circ)) \subseteq \mathcal{F}_{(\cdot)}$;

3. If $x \circ y = \alpha(x)\backslash\beta(y)$*, then* $C_{S_Q}(LM(Q,\circ)) = \mathcal{R}_{(\cdot)}$ *and* $C_{S_Q}(RM(Q,\ \circ)) \subseteq \mathcal{F}^*_{(\cdot)}$*;*
4. If $x \circ y = \beta(y)/\alpha(x)$*, then* $C_{S_Q}(LM(Q,\circ)) = \mathcal{L}_{(\cdot)}$ *and* $C_{S_Q}(RM(Q,\ \circ)) \subseteq \mathcal{F}_{(\cdot)}$*;*
5. If $x \circ y = \beta(y) \cdot \alpha(x)$*, then* $C_{S_Q}(RM(Q,\circ)) = \mathcal{R}_{(\cdot)}$ *and* $C_{S_Q}(LM(Q,\circ)) = \mathcal{L}_{(\cdot)}$*.*

Corollary 2. *Let* $(Q,\circ)$ *be a middle Bol loop. The following statements hold:*
1. if $(Q,\cdot)$ *is the corresponding right Bol loop then*
$$C_{S_Q}(RM(Q,\circ)) = \mathcal{R}_{(\cdot)} \text{ and } C_{S_Q}(LM(Q,\circ)) \subseteq \mathcal{F}^*_{(\cdot)}.$$
2. if $(Q,\cdot)$ *is the corresponding left Bol loop then*
$$C_{S_Q}(RM(Q,\circ)) = \mathcal{L}_{(\cdot)} \text{ and } C_{S_Q}(LM(Q,\ \circ)) \subseteq \mathcal{F}_{(\cdot)}.$$

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On the spacelike ruled minimal surfaces

Filiz Kanbay

Yildiz Technical University, Istanbul, Turkey
e-mail: fkanbay@yildiz.edu.tr

In three dimensional Minkowski space, the conditions that a non-developable spacelike ruled surface to be a maximal surface are given. The non-developable spacelike maximal ruled surfaces are classified and some examples are given.

About group and loop transversals in the infinite sharply 2-transitive permutation group

Eugene Kuznetsov

Institute of Mathematics and Computer Science of ASM, Chişinău, Republic of Moldova
e-mail: kuznet1964@mail.ru

Definition 1. Let G be a group and H be its subgroup. Let $\{H_i\}_{i\in E}$ be the set of all left (right) cosets in G to H, and we assume $H_1 = H$. A set $T = \{t_i\}_{i\in E}$ of representativities of the left (right) cosets (by one from each coset H_i and $t_1 = e \in H$) is called a *left (right) transversal* in G to H.
On any left transversal T in a group G to its subgroup H it is possible to define the following operation (*transversal operation*):
$$x \overset{(T)}{\cdot} y = z \quad \overset{def}{\Longleftrightarrow} \quad t_x t_y = t_z h, \;\; h \in H,$$

Definition 2. If a system $< E, \overset{(T)}{\cdot}, 1 >$ is a loop (group), then such left transversal $T = \{t_x\}_{x \in E}$ is called a *loop (group) transversal*.

An infinite sharply 2-transitive permutation group G is studied. As it was proved in [1], the set T of all elements of order 2 from G form a loop (stable) transversal in the group G to its subgroup $H = St_1(G)$. Moreover, this set T is a normal subset in the group G.

For an arbitrary (fixed) element $t_i \in T$ we can consider its centralisator $C_i = C_G(t_i)$:

$$C_i = C_G(t_i) = \{g \in G | g t_i = t_i g\}.$$

In [2] it was shown the following
Theorem 1. The following statements are true:

1. For any $i \in E - \{1\}$ the set C_i is a left transversal in G to H,
2. For any $i \in E - \{1\}$ the left transversal C_i is a group transversal in G to H,
3. For any $i, j \in E - \{1\}$ transversal operations $< C_i, \cdot >$ and $< C_j, \cdot >$ are isomorphic.

Also it can be proved the following
Theorem 2. For any $i, j \in E - \{1\}$ sets C_i and C_j are conjugated in G.

Let us remind how any two left transversals T and P in a group G to its subgroup H are connected (see [3]).
Lemma 3. Let $T = \{t_x\}_{x \in E}$ and $P = \{p_x\}_{x \in E}$- be left transversals in G to H. Then there is a set of elements $\{h_{(x)}\}_{x \in E}$ from H such that:

1. $p_x = t_x h_{(x)}\ \forall x \in E$;
2. $x \overset{(P)}{\cdot} y = x \overset{(T)}{\cdot} \hat{h}_{(x)}(y)$.

This set $\{h_{(x)}\}_{x \in E}$ is called a *derivation set* for transversal T (and for transversal operation $< E, \overset{(T)}{\cdot}, 1 >$).

For any $i \in E - \{1\}$ loop transversal $T = \{t_x\}_{x \in E}$ and group transversal C_i in G to H determine the corresponding derivation set $H^i = \{h^i_{(x)}\}_{x \in E}$.

Now it can be proved the following
Theorem 4. For any $i, j \in E - \{1\}$ derivation sets H^i and H^j are conjugated in G.

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On invariance of recursive differentiability under the isotopy of loops

Inga Larionova-Cojocaru and Parascovia Syrbu

State University of Moldova, Chişinău, Republic of Moldova e-mail: larionovainga@yahoo.com

Recursive s-differentiable quasigroups ($s \geq 1$) have been defined in [1], were they appear as check functions of complete recursive codes. Let $(Q, \cdot)$ be a quasigroup and let i be a natural number. The operation $(\overset{i}{\cdot})$ defined on Q by: $x \overset{0}{\cdot} y = x \cdot y$, $x \overset{1}{\cdot} y = y \cdot xy$, $x \overset{i}{\cdot} y = (x \overset{i-2}{\cdot} y) \cdot (x \overset{i-1}{\cdot} y)$, for $\forall x, y \in Q$, is called the recursive derivative of order i of the quasigroup $(Q, \cdot)$. A quasigroup $(Q, \cdot)$ is called recursively s-differentiable if its recursive derivatives $(Q, \overset{i}{\cdot})$ are quasigroups for all $i = 0, 1, ..., s$. The notion of core of a loop was introduced by R. Bruck [2] for Moufung loops and studied by V. Belousov in left Bol loops [3]. If $(Q, \cdot)$ is a loop then the grupoid $(Q, +)$, where $x + y = x \cdot (y \backslash x), \forall x, y \in Q$, is called the core of $(Q, \cdot)$. It is shown in [4] that in LIP-loops the core is isostrophic to the recursive derivative of order 1. Recursively differentiable left Bol loops are considered in [4]. We study the recursively differentiable di-associative loops and the invariance of recursive differentiability under the isotopy of loops in the present work.

Theorem 1. *If the cores of two loops are isotopic then the recursive derivatives of order one of these loops are isotopic.*

Corollary 1. *The recursive derivatives of order 1 of isotopic left Bol loops are isotopic.*

Corollary 2. *Every loop isotopic to a recursively 1-differentiable left Bol loop is recursively 1-differentiable.*

Theorem 2. *Let $(Q, \cdot)$ be a di-associative loop. The following statements hold:*

1. $(Q, \cdot)$ is recursively differentiable if and only if the mapping $x \to x^2$ is a bijection;

2. If $(Q, \cdot)$ is recursively 1-differentiable then its recursive derivative of order 1 is an RIP-quasigroup.

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About symmetries and conservation laws in k-symplectic formalism

Florian Munteanu

Department of Applied Mathematics, University of Craiova, Romania
e-mail: munteanufm@gmail.com

In this talk I will present the relationship between symmetries and conservation laws for k-symplectic Hamiltonian systems in first-order classical field theories. In particular, I will present a way to obtain new kinds of conservation laws for k-symplectic Hamiltonian and Lagrangian systems, without the help of a Noether type theorem and without the use of a variational principle, using only symmetries and pseudosymmetries.

AMS Subject Classification (2010): 70S05, 70S10, 53D05.
Keywords: symmetry, conservation law, k-symplectic formalism.

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On partially ordered nilpotent A-loops

Vasile Ursu

[1] *"Simion Stoilov" Institute of Mathematics of Romanian Academy, Bucharest, Romania*
[2] *Technical University of Moldova, Republic of Moldova*
e-mail: Vasile.Ursu@imar.ro

In the paper we use, as a rule, ordinary terminology [1,2].

Partial order in a loop is identify with a groupoid generated by positive elements. An abstract loop L is called free orderable, if each of its partial order can be completed up to a linear order.

Theorem 1. *For partial order P of a free orderable loop L to be an intersection of linear order is necessary and sufficient as for any element $x \in L$to have:*

$$P^{-1}\backslash\{e\} \cap S(x) \neq \emptyset \Rightarrow x^{-1} \in P^{-1}, \tag{1}$$

where $S(x)$grupoid generated by the element x and e is the unit element loop.

The A-loop (or automorphic loop) is a loop whose inner mappings are automorphisms. The study of A-loops initiated by Bruck and Paige [3]. A careful study of nilpotent A-loops is achieved in [4] and some results from here we use in the paper.

Corollary 2. *Because in an A-nilpotent local loop L without torsion a partial order P of its to be an intersection of linear order is necessary and sufficient so that L verify the condition (1).*

Leftist associator leftist, rightist associator of elements x, y, z and commutator of elements x, y of a loop L are defined through the loop operations by equalities:

$$[x, y, z] = x\backslash[(xy \cdot z)/yz], \quad (x, y, z) = x \cdot yz = [xy\backslash(x \cdot yz)]/z, \quad xy = y \cdot x[x, y].$$

Lemma 3. *In any partial ordered loop are true as quasiidentities*

$$x > e \Rightarrow x[x, y, z] > e, \quad x > e \Rightarrow (z, y, x)x > e, \quad x > e \Rightarrow x[x, y] > e.$$

Analog for groups, define metabelian loop the loop nilpotent of class two.

Lemma 4. *The inner mapping group of metabelian A-loop is a commutative.*

Lemma 5. *Any normal subloop generated by a single element of metabelian loop is commutative group.*

According to these Lemme 3-5 demonstrating:

Theorem 6. *For a metabelian A-loop L to be isomorphic sunk in product Cartesian linear ordered loops is necessary and sufficient the loop L to be true quasiidentities form*

$$x^n = e \Rightarrow x = e, \tag{2}$$

where $n > 0$ is a natural number.

Finally we built an example of nilpotent three class A-loop without torsion where quasiidentity

(2), is true but it can not be isomorphic sunk in Cartesian product of linear ordered A-loops.

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On orthogonal systems of ternary quasigroups admitting at least one paratopy

Parascovia Syrbu and Dina Ceban

State University of Moldova, Chişinău, Republic of Moldova
e-mail: cebandina@mail.ru

The n-ary operations $A_1, A_2, \ldots, A_n$ defined on a nonempty set Q are called orthogonal if, for every $a_1, a_2, \ldots, a_n \in Q$, the system of equations

$$\{A_i(x_1, x_2, \ldots, x_n) = a_i\}_{i=\overline{1,n}}$$

has a unique solution. A bijection $\theta : Q^n \to Q^n$ is called a paratopy of an orthogonal system Σ of n-ary operations defined on Q, if $\Sigma\theta = \Sigma$.

V. Belousov proved in [1] that there exist exactly nine orthogonal systems, consisting of two binary quasigroups and the binary selectors, which admit at least one nontrivial paratopy. We consider orthogonal systems consisting of three ternary quasigroup operations and the ternary selectors and prove that there exist exactly 153 such systems which admit at least one nontrivial paratopy [2]. The existence of paratopies implies 67 identities in ternary quasigroups which can be reduced to identities of one of the following four types: $I.\ {}^{\alpha}A({}^{\beta}A, {}^{\gamma}A, {}^{\delta}A) = E_1$, $II.\ {}^{\alpha}A({}^{\beta}A, {}^{\gamma}A, E_1) = E_2$, $III.\ {}^{\alpha}A({}^{\beta}A, E_1, E_2) = {}^{\gamma}A({}^{\delta}A, E_1, E_3)$, $IV.\ {}^{\alpha}A({}^{\beta}A, E_1, E_2) = {}^{\gamma}A({}^{\delta}A, E_1, E_2)$, where A is a ternary quasigroup operation and $\alpha, \beta, \gamma, \delta \in S_4$ [3].

A generalization of these results is considered for the n-ary quasigroups.

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Boolean Functions: morphisms, anti-morphisms and invariant sets

Şerban E. Vlad

Oradea City Hall, Romania
e-mail: serban_e_vlad@yahoo.com

The invariant sets $A \subset \{0,1\}^n, A \neq \emptyset$ of the Boolean functions $\Phi : \{0,1\}^n \longrightarrow \{0,1\}^n$ are defined, together with the morphisms and the anti-morphisms of such functions. We sketch the study of the relations between these concepts.

Section 6. Mathematical Modeling

Mathematical Models in the Romanian research and development of petroleum industry: achievements and perspectives

Vasile Badiu[1] and Florin Vasile Badiu[2]

[1] *Volunteer with Society of Petroleum Engineers-Romania (SPE), Romania*
[2] *Renault Romania, SPE Members*
e-mail: vasile.badiu@gmail.com

In 1857, Romania was the first country in the world to officially record an oil production of 275 tons in the international statistics and was followed by official oil production from the United States in 1859 and petroleum is still vital for to many world industries. The Petroleum Industry in Romania has a great history, which is still ongoing.

The American Institute of Mining, Metallurgical, and Petroleum Engineers (AIME) and the Society of Petroleum Engineers (SPE) publications, as well as, the books by Morris Muskat, highlighted the role of mathematics in the petroleum industry and proved the relevance of the multidisciplinary and flexible team, promoted later by Nicolae Cristea as a standard in the Institute for Research and Technology (ICPT) Campina, since 1950.

Since 1970, the application software have been developed in ICPT by Nicolae Viorel Popescu team based on the existing methodologies of using similitude and analogy models and on the existing algorithms developed by researchers and used by technicians by manual calculations. Nicolae Viorel Popescu, was a leader of the Numerical Modeling and Simulation in Romanian scientific community, he founded in ICPT Campina the Mathematical Modeling Laboratory in 1991 and the Numerical Reservoir Simulation Laboratory in 1996. The multidisciplinary teams of ICPT Campina ran for ICPT software library and have developed hundreds of application software for ICPT Campina research projects.

Since 2001, his legacy left to the Numerical Reservoir Simulation Laboratory has been taken forward in all activities: in-house Software Development, Data Base Management, Hardware Systems Administration and Integrated Reservoir Modeling using Integrated Software System as an outstanding activity in Petrom SA until 2005.

Since 2005, OMV Petrom mainly uses the commercial software as "black boxes" and only a very small fraction of ICPT software library. That "black boxes" can be successfully used only after less sophisticated methods and technologies which, constantly, proved to be more robust. Therefore, the ICPT software library needs to be continuous updated and used for the benefit of OMV Petrom and Romanian Petroleum Industry.

Our paper we will present a summary of mathematical models developed in ICPT software library with the case studies: Monte Carlo Simulation for Resources and Reserves Evaluations, Numerical Simulation of Sucker Rod Pumping System, Water Flood Streamline Model and the Integrated Reservoir Modeling with a personal view of perspective for Mathematical Modeling in Romanian Research and Development activities.

Effect of combined control policies on the optimal control of a host-vector model for malaria with infective immigrants

E.A. Bakare

Department of Mathematics, Federal University Oye Ekiti, Ekiti State, Nigeria e-mail: emmanuel.bakare@fuoye.edu.ng

We formulate and analyzed a compartmental deterministic model on the effect of combined control policies on the optimal control of a host-vector model for malaria with infective immigrants.We provided sufficient conditions for the sensitivity analysis for the basic reproduction number with respect to the model parameters for the host-vector model without the control.We also applied optimal control theory to study optimal strategies for controlling the epidemiology of malaria disease in the presence of infective immigrants using quarantine, treatments and Insecticide treated Bed-Nets as our system control variables and by deriving its necessary conditions for optimal control of the malaria disease using the Pontryagin's Maximum Principle (PMP). With the applications of optimal control theory, the optimal levels of the three controls are characterized. We carried out the Numerical simulations and extend the analytical results.

2010 Mathematics subject classification: 92B05, 93A30, 93C15.
Keywords: Optimal control, Basic reproduction number, Disease Free Equilibrium, Vaccination, Pontryagin's Maximum Principle, Optimal system, Optimal control, Hamiltonian, host-vector model, sensitivity analysis, Simulation.

Photon propagation modeling based on linear spaces and tensors applied in quantum optics

Alexandra Băluță, Diana Rotaru, Mihaela Ilie, D. Fălie, and E. Vasile

"Politehnica" University of Bucharest
e-mail: alexandraa.baluta@gmail.com

A systematized approach concerning the use of terms and symbols from linear spaces and tensors theory is presented in order to describe the photon beams propagation through quantum optics circuits designed for information processing. The "single photon" case of optical beams is considered. The main optical components of an experimental set-up are: laser diode, nonlinear optical crystal, mirrors, polarizers, filters and beam-splitters. The operatorial representation of some specific optical components (starting from the ideal cases) is emphasized for numerical simulations using MathCAD software.

Keywords: quantum optics, optical components, linear spaces, tensors, MathCAD.

Existence of minimal and maximal solutions for a second order quasilinear dynamic equation with integral boundary conditions

Mohammed Derhab[1] and Mohammed Nehari[2]

[1] *University Abou-Bekr Belkaid Tlemcen, Faculty of Sciences, Tlemcen, Algeria*
[2] *Dynamic Systems and Applications Laboratory, Preparatory School of Economy, Tlemcen, Algeria*
e-mail: derhab@yahoo.fr, nehari_72@yahoo.fr

This work is concerned with the construction of the minimal and maximal solutions for a second order quasilinear dynamic equation with integral boundary conditions, where the nonlinearity is a continuous function. We also give an example to illustrate our results.

2010 Mathematics Subject Classification: 34B15, 39A10.
Keywords: Integral boundary conditions, upper and lower solutions, monotone iterative technique, time scale, p-Laplacian.

Modeling of air velocity, temperature and humidity distribution in free convection

Adrian-Gabriel Ghiaus

Technical University of Civil Engineering - Bucharest, Romania
e-mail: ghiaus@instal.utcb.ro

A very common industrial application of free (natural) convection is solar drying in which the air motion is not generated by the aid of fans but only by density differences occurring due to temperature gradients. The greatest drawback of the tray dryer is uneven drying because of poor airflow, temperature and water vapor concentration distribution in the drying chamber. Analysis of tray drying systems may reduce or even eliminate non-uniformity of drying and increases dryer efficiency. The fluid's motion is mathematically described as a vector field, and the transported material is described by a scalar one, showing its distribution over space.Convection can be described by the convection-diffusion equation, also known as the generic scalar transport equation. The convection-diffusion equation is a combination of the diffusion and convection (advection) equations, and describes physical phenomena where particles, energy, or other physical quantities, such as moisture content, are transferred inside a physical system. The term advection sometimes serves as a synonym for convection, but technically, convection covers the sum of transport both by diffusion and by advection. The advection equation is the partial differential equation that governs the motion of a conserved scalar field as it is advected by a known velocity vector field.

In this study, because of its capability to solve equations for the conservation of momentum, heat and mass, numerical simulation was extensively used to predict the air velocity, temperature and humidity profiles inside a five trays solar dryer in unsteady-state conditions for a whole sunny day but also during the night. The coupled partial differential equations were solved by finite element method using Comsol Multiphysics commercial code. Prediction of operation parameters allowed the identification of unwanted recirculation regions, saturation moist air zones and optimum operation periods for an efficient drying process.

Two-Phase flow of fluid-particle interaction over a stretching sheet in the presence of magnetic field

Jafar Hasnain[1], Z. Abbas[1] and M. Sajid[2]

[1] *Department of Mathematics, The Islamia University of Bahawalpur, Pakistan*
[2] *Theoretical Physics Division, PINSTECH, Islamabad, Pakistan*
e-mail: jafar_hasnain14@yahoo.com

This article presents the study of magnetohydrodynamic boundary layer flow of a dusty viscoelastic fluid over a porous stretching sheet. The basic steady equations of the viscoelastic second grade fluid and dust phases are in the form of partial differential equations. A set of coupled non-linear ordinary differential equations is obtained by using suitable similarity transformations. The approximate first order solutions of the resulting equations are obtained using the perturbation technique. The results are also verified with the well-known finite difference technique known as Keller box method. The physical insight of the involved parameters on the velocity of both fluid and dust phases and the skin-friction coefficient is shown through graphs and tables and discussed in detail. The study shows that an increased effective viscosity increases the velocity of both fluid and particle phase.

Asterix - a theoretical, numerical and computational model for erosion processes

Stelian Ion, Ştefan-Gicu Cruceanu, and Dorin Marinescu

"Gheorghe Mihoc - Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics of Romanian Academy, Bucharest, Romania

e-mail: stelian.ion@ima.ro, stefan.cruceanu@ima.ro, dorin.marinescu@ima.ro

Flooding and erosion phenomena have become very violent in recent years and are of great interest in the current climate context. It is known that water dynamics, density and type of the plant cover, as well as soil erosion are strongly correlated. We propose a mathematical model to study the topography influence on water dynamics, the interactions plant cover - water flow, soil particle detachment - plant cover. In order to illustrate the model behavior, we present different scenarios with real topography data.

Measurement and evaluation of customer satisfaction in hotel industry. forecasting the future of business. A case study from Durres, Albania.

Robert Kosova, Teuta Thanasi, and Lindita Mukli

Department of Mathematics, Faculty of Information Technology, University "Aleksander Moisiu", Durres, Albania

e-mail: robertko60@yahoo.com, teutamatematika@hotmaul.com, linditamukli@gmail.com

In order to be successful in the complex market of hotel services, you need to do more than trying to attract new customers. Hotel managers also need to focus on keeping existing customers by implementing successfully effective policy of evaluation, satisfaction and fulfillment of their demands and expectation. Hotel industry services, rating and customer service satisfaction are almost built on the basis of service quality. A successful approach to service management focused on customer satisfaction can improve his loyalty and improve the positive image. For this reason, assessment and customer service hospitality will help determine their choice of the future.

The study on the topic of customer satisfaction will determine the impact on them and their behavior. It will help to answer the questions; would they return again, would they recommend the hotel to their friends. Forgetting these necessary attributes to tourism services can lead to negative reviews of the hotel, limiting the possibility of repeating customers and difficulties to have newcomers.

We will conduct a qualitative analysis of some successful hotels, with an experience of more than 20 years in tourism industry. Through analysis we will evaluate the overall level of satisfaction of tourists for the hotel and for every service closer to it. We will conclude by discussing the results and proposing ways to improve the hotel management services for the hospitality of the tourists.

Keywords: turism, industry, evaluation, customer, service.

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Ranking DNA subsequences using nonextensive statistical mechanics

H. Moghaddasi[1], K. Khalifeh[2], and A.H. Darooneh[1]

[1] *Department of Physics, University of Zanjan, Iran*
[2] *Department of Biology, University of Zanjan, Iran*
e-mail: h.moghaddasi@znu.ac.ir

Functional DNA subsequences and genome elements are spatially clustered through the genome just as keywords in literary texts. Therefore; some of the methods for ranking words in texts can also be used to compare different DNA subsequences. One of these methods is based on nonextensive statistical mechanics [1]. Here we claim that the distribution of distances between the successive subsequences (words) has q-exponential form which is the Tssalis distribution function in nonextensive statistical mechanics [2]. Thus we introduce the q-parameter as a measure for estimating the clustering level of words.

In this study, we analyzed the distribution of distances between consecutive occurrences of 16 possible dinucleotides in human chromosomes by comparing their corresponding q-parameters and found that the CGs have the highest clustering level which is a biologically important word concerning it's methylation. We extended our study by comparing the genome of some selected organisms and concluded that the clustering level of CG dinucleotides increases in higher evolutionary organisms compared to lower ones.

Keywords: Statistical mechanics, DNA sequences, q-exponential function.

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The formulation design of palatine rugae prints subraces deuteromelayu in forensic odontology

Intan Nursamsi[1], Sudradjat Supian[2], and Fahmi Oscandar[1]

[1] *Forensic Odontology Science, Faculty of Dentistry*
[2] *Mathemathic Science, Faculty of Mathematics and Natural Sciences Universitas Padjadjaran, Indonesia*
e-mail: intannursamsi@yahoo.com

The analysis of the DNA, fingerprints and teeth comparison are probably the most used techniques in human identification. However, in some cases these techniques cannot always be applied, therefore another alternative technique is needed; one of them is palatoscopy which is the study of palatine rugae prints. Palatine rugae prints is a different morphological characteristic and owned by every individual that can be used as an individual identification method in forensic odontology.

The purpose of this research is to find the simple formulation design of the palatine rugae prints which applicable as a reference for the individuals identification of subraces Deuteromelayu. indent This research used descriptive analytic method. Population in this research are positive models of college student class of 2010-2013 in FKG Unpad. Technique used in this research is purposive random sampling, and obtained 42 positive models samples (10 men, 32 women). The result showed that palatine rugae print scan be made in a simple mathematical formulation with parameters MFw is main form of palatine rugae, based on Martin dos Santos's classification; RCn is rugae counting; δy rugae tracing, based on Carrea's classification; Fz is each form of palatine rugae, based on Martin dos Santos's classification; and Mx is each measurement of palatine rugae, based on Lyssel's classification; and if the formula tested by rugae palatine of every individual, there is no same palatine rugae prints among individuals with ratio 1 : 71.425.670.400.

The conclusion of this research is the formulation design of palatine rugae print creating formulation of each side palatum with formulation of each palatine rugae form and can be symbolized MFw; RCn; δy;[(Fz;Mx)$_d$].

Keywords: Palatine rugae print,Formulation design, Subraces Deuteromelayu, Mathematical modeling, forensic science.

Hybrid method for construction of earthquakes network: active and passive points

S. Rezaei, and A.H. Darooneh

Department of Physics, University of Zanjan, Iran
e-mail: s.rezaee@znu.ac.ir

Recent results in complex systems have indicated that network theory, which is known as graph theory in mathematics, is a powerful method to handle unsolved complex problems. Earthquakes manifest spatio-temporal complex behaviorthat can be studied using complex networks.It is so essential for these studies to construct an appropriate network.Then we use the statistical mechanics to study the property of such a network.

We propose a new combination method of Abe - Suzuki method [1] and Telesca - Lovallomethod[2]for constructing earthquakes network. We divide a geographical region into small square cells that these cells cover the entire region without any overlapping. If an earthquake with any value of magnitude occurs in a cell, we identify it as a vertex of a network. Then for connecting links between vertexes we use the visibility condition. Indeed in our method two events are connected to each other if visibility condition holds true between them.

For Iran, California and Italy-Greece earthquakes we divide the longitudinal and latitudinal ranges into cells (cell sizes changes from 4km- 220km). We show that the constructed networks are scale free and theirs degree distribution obey the q-exponential function which is used in non-extensive statistical mechanics [3]. The diagram of q parameter in terms of the cell size has a peak at 31km for Iran and 44kmfor California and Italy. Due to dependence of network characteristics on each other for cell sizes less than peak size, only one of the cell sizes is enough to describe the earthquakes network. We found that such model network results in both the Gutenberg-Richter and Omori laws.

Also we find the universal behavior of links and nodes number with time for same areas and same resolution of Iran, California and Italy-Greece earthquakes. This behavior is power law and similar to Omori law but there are differences between them. Furthermore, analogy to Darooneh-Lotfi [4] by using the concept of PageRank, we find the passive and active points in the geographical region of Iran.

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Numerical methods for pricing European call options using the Black-Scholes economic model

Murat Sari and Tulay Yildirim

Yildiz Technical University, Istanbul, Turkey
e-mail: tyildirim@gtu.edu.tr

This paper considers some fundamental numerical algorithms based on finite difference schemes in analyzing the Black-Scholes equation. The algorithms are implemented by taking into account various spatial and temporal discretizations. The obtained results revealed that the suggested schemes are efficient and easily applicable. The validity of the current numerical models has been verified through the obtained results and the literature.

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Section 7. Computer Science

On the consistency of a labeled graph

Beatrice Daniela Bucur

PhD Student, Department of Computer Science, University of Piteşti, Romania
e-mail: ciolan_b@yahoo.com

The aim of this paper is to establish a connection between modal logics and labeled graphs, which is useful in solving the problem of *undeterminism.* Also, we construct a canonical, maximal and consistent model of a labeled graph associated to a transition system.

Nontrivial convex cover of a tree

Radu Buzatu

Moldova State University, Moldavia
e-mail: radubuzatu@gmail.com

The concept of *nontrivial convex p-cover* of a graph is defined in [1]. Also, we know from [1] that it is NP-complete to decide whether a graph has a nontrivial convex p-cover, for a fixed $p \geq 2$.

The nontrivial convex p-partition, as a particular case of nontrivial convex p-cover, of a tree was examined in [2]. Here, we study nontrivial convex p-cover of a tree.

The greatest $p \geq 2$ for which a graph G has a nontrivial convex p-cover is called *maximum nontrivial convex cover number* of G and is denoted by $\varphi_{cn}^{max}(G)$. The *maximum nontrivial convex cover* of G, denoted $\boldsymbol{\mathcal{P}}_{\varphi_{cn}^{max}}(G)$, is the nontrivial convex p-cover of G such that $p = \varphi_{cn}^{max}(G)$.

A vertex x of a graph G is called *resident* in $\boldsymbol{\mathcal{P}}_{\varphi_{cn}^{max}}(G)$ if x belongs to only one set of $\boldsymbol{\mathcal{P}}_{\varphi_{cn}^{max}}(G)$.

Theorem 1. Let T be a tree with $diam(T) \geq 3$. Then, there exists a maximum nontrivial convex cover $\boldsymbol{\mathcal{P}}_{\varphi_{cn}^{max}}(T)$ such that every terminal vertex of T is resident in $\boldsymbol{\mathcal{P}}_{\varphi_{cn}^{max}}(T)$ and any two terminal vertices do not belong to the same set of $\boldsymbol{\mathcal{P}}_{\varphi_{cn}^{max}}(T)$.

Corollary 1. Let T be a tree with $diam(T) \geq 3$ and p terminal vertices. Then, $\varphi_{cn}^{max}(T) \geq p$.

Corollary 2. Let T be a tree with $diam(T) \geq 3$ and p terminal vertices, where every nonterminal vertex of T is adjacent to at least one terminal vertex. Then, $\varphi_{cn}^{max}(T) = p$.

Corollary 3. Let T be a tree with $3 \leq diam(T) \leq 5$ and p terminal vertices. Then, $\varphi_{cn}^{max}(T) = p$.

Theorem 2. A tree T on $n \geq 4$ vertices has a nontrivial convex p-cover, for every p, $2 \leq p \leq \varphi_{cn}^{max}(T)$.

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A system for processing unstructured text data by means of lexicon of keywords

Radu Dumbrăveanu, Mircea Petic, and Svetlana Melnic

Alecu Russo Balti State University, Republic of Moldova e-mail: rndumbraveanu@gmail.com

Rapid development of information technologies and increase of their usage in day-to-day activities have resulted in a large volume of unstructured text data available on World Wide Web. The grow of this kind of information represents an interest and a challenge for Natural Language Processing (NLP).

While the volume of unstructured data is increasing the number of software specialized in text processing and extracting insightful intelligence is growing slowly [3], especially for Romanian language.

Taking these (and not only) into account, a software system was written [1] as a result of synthesis, intermediation and enhancement of existing instruments in the field of NLP for Romanian language. This tool allows a more coherent use of intelligent mechanisms described in [2], facilitating its access to a bigger range of users and contributing to spreading of competencies in this field.

The software is called SofFCrates. It uses a POS tagger to extract the most frequent words from the unstructured text data. For every of these words, the context, where it is present, is highlighted and rules of inflection and derivation with a high degree of accuracy are applied to generate more semantically related words, thus enriching the set of markers.

This software will be helpful to different users, but especially to philologists in their work on literary text resources for research purposes.

In future we will try to implement mechanisms of diversification of the found words by means of derivation and WordNet semantic net. Moreover, we will optimize the interface to have the possibility to search not only by a single word, but also by several words that the user considers to be more relevant to the text.

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Mathematical aspects of using neural approaches for information retrieval

Iuliana Iatan[1] and Maarten de Rijke[2]

[1] *Department of Mathematics-Computer Science, Technical University of Civil Engineering Bucureşti, UTCB, Romania*

[2] *ISLA, University of Amsterdam*

e-mail: iuliafi@yahoo.com

Information Retrieval Systems (IRS) are nowadays very popular, mainly due to the popularity of the Web. Web classification has been attempted through many different technologies. In this paper we will treat the possibility of using neural networks in Information Retrieval (IR) and highlight the advantages of applying two neural networks models for solving the problem of simplifying the complex structure of an IR system, by substitution of the relations between its subsystems by neural networks: the first neural network, which is a Fuzzy Multilayer Perceptron solves the relation between the user query and the keywords of documents and the second neural network, a Spreading Activation Neural Network simulates the relation between the keywords and the relevant documents. In this work, we shall reduce the text documents based on the Discrete Cosine Transformation (DCT), by which the set of keywords reduces to the much smaller feature set.

On crown structures in trees

Vadim E. Levit[1] and Eugen Mandrescu[2]

[1] *Ariel University, Israel*
[2] *Holon Institute of Technology, Israel*
e-mail: levitv@ariel.ac.il, eugen_m@hit.ac.il

Let G be a graph with $V(G)$ as a vertex set, and $E(G)$ as an edge set. If no two vertices from $S \subseteq V(G)$ are adjacent, then S is a *stable* (or an *independent*) set of G. A *matching* is a set $M \subseteq E(G)$ of pairwise non-incident edges; M is a *perfect matching* if it covers all the vertices. The *matching number*, denoted $\mu(G)$, is the size of a largest matching. If S is stable and there is a matching from $N(S)$ into S, then S is a *crown* of order $|S| + |N(S)|$ (see [1]).

While, for general graphs, it is **NP**-complete to decide if a graph has a crown of a given order (for details, see [3]), in [2] it is proved that in a bipartite graph with a unique perfect matching there exist crowns of every possible even order.

In this talk we show that if T is a tree on $n \geq 3$ vertices, then T contains a crown of order k, for every $k \in \{0, 2, 4, \cdots, 2\mu(T), 2\mu(T) + 1, 2\mu(T) + 2, \cdots, n\}$.

We conjecture that: every tree on $n \geq 3$ vertices, that has two leaves with the same neighbor, contains a crown of order k, for every $k \in \{0, 2, 3, ..., n\}$.

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Study on two intermodal transport simulation scenarios for Romania using a TSP exact solver

Elena Nechita and Gloria-Cerasela Crişan

"Vasile Alecsandri" University of Bacău, Romania
e-mail: enechita@ub.ro

The inter-modal transport is identified as an indispensable element of the logistics system of any country in Europe. The *Inter-Modal Strategy Romania 2020* points on the role of freight transport logistics in the growth of the Romanian economy. The volume of the total transport in Romania (where the road transport prevails) is expected to increase in 2020-2025, leading to a doubling of today's transport by road. Therefore, special attention needs to be given to alternative solutions.

Our paper presents two intermodal transport scenarios, assessing their efficiency with Concorde, a TSP exact solver. The aim is to highlight some alternatives that entail a more environmental friendly and a more suitable transport mode than road transport only. Moreover, these variants would provide more flexibility in situations of unexpected events or demands, such as the shipment of emergency medical supplies.

The neutrosophic cube an eight valued fuzzy space

Vasile Pătraşcu

Tarom Information Technology, Otopeni, Romania
e-mail: patrascu.v@gmail.com

The neutrosophic representation of information was proposed by Florentin Smarandache as an extension of fuzzy representation proposed by Lotfi Zadeh and intuitionistic fuzzy representation proposed by Kasimir Atanassov. The primary reprezentation of neutrosophic information is defined by three parameters: degree of truth T, degree of falsity F and degree of indeterminacy I. For each triplet there exist a point in the framework of the neutrosophic cube $[0,1]^3$. This paper presents a multi-valued representation of neutrosophic information. Starting from the primary triplet it is constructed a multi-valued representation that nuances better the specific features of neutrosophic information. This representation emphasizes features of uncertainty like saturation, contradiction, ignorance, neutrality, features of certainty like truth and falsity and some features that are between certainty and uncertainty like weak truth and weak falsity. The eight features enumerated above are defined by the following indexes:

Index of truth:

$$t = \max(\mu, 0) + \min(\nu, 0) + \min(\omega, 0) + \min(\delta, 0)$$

Index of falsity:

$$f = \min(\mu, 0) + \max(\nu, 0) + \min(\omega, 0) + \min(\delta, 0)$$

Index of neutrality:

$$n = \min(\mu, 0) + \min(\nu, 0) + \max(\omega, 0) + \min(\delta, 0)$$

Index of saturation:

$$s = \min(\mu, 0) + \min(\nu, 0) + \min(\omega, 0) + \max(\delta, 0)$$

Index of contradiction:

$$c = -2\min(\omega, 0)$$

Index of ignorance:

$$u = -2\min(\delta, 0)$$

Index of weak truth:

$$t^- = -2\min(\nu, 0)$$

Index of weak falsity:

$$f^- = -2\min(\mu, 0)$$

where:

$$\mu = \frac{T-F-I+1}{2}, \quad \nu = \frac{F-T-I+1}{2},$$
$$\omega = \frac{I-T-F+1}{2}, \quad \delta = \frac{T+F+I-1}{2}.$$

The eight indexes define a fuzzy partition of unity, namely:

$$t + t^{-} + f + f^{-} + s + n + u + c = 1$$

Also it is presented the inverse transform defined by:

$$T = t + t^{-} + c + s,$$

$$F = f + f^{-} + c + s,$$

$$I = n + t^{-} + f^{-} + s.$$

The inverse transform shows the complexity of neutrosophic information.

Automatic text analysis in the field of social disasters

Mircea Petic[1,2] and Grigore Horoş[1]

[1] *Institute of Mathematics and Computer Science of ASM, Chişinău, Republic of Moldova*
[2] *Alecu Russo Balti State University, Republic of Moldova*
e-mail: petic.mircea@gmail.com

This research is made within a project [1] whose goal is to find the way to have access to methods to detect social disasters on the Web. The start is to monitory networks information sources in Ukraine, Romania and Republic of Moldova and to collect pertinent texts. The collected texts would be processed at Situation Analytical Center established in Kiev to suggest to decision makers the appropriate resilience scenarios.

We used methods of texts sources processing that permitted to extract keywords of social disasters. We gathered more than 740 news articles and started to create a lexicon which comprise keywords referred to technological, social or natural disasters. Then based on these triggers we formed a lexicon that serves as basis for our future system of identification and classification of texts from the Internet.

In order the processing phase be more rapid we elaborated a Crawler-based [2] application service. It search through web news articles, downloads and extracts the text of this news, and stores them in the database. As every news site has its own structure we should take into account its particularity. We worked on 4 different news websites. Another phase is needed in order to extract from web page only the text part of the article.

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Symbolic-Numeric tool for pricing a multi-SDE framework with Milstein discretization

Tiberiu Socaciu and Paul Pascu

University of Suceava, Romania

e-mail: socaciu@seap.usv.ro

We present a tool with a (symbolic) formula generator for Milstein discretization in a multi-SDE framework (for a financial derivative pricing) coupled to a MonteCarlo based tool (numeric) for simulation. Example in paper is for a double-Heston model.

A closed form formula for Heston-S PDE in Heston framework with digital option

Tiberiu Socaciu and Paul Pascu

University of Suceava, Romania
e-mail: socaciu@seap.usv.ro

Using methods from [Heston] (Heston PDE with European options) and [Lazar] (Heston PDE with digital options), we build a closed form formula for Heston-S PDE. We try to validate (in Popper sense) our formula.

Quantitative and qualitative analysis of evacuation system by using GSPNs

Inga Titchiev

Institute of Mathematics and Computer Science of ASM, Chişinău, Republic of Moldova
e-mail: inga.titchiev@gmail.com

The aim of this research is to provide potential solutions to respond in case of disaster [2] and assist in the decision-making process by using Petri nets model [1,4]. In order to provide these solutions logical and temporal dependencies have to be considered. Petri Nets and their extensions are applied successfully in various fields. Especially in the area of emergency and disaster management [3]. For modeling of sequences of countermeasures, will be discussed information about the influencing factors and endangered objects in order of adequate response to the event. In these order quantitative and qualitative analysis of case study (evacuation system) will be done.

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Section 8. Education

Graphical resolution of engineering problems using mathematical software

Elisabete Alberdi Celaya, Isabel Eguia Ribero, Josefa González Gómez, and Judit Muñoz Matute

University of the Basque Country (UPV/EHU), Bilbao, Spain
e-mail: elisabete.alberdi@ehu.es

This work is focused on the digital competence in the subjects of two Engineering Degrees taught in the University College of Technical Mining and Civil Engineering (University of the Basque Country): the Technical Civil Engineering Degree and the Technical Mining Engineering. This design expects to provide the students with a learning methodology based in the use of digital tools, making possible the development of visualizations that will lead them from the theoretical aspects to the resolution of problems related to engineering. To achieve this aim, the professors have selected some topics of the basic subjects of the engineering degrees and they have suggested some activities in which mathematical concepts have been developed using graphical representations. The selected topics are the following: the graphical representation of functions given in different form (explicit form, implicit form, parametric form and polar form), the analytical and numerical solution of Ordinary Differential Equations (ODEs) and Partial Differential Equations (PDEs) and their graphical representation. These topics have been selected because they are present in the majority of the engineering degrees and because many phenomena of science and engineering are modelled using ODEs and PDEs. Two softwares have been selected to accomplish this work: Mathematica and MATLAB. The graphical representation of functions by means of digital tools allows for doing simulations changing the data in an easy and fast way. This is a good way to capture students' attention, reinforcing their learning. From this work it can be concluded that the development of the digital competence means an improvement of the students' abilities, favouring their self-learning and their decision-making.

Keywords: digital competence, digital tools, autonomous learning, Mathematica, MATLAB.

Elemente de Matematici Financiare în liceu

Olga Benderschi

Moldova State University, Moldavia
e-mail: obenderschi@yahoo.com

În Programa şcolară pentru disciplina opţională educaţie financiară [1] se menţionează: Educaţia financiară şi incluziunea financiară reprezintă priorităţi pentru Uniunea Europeană, în scopul creării unei pieţe integrate a serviciilor financiare la nivel comunitar, care să fie accesibilă pentru toţi cetăţenii statelor membre. Ţinând cont de acest lucru şi din multe alte considerente în ultimii ani s-au introdus elemente de calcul financiar în curriculumul pentru liceu. În Republica Moldova elemente de calcul financiar apar în manualul de clasa a XII-a în modulul "Elemente de statistică matematică şi de calcul financiar" [2].

În liceu predarea elementelor de calcul financiar are mai multe obiective. În primul rând, elevul trebuie să se familiarizeze cu noţiunile financiare: procent, dobândă, TVA, preţ de cost, profit, tipuri de credite, buget etc. În al doilea rând, elevul trebuie să înveţe să calculeze sumele finale şi dobânzile aferente în regim de dobândă simplă şi regim de dobândă compusă. În al treilea rând, elevul trebuie să poată utiliza unii algoritmi specifici calculului financiar pentru analiză de caz. Aceste competenţe trebuie să asigure formarea la elevi a unei gândiri financiare.

Obiectivele menţionate urmează să fie atinse cu ajutorul unui sistem complex de exerciţii (rezolvate sau propuse spre rezolvare) [2]. Primele exemple şi probleme se referă la calculul procentelor: determinarea numărului T care constituie $p\%$ din numărul G; determinarea numărului G dacă se cunoaşte numărul T care constituie $p\%$ din G etc. Aceste probleme trebuie să fie foarte diverse şi aplicative, fapt care va contribui la înţelegerea profundă de către elevi a noţiunii de procent. Apoi se definesc noţiunile de capital iniţial, capital final, rata dobânzii, regim de dobândă simplă şi regim de dobândă compusă. Probleme propuse trebuie să ajute elevul să deosebească rezultatul aplicării regimului de dobândă simplă şi regimului de dobândă compusă asupra capitalului iniţial. Un lucru foarte util elevilor este faptul de a putea forma bugetul familial. În acest scop se propun probleme de calculul bugetului, indicându-se cheltuielile, profiturile şi economiile unei familii. Este bine ca elevul să ştie că nu toată suma de bani plătită pentru un produs îi revine producătorului, deoarece în preţ intră şi anumite taxe. Astfel sunt propuse probleme de calcul a preţului de cost, venitului producătorului, veniturile vânzătorului etc. ţinând cont de TVA.

Introducerea elementelor de calcul financiar contribuie la pregătirea elevilor pentru asumarea, în cunoştinţă de cauză, a calităţii de consumatori care îşi cunosc nevoile, precum şi la faptul că vor fi conştienţi de riscuri şi de oportunităţi, şi vor putea face alegeri informate, precum şi exersa unele deprinderi şi atitudini corecte în ceea ce priveşte administrarea bugetului personal şi familial [1].

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Personalized approach in geometry learning for grades 4-6

Olesea Caftanatov

Institute of Mathematics and Computer Science, Academy of Sciences of Moldova, Moldavia
e-mail: caftanatov@gmail.com

The aim of our work is to figure out the difficulties faced by pupils in the study of school geometry courses and designation of the possible approaches to overcoming them.

We assume that all the teachers perceive their pupils as individuals. However, traditional schooling often doesn't provide time, space or resources for this. Good teachers skillfully adapt what they're teaching to their pupils/students' interests and abilities. Unfortunately, in process of studing, teachers can't adapt a model of learning style for each student individually.

Because of that, usually pupils who can't accommodate to teacher style don't assimilate the information that they get. We all know by our experience that when we can't understand something we get bored. What is more frustrated then a bored pupil? When it's not possible to provide a great 1:1 instructor for every learner, technology can play a crucial role. Thinking about this we thought of aproaching personalization in education.

Looking through different kind of surveys we observed that students have difficulties in learning such subject like geometry or math in general. This problem seems to be a common one for differnet countries. For example: in a survey operated by Ogilvy Public Relations international company [1] three in ten Americans (29%) report that they are not good at math. Further, over a third of Americans (36%) admit that there have been many times that they've found themselves saying they can't do math. Also the 2003 TIMMS report showed that the learners from South Africa did extremely poorly in geometry [2].

Many persons admit having difficulty when faced with applying geometry in everyday situations, and this is a main problem. Learners are surrounded by spatial settings and the ability to perceive spatial relations is regarded as important for everyday interaction in space. For example, in [3] the importance of these skills is stressed: "Without spatial sense it would be difficult to exist in this world - we would not be able to communicate about position, relationships between objects, giving and receiving directions or imagine changes taking place regarding the changes in position and size of shapes".

It was observed, that pupils meet difficulties in understanding of symmetry or congruence concepts, in angles calculation, etc.

One of one possible solution would be to use technologies in learning geometry. Modern computer tools make it easy to create software for geometric shapes and their spatial (3D) representation. Such kind of applications may be dynamic, using a game elements and concepts of adaptive learning and personalization.

These "virtual manipulatives" can assist students in learning important geometric concepts, allowing them to operate quickly and easily with models of geometric shapes and figures in ways that would be more difficult to demonstrate with specific materials or hand-drawn representations [4]. Also, through this type of application, students can develop a deeper understanding of the relationship between two-dimensional shapes and three-dimensional objects.

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Aplicarea modelelor matematice la determinarea gradului de poluare a solului prin prisma studierii speciei Trifolium repens

Eugenia Chiriac

Universitatea de Stat Tiraspol, Chişinău, Moldova
e-mail: eugeniachiriac@gmail.com

Trifolium repens (trifoiul alb) aparţine genului Trifolium, familia Fabaceae, clasa Magnoliopsida, încrengătura Magnoliophyta. Este interesant faptul, că specia dată (comparativ cu cele 22-24 specii, întâlnite în Republica Moldova), prezintă un şir de însuşiri specifice, cum ar fi: uşor se adaptează la un spectru larg de factori abiotici, inclusiv factorul antropic; posedă o plasticitate fenotipică (în dependenţă de condiţiile în care creşte) exprimată în forma şi mărimea "petelor albe" de pe frunze; în condiţiile ecologice curate este expusă stresului cenotic (este mai "oprimată" în concurenţa cu celelalte specii de plante din comunitatea dată); în condiţiile poluate, concurenţa dintre speciile de plante lipseşte [1-3].

Cercetările histologice (Schwartzman, 1986) au arătat că apariţia aşa numitor "pete albe"este legată de o grupă de celule palisadice din mezofilul frunzei de trifoi, în care clorofila se găseşte într-o cantitate foarte mică sau chiar lipseşte. Astfel în aceste porţiuni, celulele palisadice sunt mai mici în dimensiuni, mai puţin alungite şi cu spaţii mai mari între ele, comparativ cu partea verde a frunzei. Koncina şi Marina (2010), arată că forma şi gradul de dezvoltare al acestor " pete albe", pot varia în dependenţă de starea ecologică a solului [4].

În corespundere cu cele expuse mai sus, autorul a studiat arealuri de trifoi alb, cu grad diferit de influenţă a factorului antropic, situate unele de altele la distanţe apreciabile. Pe fiecare areal de cercetare s-a delimitat 3-5 suprafeţe mai mici (în dependenţă de densitatea răspândirii reprezentanţilor speciei date). Pe fiecare suprafaţă s-a identificat fenotipul frunzelor la un număr nu mai mic de 200 de indivizi. În cazul când abundenţa este mare şi este dificil de delimitat o plantă de alta, s-au folosit pătrate de sârmă zincată (D. Ivan, 1979) cu mărimea de 25 x 25 cm. Aceste pătrate se aşează la distanţe de 5 m., şi se numără plantele cu fenotipul respectiv. Numărarea se repetă de 20 de ori.

Pentru populaţiile de Trifolium repens de pe fiecare suprafaţă s-au utilizat instrumente matematice, cu ajutorul cărora s-a determinat frecvenţa răspândirii a unui anumit fenotip conform relaţiei:

$$Pi = (100 \times Hj)/N,$$

cât şi suma frecvenţei tuturor formelor fenotipice, unde indicele coraportului fenotipurilor (ICF) poate fi determinat utilizând următoarea relaţie:

$$ICF = 100 \times (n1 + n2 + n3...)/N.$$

Astfel, Pi reprezintă frecvenţa unui anumit fenotip; Hj -numărul de plante cu "desenul de tip i" pe limbul foliar (n1 - numărul de plante fără "pată albă", n2 - numărul de plante cu o anumită formă a "petei albe", etc.); N -numărul total de plante numărate.

Koncina şi Marina (2010), propun o scară de utilizare în bioindicaţie. Astfel, dacă ICF-ul variază intre: $20\% - 30\%$, atunci solurile examinate sunt considerate foarte ecologice; $31\% - 40\%$, atunci solurile sunt considerate ecologice; $41\% - 60\%$, atunci solurile respective au un grad mediu de poluare; $61\% - 80\%$, atunci solurile sunt considerate cu un grad mare de poluare; $81\% - 100\%$, atunci solurile sunt incluse în categoria celor foarte poluate.

Pe parcursul anilor 2012-2016, s-au efectuat ieşiri în teren, în scopul studierii plantelor de Trifolium repens, inclusiv, variaţia desenelor pe care le formează "pata albă" pe frunzele speciei. Astfel, s-a calculat indicele coraportului fenotipurilor (ICF), după care s-a determinat gradul de poluare în trei zone diferite: lunca râului Ichel, din amonte de satul Goian, comuna Ciorescu, municipiul Chişinău, unde se găseşte o cariera de extragere a calcarului; zona parcului Râşcani, în vecinătate cu fabrica Viorica -Cosmetic, SA, municipiul Chişinău şi o zonă dintre cartierele de locuit, de pe strada V. Alecsandri, municipiul Chişinău.

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Integrarea evaluării interactive în procesul de studiere a cursului universitar "Tehnici de programare"

Liubomir Chiriac and Angela Globa

Universitarea de Stat din Tiraspol, Republica Moldava e-mail: llchiriac@gmail.com, angelagloba@gmail.com

Aplicarea raţională a TIC contribuie esenţial la eficientizarea procesului de învăţământ şi la intensificarea relaţiilor de colaborare şi cooperare: profesor - student, student - student.

Instruirea interactivă, învăţarea interactivă sunt asistate de o evaluare interactivă, care se prezintă ca o evaluare formativă şi formatoare.

Utilizarea sistemelor digitale de testare interactivă în combinaţie cu alte mijloace tehnice permit modelarea activităţilor didactice în cadrul orelor facilitând, astfel, obţinerea unei calităţi ridicate de însuşire a cursului universitar *Tehnici de programare*.

Integrarea evaluării interactive în procesul de studiere a cursului universitar *Tehnici de programare* a permis monitorizarea procesului de învăţare a studenţilor, ajustarea interactivă imediată a demersului didactic în funcţie de rezultatele înregistrate la testele propuse, anticiparea unor situaţii dificile în procesul didactic, contribuind, astfel, la sporirea succesului academic al studenţilor.

În cercetarea noastră evaluarea interactivă se aplică cu scopul de a scoate imediat în evidenţă calitatea însuşirii subiectelor examinate de către studenţi şi, în dependenţă de rezultatele obţinute, să se poată continua, schimba, modifica ori ajusta abordările şi strategiile didactice.

Obiectul evaluării este constituit din: cunoştinţele acumulate şi nivelul de dezvoltare a competenţei măsurate prin abilităţile studentului de a folosi aceste cunoştinţe în diverse situaţii practice. În timpul evaluării complexitatea situaţiilor în care este plasat studentul se află în creştere, adica itemii după gradul de dificultate sunt ordonaţi în ordine crescătoare. Gradul de dificultate a unui item este determinat de "concentraţia" noţiunilor teoretice şi abilităţilor practice necesare pentru soluţionarea itemului respectiv. Astfel, studentul este pus în condiţia de a aplica cunoştinţele acumulate şi abilităţile practice gradual, de la simplu la compus.

Referindu-ne la cursul universitar *Tehnici de programare*, evaluarea interactivă s-a aplicat mai mult la etapele de învăţare de bază (integrare şi transfer) în cadrul orelor de curs, în mod special, la desfăşurarea prelegerilor intensificate. Evaluarea interactivă s-a utilizat în procesul de predare, la o anumită etapă, atunci când profesorul a decis să clarifice dacă noţiunile teoretice şi conceptele studiate, până la o anumită etapă, care ţin de tehnicile de programare sunt înţelese la nivelul prevăzut de program. În funcţie de rezultat profesorul poate reveni la abordarea suplimentară a subiectelor examinate anterior, mai puţin înţelese, utilizând alte instrumente şi abordări didactice, ori poate trece la următoarea etapă a prelegerii intensificate care presupune studierea subiectelor noi, de o complexitate mai avansată. În calitate de *instrument* de evaluare a servit testul.

În concluzie, se poate sublinia că, evaluarea interactivă are un caracter formativ, de estimare în baza unor criterii bine determinate, şi se încadrează organic în procesul didactic, accentuând centrarea pe student a întregului proces de formare.

Working the descriptive geometry and the affine geometry from the graphical and mathematical point of view

Olga Danilkina[1], Elisabete Alberdi Celaya[2], Isabel Eguia Ribero[2], José García López[2], Aitziber Unzueta Inchaurbe[2], Irantzu Alvarez González [2]

[1] *University of Dodoma (UDOM), Tanzania*
[2] *University of the Basque Country UPV/EHU, Bilbao, Basque Country (Spain)*
e-mail: olga.danilkina@gmail.com, elisabete.alberdi@ehu.es

This is a project to develop together the subjects "Algebra and Geometry" and "Graphical Expression" taught is many technical degrees, joining the mathematical formulation with the graphical representation.

We have realised that approximately the 25The common lessons form part of the area called descriptive geometry in the subject of Graphical Expression, and they belong to the area of affine geometry in the subject of Algebra and Geometry. The identified common areas have been the following: the lesson related to angles, representation of different type of elements in the 3-dimensional space, relative positions between elements, distances and symmetries. After having accomplished this first step, the next stage has consisted in the design and creation of didactic material taking into account the mathematical and the graphical point of views, and the use of this material in the engineering degrees. The process has finished doing an evaluation which has been carried out by asking the students about this new experience.

Keywords: Algebra, graphical expression, descriptive geometry, affine geometry.

Collaboration project to improve the mathematical academic performance of the first year degree students

Olga Danilkina[1], Elisabete Alberdi Celaya[2], and Jefta Sunzu[1]

[1] *University of Dodoma (UDOM), Tanzania*
[2] *University of the Basque Country UPV/EHU, Bilbao, Basque Country (Spain)*
e-mail: olga.danilkina@gmail.com, elisabete.alberdi@ehu.es

This is a collaboration project between the University of the Basque Country (UPV/EHU, Spain) and the University of Dodoma (UDOM, Tanzania) to improve the academic performance of the first year students in the scientific and technical degrees in the subjects related to mathematics. A previous experience of the University of the Basque Country has been taken into account, in which the formative lacks of the students that start some technical degrees were detected and a zero course was designed in order to improve the results of the first year students. The aim of this project is to repeat a similar experience in UDOM with the aim of correcting the first year degree students' deficiencies and reinforcing the acquired knowledge in the mathematical subjects.

Keywords: self-learning, mathematics, zero course, academic performance.

Points and vectors in the three-dimensional Euclidean space studied with Maple

Raluca Mihaela Georgescu

University of Piteşti, Romania
e-mail: gemiral@yahoo.com

The paper presents some types of homogeneous and nonhomogeneous linear systems of two ordinary differential equations with variable coefficients. There are presented systems where the matrix of the homogeneous system is symmetric, systems with a known solution or systems which can be reduced to systems with constant coefficients.

Unele abordări didactice privind studierea Roboticii în sistemul de învăţământ preuniversitar

Lilia Mihălache and Liubomir Chiriac

Universitatea de Stat Tiraspol, Chişinău, Moldova e-mail: lilia.mihalache@gmail.com, llchiriac@gmail.com

Abstract. *Sunt analizate unele oportunităţi şi avantaje privind studierea roboticii în şcoală. Se propun unele aspecte didactice vizînd tematica curriculumului la robotică.*

O abordare nouă a educaţiei pentru tehnologiile informaţionale, care câştigă popularitate, constă în implementarea roboticii în şcoală. Astăzi, în Republica Moldova peste 20 de instituţii şcolare au la dispoziţie seturi de roboţi şi pot studia robotica conform curriculumului pentru disciplina opţională Robotica. Robotica încurajează elevii să gândească creativ, să analizeze diverse situaţii şi să folosească gândirea critică în rezolvarea problemelor din lumea reală. Elevii învaţă că este acceptabil să fie comise erori, iar acest fapt, poate, conduce la găsirea unor soluţii mai bune pentru soluţionarea problemelor examinate. În acest context, este necesar de menţionat, de exemplu, că metodologia educativă bazată pe kitul de robotică *LegoMindstorm* se utilizează în peste 30000 de instituţii de învăţământ, de la şcoli la universităţi.

Cunoscutul expert, Seymour Papert, consideră că tehnologiile pot oferi noi modalităţi de învăţare. Deasemenea el a raţionalizat că conceptele teoretice, în multe cazuri, sunt dificile de înţeles copiilor, deoarece nu există materiale reale care să le materializeze. Totodată, a identificat că roboţii sunt un excelent mijloc pentru punerea teoriei constructivismului în practică, mai ales acei cu potenţial în arta programării.

Luând în vedere propria experienţă, în cadrul procesului instructiv-educativ, implementarea roboticii, credem noi, generează noi oportunităţi şi oferă avantaje suplimentare la însuşirea informaticii, tehnologiilor informaţionale cât şi altor discipline de studiu. În continuare ne vom referi succint, în acest context, la avantajele didactice care ţin de *creşterea capacităţilor şi abilităţilor privind analiză, proiectare/modelare şi programare.*

Robotica modernă, mai ales sistemul *LegoMindstorm*, permite elevilor să proiecteze şi să execute artefacte interactive care utilizează instrumente de orientaţie tehnică, motoare şi senzori, astfel cercetând în mod activ prin intermediul creării unor experimente prin joc.

Având la dispoziţie elemente mecanice cu care se pot construi structuri şi mecanisme, elemente electronice cum ar fi senzori, servomotoare, rezultă că setul EV3 Education permite construirea si programarea unui robot mobil reconfigurabil deosebit de versatil. Elementele mecanice ale sistemului respectă binecunoscutul principiu LEGO al interconexiunii dintre elemente modularizate. Astfel:

- **Analiza problemelor.** Robotica încurajează elevii să privească problemele în ansamblu, să identifice care sunt problemele pe care trebuie să le rezolve. Se pot găsi uşor probleme din lumea reală pentru proiecte, oferind elevilor contexte pentru proiectele lor. Înainte de începerea construcţiei robotului, elevii trebuie să determine ce aşteptări poate îndeplini robotul din perspectiva problemei date, şi având în vedere aşteptările respective trebuie să ştie să analizeze minuţios datele problemei pentru ca ulterior să se focuseze pe proiectarea robotului.

- **Proiectare.** Clarificând exact, în urma analizei efectuate, ce îşi doresc să obţină, elevii pot începe proiectarea propriu zisă. Această etapă de lucru oferă elevilor posibilitatea de a realiza în practică ideile generate teoretic. La această etapă văd nemijlocit aplicabilitatea conceptelor teoretice care păreau abstracte şi neabordabile. Tot odată, elevii pot să perfecţioneze şi să îmbunătăţească anumite aspecte ale ideilor lansate.

- **Programare.** După ce etapa care ţine de proiectare este finalizată cu succes, elevii pot să se concentreze pe programare. Având la dispoziţie o varietate de limbaje de programare pentru programarea roboţilor, de la limbaje cu interfeţe grafice până la limbaje bazate pe text, elevii pot să-şi desăvârşească abilităţile de programare. Experienţa a demonstrat că competenţele de programare învaţă elevii să gândească logic şi să prevadă situaţi multiple, astfel încât robotul să poată execută exact obiectivele proiectării, astfel încât informaţiile parvenite de la diferiţi senzori să fie prelucrate logic şi corect.

Astfel, această joacă inteligentă, extrem de interesantă, pune în valoare potenţialul intelectual şi imaginativ al elevilor şi încurajează iniţiativele raţionale, bine gândite şi argumentate. Robotica stimulează interesul elevilor pentru studierea suplimentară a informaticii, matematicii, fizicii, biologiei etc., iar munca în echipă pe parcursul realizării proiectului de robotică, după cum ne demonstrează practica, consolidează relaţiile de cooperare şi amiciţie intre elevi.

Atractivitatea specială a disciplinei Robotica, în opinia noastră, este susţinută şi de faptul că elevii văd imediat aplicabilitatea noţiunilor teoretice studiate cât şi conexiunea intre diverse discipline, interdisplinaritatea, care presupune corelarea şi implementarea cunoştinţelor şi abilităţilor acumulate în procesul studierii diverselor discipline şcolare.

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Aspecte privind compunerea ecuaţiilor funcţionale pentru concursurile matematice

Larisa Sali and Mitrofan Ciobanu

Tiraspol State University, Republic of Moldova
e-mail: salilarisa@yahoo.com, mmchoban@gmail.com

Procesul de compunere a problemelor are drept scop formarea capacităţii de analiză a noţiunilor şi a proprietăţilor lor, consolidarea cunoştinţelor, formarea premiselor pentru aplicarea acestora, dezvoltarea creativităţii matematice a elevilor etc. Noţiunile de ecuaţie şi de funcţie sunt fundamentale în cursul de matematică elementară şi înglobeaza un bogat potenţial pentru soluţionarea problemelor cu caracter inter- şi transdisciplinar. Relevanţa formarii competenţelor de rezolvare şi compunere a ecuaţiilor funcţionale la elevii capabili de performanţe înalte rezultă din analiza listei domeniilor de activitatate profesionala pentru care opteaza ei.

Prezenta lucrare reflectă analiza generală a ecuaţiilor funcţionale de tipul $af(p(x)) + bf(q(x)) = g(x)$, unde $a, b \in R$, $p(x)$, $q(x)$, $g(x)$ sunt polinoame şi $grad(p(x)) + grad(q(x)) \geq 3$.

Abordarea generală permite structurarea algoritmului de compunere a unor ecuaţii funcţionale concrete, al caror caracter "picant" provine de la modul de selectare a coeficienţilor a, b şi a gradelor polinoamelor $p(x)$, $q(x)$, $g(x)$. În particular, sunt examinate ecuaţiile de tipul $mf(ax + b)+ nf(-ax + c) = g(x)$, $mf(ax) + nf(-1/ax) = g(x)$ şi cazul cand $p(x) = q(x) + c$.

Lucrarea conţine indicaţii metodice pentru cadrele didactice şi elevii interesaţi de matematica competitivă.

The course "Theory of Probability and Mathematical Statistics" on e-learning platform MOODLE

Vladislav Seiciuc[1] and Victor Seiciuc[2]

[1] *Trade Co-operative University of Moldova, Chişinău*
[2] *State University of Moldova, Chişinău, Republic of Moldova*

e-mail: seiciuc@mail.ru, runsmar@yahoo.com

In the paper is presented the university course Theory of Probability and Mathematical Statistics (TP and MS), created on the site of Trade Co-operative University of Moldova (TCUM): HYPERLINK "http://www.uccm.md/" www.uccm.md/ for distance learning. This E-course contain the next twelve themes: 1. Classical and geometric probabilities; 2. The addition and the multiplication of probabilities; 3. The total probability formula. Bayes's formula; 4. Repeated experiments; 5. Discrete random variables; 6. Continuous random variables; 7. Classical random variables; 8. Two-dimensional discrete random variables; 9. Two-dimensional continuous random variables; 10. Statistic population. Selection; 11. Parameter estimation; 12. Correlation. Regression lines.

Each of the twelve themes of the course TP and MS, developed on the Moodle platform contains the following components: Lecture Notes - contains the theory, Solved Exercises - are presented the solved issues and exercises, Suggested Exercises - contains problems to be solved, Answers - are given the answers to all outstanding issues, Glossary - contains a list of terms with their definitions, Self-Assessment Tests - for training and self-assessment, Game-Test - for training and self-assessment using the animated games, Chat - for conversations on the topic, Assessment Tests - for estimating knowledge on the topic.

The General compartment include the following components: Forum, Annotation of the E-learning course TP and MS, Curriculum of the course TP and MS at TCUM, the Manual of the course reissued in 2013, Glossary, Chat, Initial Test for the initial evaluation, Questionnaire where users have the opportunity to express their opinions on the usefulness and effectiveness of this course and which are always welcome and expected by the authors of this E-learning course presented.

At the end of the course themes TP and MS are presented the Tables and Bibliography, which are necessary to solve problems and further documentation in the field.

The same, it is organized [1] the Base of Questions, having the same structure as the Self-Assessment Tests, which also made the completion and operational processing of its contents. At the end of Base of Questions is organized the category Probation, which allows to facilitate the work with tests. Here tests are analyzed and prepared. At the end of each compartment are proposed Summative Evaluation of Knowledge Tests. Finally, consulting the connections, readers will find many other titles of books and electronic sources that will effectively serve to strengthen and deepening of the acquired knowledge from studying this course.

Primarily, the course it is addressed to students of higher education institutions. So, it can be used by high school students, as well of economists, engineers, which use probabilistic and statistical methods in their work.

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Metoda vectorială la rezolvarea problemelor de algebră

Boris Ţarălungă and Zinaida Ghilan

UPS "Ion Creangă" , Chişinău, Republica Moldova
e-mail: Borisstar@mail.ru, ghilan_z@mail.md

În lucrare se abordează metoda vectorială de rezolvare a unor probleme de algebră. Aplicarea proprietăţilor vectorilor permite economie a timpului, dar şi evitarea calcului voluminos prin utilizarea metodelor clasice. În lucrare se discută următoarele tipuri de probleme:

1. Ecuaţii iraţionale.
2. Inecuaţii trigonometrice.
3. Sisteme de ecuaţii.
4. Probleme de minim şi maximum ale funcţiei.

La rezolvarea sistemelor de ecuaţii se indică algoritmul de rezolvare:

1. Se definesc vectorii m şi n .
2. Se determină produsul scalar (m, n) şi modulul acestor vectori $|m|, |n|$.
3. Se compară valorile obţinute pentru produsul scalar şi produsul modulelor acestor vectori:

a) dacă produsul scalar este egal cu produsul modulelor, atunci sistemul are o singura soluţie;

b) dacă produsul scalar este mai mic decât produsul modulelor, atunci sistemul are o infinitate de soluţii;

c) dacă produsul scalar este mai mare decât produsul modulelor, atunci sistemul este incompatibil.

Mathematica, Wolfram language and computer based mathematics

Valeriu Ungureanu

Moldova State University, Moldavia
e-mail: `v.a.ungureanu@gmail.com`

IT development makes a major impact on all the domains of human activity, including the domain of mathematical and computer science education. Mathematica and Wolfram Language [1] are two featured products of the Wolfram Research Inc. which are leaders in this context. WRI has a lot of other important education aimed products, such as WolframAlpha, WolframLab, Wolfram Problem Generator, Demonstrations Project, Wolfram Community, etc., which are intensively used both by professors and students.

In our presentation we intend to expose main features of these products as well main approaches to their applying in teaching of different primary, secondary, pre-university and university courses such as, e.g. arithmetic, algebra, geometry, calculus, optimization methods, mathematical logic, probability and statistics, differential equations, computer programming, programming paradigms etc.

A series of examples has to highlight the importance of interactive, visual and symbolic expositions of materials in the process of teaching and learning. The examples are representative for the computer algebra system Mathematica and points out its featured characteristics:

1. Symbolic computation/calculation,
2. Numerical calculations,
3. Data representation,
4. Graphics and visualization,
5. Geo-computation,
6. Programming paradigms,
7. Real World Data,
8. Machine Learning,
9. Entities
10. Etc.

Our presentation has also the purpose to announce forthcoming opening of the Wolfram Research Center at Moldova State University.

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Conexiuni ale matematicii cu teoria informaţiei, din punct de vedere metodologic şi predare, învăţare

Ion I. Valuţă

Universitatea Tehnică a Moldovei, Moldova
e-mail: valutse@mail.md

Dezvoltarea matematicii, de altfel ca şi a oricăror altor ramuri ale ştiinţelor, decurge în întreaga sa istorie, prin diverse influenţe reciproce, conexiuni cu celelalte ştiinţe şi se manifestă prin diferite modalităţi: activităţi practice, teoretice, sau din punct de vedere metodologic. Cum a menţionat R. Descartes diversele activităţi intelectuale ale omenirii, ce şi constituie lumea ştiinţelor, exercită influenţe asupra dezvoltării fiecăreia dintre ele, deci şi a matematicii şi viceversa, fiecare cu anumite intensităţi în diferite perioade (în privinţa perioadelor de dezvoltare a matematicii se poate vedea [1], [2]). Astfel, în ştiinţa antică s-ar putea menţiona, în ceababiloniană influenţa astronomiei asupra matematicii (sistemul de nmumeraţie poziţional în baza şasezeci), în cea egipteană, a construcţiilor (Imhotep), în cea greacă, a filozofiei (Platon), a mecanicei (Arhimede), ceia ce este mai puţin amintit influenţa teologiei în evul mediu (N. Oresm, Th. Bradwardine) etc. Dar, există o ramură a ştiinţelor - Teoria Informaţiei în cel mai larg sens actual al acestui cuvânt- care prin categoriile sale e prezentă în orice ramură a ştiinţelor începând cu primele etape ale cunoaşterii în genere, legate de cele mai primordiale modalităţi de codare a informaţiei, un deosebit rol jucându-l în epoca preistorică cea sonorică: Întâi a fost cuvântul! O deosebită situaţie, crucială în sensul menţionat, se stabileşte începând cu mijlocul secolului trecut, când s-a format Teoria Informaţiei ca atare prin introducerea modalităţilor de măsurare a informaţiei. Providenţiale au fost cuvintele lui Norbert Wiener (publicate în 1948): "Informaţia este informaţie, nu este nici materie, nici energie", urmate de "Materialismul care nu recunoaşte acest lucru, nu poate fi apt de viaţă în timpurile noastre", cu care finisează capitolul "Maşinile de calcul şi sistemul nervos" din [3].

Astfel, conform lui N. Wiener, universul se poate vedea ca fiind alcătuit din trei părţi constituiente: materia, energia şi informaţia, acestea, spunem în continuare, manifestându-se, în modalităţile respective, prin stările veşnice ale lumii: MATERIA, ENERGIA şi VITALITATEA.

Situaţia formată la ora actuală în plan teoretic, împreună cu succesele tehnicii electronice în producerea calculatoarelor, generează o nouă stare socială - Civilizaţia Informaţională - operă în care matematica a jucat şi continuă să joace un rol primordial. Deoarece civilizaţia informaţională este civilizaţia viitorului se pun probleme noi în primul rând în sfera de educaţie, predare, învăţare în genere, dar, cu un deosebit imperativ apar aceste întrebări în faţa procesului de învăţare-predare a matematicii, nu în ultimul rând şi pentru că anume în cadrul matematicii au fost iniţiate aceste tehnici. începând cu anticul abac (simplu, dar fundamental, autori - toate popoarele, manifestându-se şi aici diferite personalităţi, de ex. Gerbert), perfecţionat prin aritmometre (W. Schiccard, B. Pascal, G. Leibniz, L. Cebâşev ş.a.) şi continuând cu sistemele contemporane (Ch. Babbege, A. Turing, J. von Neumann , D. Knuth, V.M. Gluşkov, B. Gates, J. Gosling etc, etc, cei enumăraţi fiind matematicieni). Sigur că tehnica ca atare joacă rolul său, tehnicienii, cu interese aplicative au acaparat domeniul, matematicienii, în primul rând cei preocupaţi de predare, dar în anumită măsură şi teoreticienii, nu s-au inclus activ în procesul de exploatare a celora ce constituie civilizaţia informaţională. Situaţia trebuie schimbată pentrucă e clar că în matematică în mod natural pot şi trebuie aplicate aceste tehnici de calcul şi prelucrare a informaţiei, formând la elevi deprinderile cuvenite, deschizând astfel posibilităţi de implementare ulterioară conştientă şi în alte domenii.

Menţionări în privinţa beneficiului aplicării tehnicii de calcul în învăţământ întâlnim adesea, de exemplu, în [4]. Aici se observă o analogie cu situaţia din Europa sec. XVII legată de introducerea sistemului poziţional de scriere (codare) a numerelor naturale şi efectuarea calculelor în acest sistem. Atunci unii considerau că învăţareea pe de rost a tăbliţei de înmulţire nu dezvolta inteligenţa aşa ca calculul oral (ce nu e lipsit de sens), acum se cere cunoaşterea tăbliţei de înmulţire (ce, evident, nu e lipsit de sens) dar se interzic adesea aparatele electronice.

Problemele care apar sunt diverse şi complicate, de la baza materială, adică pregătirea manualelor electronice pentru fiecare obiect şi clasă, până la pregătirea cadrelor şi multe de ordin ideologic şi organizatoric, care sunt costisitoare, dar nu este altă cale decât de a le rezolva.

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The factors leading to the determination of the educational institution

Sonila Zerelli, Lindita Mukli, and Teuta Thanasi

Department of Mathematics, Faculty of Information Technology, "Aleksandër Moisiu" University, Durrës, Albania

e-mail: sonilazerelli@yahoo.com, linditamukli@gmail.com, teutamatematika@hotmaul.com

Nowadays there are a growing problem in relation to education level and type of institution where an individual is educated. Given this, I think to carry out a study regarding factors which may significantly determine the origin where the student is educated. By theory, we know that to get the same qualification in different universities, they do not have the same requirements. To do this study I will take into account three universities and two generations last of students that they have certificated. The factors of taking into consideration are: the average note for student admission to university, the average note of exact sciences, the age of the student, the annual fee of university, the number of years of existence of this university, the number of graduates, etc. Through discriminating analysis I will tested which of them is more important and which are not yet. Also, I will do the comparative analysis between them in relation to the average number of students employed after graduation. Data for this study I have received from databases that have created these universities regarding students who finish their studies at that university. I hope to give a contribution no matter how small in this area.

Keywords: average note, age, annual fee, type of university.
JEL Classification: Education.

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