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1. Real, Complex, Functional and Numerical Analysis

Convergences and Cauchy conditions on semilinear topological spaces

Gabriela Apreutesei

Faculty of Mathematics, "Alexandru Ioan Cuza" University of Iaşi, Romania
e-mail: gapreutesei@yahoo.com

One defines semilinear space and semilinear topology, as well as different types of convergences and Cauchy conditions associated with them. One compares and establishes links between them. Cauchy nets are characterized by small sets and the "completeness" is described by Cantor type theorems.

Convergence of iterates of asymptotically quasi-nonexpansive mappings

Amitabh Banerjee

Govt D.B. Girls' P.G. College, Raipur, Chhattisgarh, India
e-mail: amitabh_61@yahoo.com

The iterative approximation problem for various nonexpansive were studied extensively by various researchers. In most of the above, authors have mainly studied Mann and Ishikawa type iteration process for iterative approximation of above class of mappings. The purpose of this paper is to study the problem of approximation of fixed point of the more general class of asymptotically quasi nonexpansive type mappings using generalized iteration process.

The category of semi-simplicial analytic spaces

Andrei Baran

"Simion Stoilov" Institute of Matematics of Romanian Academy, Bucharest, Romania
e-mail: andrei.baran@imar.ro

The term "semi-simplicial analytic space" is shorthand for a family of analytic spaces indexed by a simplicial complex, together with a family of connecting morphisms compatible with the inclusions in the simplicial complex. Semi-simplicial (s.s.) analytic spaces and the coresponding s.s. analytic modules have appeared, under different guises, in Forster - Knorr's proof of Grauert's direct image theorem, and in the definition of the natural topology on the hypercohomology groups of a complex of analytic sheaves with coherent cohomology given by J.P.Ramis, etc. Essentially, s.s. analytic spaces represent a flexible tool to produce global analytic objects starting from objects that normally can be proven to exist only locally. - in particular one can construct global resolutions with good projective properties for coherent analytic sheaves, and hence, compute Ext groups, generalize the Dolbeault resolution to spaces with singularities, construct the dualizing complex of a complex space etc. The talk will describe the basic properties of the category of s.s.analytic spaces with an emphasis on the direct image and exceptional direct image functors.

Banach algebras generated by singular integral operators

Diana Biclea[1], Vasile Neagu[2]

[1] *Tiraspol State University, Chişinău, Republic of Moldova*

[2] *Moldova State University, Chişinău, Republic of Moldova*
e-mail: vasileneagu45@gmail.com

Let Γ be an oriented contour in the complex plane, consisting of a finite number of closed simple Lyapunov curves, and $L(L_p(\Gamma,\rho))$ the Banach algebra of all linear bounded operators acting in space $L_p(\Gamma,\rho)$. We shall denote by $\Sigma(p,\Gamma,\rho)$ $(1<p<+\infty)$ the smallest closed sub algebra of $L(L_p(\Gamma,\rho))$ containing all one-dimensional singular integral operators A of the form

$$(A\varphi)(t)=c(t)\varphi(t)+\frac{d(t)}{\pi i}\int_\Gamma \frac{\varphi(\tau)}{tau-t}d\tau \quad (t\in\Gamma) \tag{1}$$

with arbitrary continuous coefficients $c(t)$, $d(t)$.
The following propositions are valid:
1^0. The algebra $\Sigma(p,\Gamma,\rho)$ is the linear hull of all operators of the form (1) and of the set $\mathrm{T}=\mathrm{T}(L_p(\Gamma,\rho))$ of all completely continuous operators belonging to $L(L_p(\Gamma,\rho))$.
2^0 The factor algebra $\Sigma(p,\Gamma,\rho)/\mathrm{T}$ is isomorphic (and in the case p=2, $\rho(t)\equiv$ 1 isometric, too) to the algebra of all continuous functions defined on the compact $\Gamma\times J$ where J is the two-point set ±1. The function corresponding to the operator A of the form (1) under this isomorphism is $A(t,j)=c(t)+jd(t)$ $(t\in\Gamma,\ j\in J)$.
3^0 The operator A of the form (1) is Noetherian in the space $L_p(\Gamma)$ if and only if the function $A(t,j)$ does not vanish at any point of $\Gamma\times J$.
The function $A(t,j)=c(t)+jd(t)$ is called (see [1]) the symbol of the singular integral operator$A$.
The proof of these propositions is based on following properties of the singular integration operator S and the operator of multiplication by an arbitrary continuous function $a(t)$:

$$S^2=I,\ S-S^\bullet\in\mathrm{T},aS-SaI\in\mathrm{T}. \tag{2}$$

Let us make clear that the operator S is considered in L_p, while $S^\bullet$ is understood to be the adjoint of S as an operator acting in $L_q(\Gamma,\rho^{1-q})$ $(p^{-1}+q^{-1})$.
The propositions 1^0-3^0 are no more valid in the case where the contour Γ is non-closed, non Lyapunov or non-simple, nor for algebras containing singular integral operators of the form (1) with discontinuous coefficients. In these cases not all of the properties (2) hold true. The latter circumstance leads to substantial complications. The algebras generated by such operators consist not only of the linear hull of the operators (1) and those belonging to T. The symbol and the set where its argument varies can be also constructed according to more complicated rules. Sometimes the symbol is a matrix function depending substantially on p and ρ .

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Modified Han algorithm for maritime containers transportation problem

Doina Carp[1], Constantin Popa[2], Cristina Şerban[2]

[1] *Constanţa Maritime University, Romania*

[2] *Ovidius University of Constanta, Romania*
e-mail: doina.carp@gmail.com; cpopa1956@gmail.com; cgherghina@gmail.com

The most important particularity of the maritime containers transportation is the fact that the capacities of different units of cargo are different, so the cost of a unit of transport is not the same for a particular route. If we consider the unit cost of the transport of a Twenty-foot Equivalent Unit (TEU, for short) between the supplier to the warehouse, the difference consists in that the costs are for TEU and not for each container. The mathematical model of this will provide us a problem similar to an inconsistent transportation problem. In this respect, we reformulate the problem as an inconsistent system of linear inequalities, for which we propose a modified version of Han iterative algorithm from [1]. Numerical experiments and comparisons with the classical Simplex algorithm are also presented.

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Remarks on Birkhoff integrability

Anca Croitoru, Alina Gavriluţ

Faculty of Mathematics, "Alexandru Ioan Cuza" University of Iaşi, Romania
e-mail: croitoru@uaic.ro

In 1935, Birkhoff introduced an integral via countable sums for vector functions with respect to complete finite measures. In the past years, the Birkhoff integral has been studied and generalized by many authors (e.g. Fremlin, Cascales, Rodriguez, Maraffa, Fernandez, Mayoral, Naranjo, Potyrala, Boccuto, Sambucini). We present some results on Birkhoff integral for real bounded functions with respect to a monotone measure.

Cellular automaton in flow routing

Ştefan Gicu Cruceanu, Stelian Ion, Dorin Marinescu

"Gh. Mihoc-C. Iacob" Institute of Mathematical Statistics and Applied Mathematics, Bucharest, Romania
e-mail: cruceanu@gmail.com, ro_diff@yahoo.com, marinescu@gmail.com

We present a simplified model for the water flow (based on Manningś law) on a landscape given by a GIS raster data. The water flow process is approximated by using a cellular automaton with hexagonal structure for which the input data is provided from the GIS data using interpolation techniques. This model is applied to elevation real data from Romanian Ampoi's Valley.

Propriétés fines des applications de dualité et géométrie des espaces de Sobolev à exposants variables

George Dincă

Faculty of Mathematics and Computer Sciences, University of Bucharest, Romania
e-mail: dinca@fmi.unibuc.ro

Certaines propriétés géométriques des espaces de Banach(lissitude, convexité uniforme, différentiabilité de la norme) sont mises en évidence moyennant des propriétés correspondantes pour les applications de dualité définies sur ces espaces. Les exemples envisagent certains espaces de Sobolev à exposants variables intervenant dans la théorie des equations aux dérivées partielles.

Continuity properties in hypertopologies

Alina Gavriluţ

Faculty of Mathematics, "Alexandru Ioan Cuza" University of Iaşi, Romania
e-mail: gavrilut@uaic.ro

We present some results concerning different continuity properties (increasing/decreasing convergence, order continuity, regularity) and the relationships among them in different hypertopologies for monotone set multifunctions.

On spectral problems for Laplace operator in domains with perturbed boundaries

Anca-Veronica Ion[1], Boris V. Loginov[2], Davran G. Rakhimov[3]

[1] *"Gh. Mihoc-C. Iacob" Institute of Mathematical Statistics and Applied Mathematics, Bucharest, Romania*

[2] *Ulyanovsk State Technical University (UlGTU), Ulyanovsk, Russia*

[3] *V.I. Romanovsky Institute of Mathematics and Information Technologies Acad.Sci of Uzbekistan, Tashkent, Uzbekistan*
e-mail: averionro@yahoo.com; bvllbv@yandex.ru; davranaka@yandex.com

Originating in the works of F. Rellich, T. Kato and B.Sz-Nagy, the study of perturbation theory in spectral problems is extending. The spectral problems for differential operators in domains with perturbed boundaries are connected to the relatively new new subject of this theory [1]. The most powerfull approach to the resolving of spectral problems in perturbation theory of Fredholm operators based on Newton diagram method of bifurcation theory was suggested by V.A. Trenogin [2]. M.K. Gavurin has suggested [3] the pseudoperturbiteration method (developed later by the authors [4], [5]) based on the special perturbation construction such that the known approximation to the eigenvalues, eigenvectors and generalized Jordan chains become exact for the perturbed operator. Here the perturbation and pseudoperturbation methods are applied to two spectral problems for Laplace operator in domains with perturbed boundaries at the strong restrictions on the perturbation terms arising at the transformation of the perturbed domains to unperturbed

ones. These consist in the spectral problem $A(w,\lambda) \equiv Au + \lambda u = 0$ in $\Omega_\varepsilon, u = 0$ on $\Omega_\varepsilon = \left\{(x,y)|0 \le x \le \pi,\ -\varepsilon f(x) \le y \le \pi + \varepsilon f(x),\ f(x) \in C^2(0,\pi) \cap C\left[0,\pi\right],\ \ f(0) = f(\pi) = 0\right\}$ and the problem $A(w,\lambda)$ in elliptical domain

$$D_\varepsilon = \left\{(x,y)|\frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\}, D_0 = \left\{(\xi,\eta)|\xi^2 + \eta^2 \le b^2\right\}, b^2 = (1-\varepsilon^2)a^2.$$

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Porting data. Real data terrain applications

Stelian Ion, Dorin Marinescu, Ştefan Gicu Crucean

"Gh. Mihoc-C. Iacob" Institute of Mathematical Statistics and Applied Mathematics, Bucharest, Romania
e-mail: ro_diff@yahoo.com, marinescu@gmail.com,

We present a method for porting GIS raster data on other type of 2D networks. For this purpose we develop a procedure of extending a 2D reticulated function (defined on networks with certain regularity) by a bi-cubic spline function which can exactly reconstruct polynomials up to degree 3 in each variable. Using this extension we generate the reticulated function on the new network. We also analyze properties of continuity and Gibbs phenomenon of the extended function. Finally we present numerical applications using elevation real data from the valley of the Romanian river Ampoi.

Degree of approximation by the $(T.E^1)$ means of conjugate Fourier series in the Hölder metric

Kejal Khatri, Vishnu Narayan Mishra

Department of Applied Mathematics and Humanities, Sardar Vallabhbhai National Institute of Technology, Gujarat, India
e-mail: kejal0909@gmail.com, vishnunarayanmishra@gmail.com

We compute the degree of approximation of functions $\widetilde{f} \in H_\omega$, a new Banach space, using $(T.E^1)$ summability means of conjugate Fourier series. In this paper, we extend the results of Singh and Mahajan [T. Singh and P. Mahajan, Error bound of periodic signals in the Hölder metric, Int. J. Math. Math. Sci. Volume 2008 (2008), Article ID 495075, 9 pages] which in turn generalizes the result of Lal and Yadav [S. Lal and K. N. Yadav, On degree of approximation of function belonging to the Lipschitz class by $(C, 1)(E, 1)$ means of its Fourier series, Bull. Cal. Math. Soc. Vol. 93 (2001) 191-196]. Some corollaries have also been deduced from our main theorem and particular cases.

Some consequences of the generalized Hamilton-Cayley theorem for matrices polynomially dependent on E.Schmidt spectral parameter

Anastasia N. Kuvshinova, Boris V. Loginov

Ulyanovsk State Pedagogical University (UlGPU), Ulyanovsk, Russia
e-mail: erasya7@rambler.ru, bvllbv@yandex.ru

In a series of works at the beginning of XXth century on linear and nonlinear integral equations, E. Schmidt had introduced for linear operators in Hilbert space H systems $\{\lambda_k\}_1^\infty$, counting with their multiplicities and eigenelements $\{\varphi_k\}_1^\infty$ and $\{\psi_k\}_1^\infty$ satisfying the relations $B\varphi_k = \lambda_k\psi_k$, $B^*\psi_k = \lambda_k\varphi_k$ allowing to extend Hilbert-Schmidt theory on nonselfadjoint completely continuous operators in abstract separable Hilbert space [1], [2]. Under the name s-numbers this system has found many applications in computational mathematics and the theory of ill-posed problems. I.S. Arjanykh [3] had proved the generalized Hamilton-Cayley theorem for matrices polynomially dependent on spectral parameter λ with identity matrix at the highest degree of λ with the aim of its applications to numerical methods in linear algebra [3]. In this work it is given the extension of Hamilton-Cayley theorem for E. Schmidt matrix spectral problems polynomially dependent on spectral parameter with identity matrix at higher (lowest) parameter degree

$$(A_s + \lambda A_{s-1} + \lambda^2 A_{s-2} + ... + \lambda^{s-1} A_1)\varphi = \lambda^s \psi$$

$$(A_s^* + \lambda A_{s-1}^* + \lambda^2 A_{s-2}^* + ... + \lambda^{s-1} A_1^*)\psi = \lambda^s \varphi$$

Follow to I.S. Arjanykh and V.I. Gugnina [3], [4] we give here some consequences of the obtained results to development of the relevant characteristic polynomial and some numerical methods for computation of E.Schmidt spectrum $\{\lambda_k\}$.

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On direct approximate methods in solving of weakly-singular integral equations

Vladimir Racu

State University of Moldova, Chişinău, Republic of Moldova
e-mail: vlracu@yahoo.com

We develop mathematical methods and algorithms for collocation, quadrature, spline collocation and spline quadrature methods on solving weakly-singular Fredholm and Volterra integral equations of the second kind.
Some improvements of the traditional approach in constructing interpolation quadratures computational schemes for solving integral equations are obtained. These new computational schemes are effective and better adapted to their theoretical justification. As an example, the computational schemes of spline quadrature method for solving integral equations of the second kind are considered.

On direct methods in solving of some nonlinear singular integral equations on closed contours

Vladislav Seichiuc

Moldova Trade Co-Operative University, Chişinău, Republic of Moldova
e-mail: seiciuc@mail.ru

We study the approximate solving of the following nonlinear singular integral equations (NSIE) defined on a arbitrary closed Lyapunov contour in the complex plane:
(1) NSIE with Carleman moved shift $\gamma(t) : \gamma(\gamma(t)) = t$,

$$\Phi\Big(t, \varphi(t), \varphi(\gamma(t)), \frac{1}{\pi i}\int\limits_{\Gamma} \frac{h_1(t,\tau,\varphi(\tau),\varphi(\gamma(\tau)))}{\tau - t}d\tau, \frac{1}{\pi i}\int\limits_{\Gamma} \frac{h_2(t,\tau,\varphi(\tau),\varphi(\gamma(\tau)))}{\tau - \gamma(t)}d\tau\Big) = 0,$$

$t \in \Gamma;\ \gamma^{'}(t) \in H_{\mu}(\Gamma),\ 0 < \mu \leq 1,\ \forall t \in \Gamma;\ h_1,\ h_2,\ \Phi$ are known functions which verify the certain Hölder conditions on Γ and φ is unknown function; the Lyapunov contour Γ verifies the Hölder condition with the exponent $0 < \nu \leq 1$;
(2) NSIE with conjugation of unknown function $\overline{\varphi(t)}$:

$$\Phi\Big(t, \varphi(t), \overline{\varphi(t)}, \frac{1}{\pi i}\int\limits_{\Gamma} \frac{h_1(t,\tau,\varphi(\tau),\overline{\varphi(\tau)})}{\tau - t}d\tau, \overline{\frac{1}{\pi i}\int\limits_{\Gamma} \frac{h_2(t,\tau,\varphi(\tau),\overline{\varphi(\tau)})}{\tau - t}d\tau}\Big) = 0.$$

Next the computing algorithms for collocations and quadratures methods are elaborated and the substantiation theory of these algorithms in Hölder spaces $H_{\beta}(\Gamma),\ 0 < \beta \leq 1$ is obtained.

Some function approximations on the real axis in spaces of summable functions

Natalia V. Snizhko

Zaporozhye National Technical University, Ukraine
e-mail: n_snizhko@mail.ru

The presented work is dedicated to determination of the approximate properties of the truncated Fourier series and of the interpolating Lagrange polynomials in the spaces $L_{p\rho}$, $\rho(x) = \left(1 + x^2\right)^{-1}$, $p > 1$, L_p, $p \geq 2$ on the real axis R by systems of functions

$$\omega_k(x) = \left(\frac{x-i}{x+i}\right)^k, \qquad k = 0, \pm 1, \ldots, \tag{1}$$

$$\psi_k(x) = \frac{2i}{x+i}\left(\frac{x-i}{x+i}\right)^k, \qquad k = 0, \pm 1, \ldots \quad . \tag{2}$$

Some integrals for vector valued multifunctions

Cristina Stamate

"Octav Mayer" Institute of Mathematics, Romanian Academy, Iaşi, Romania
e-mail: cstamate@ymail.com

We presents some kinds of vector integrals for vector valued multifunctions by respect to vector multisubmeasures, some properties of these integrals and convergence results for sequences of vector integrals.

On reduction method for solving singular integral equations given on closed smooth contours

Lilia Tabără

State University of Moldova, Chişinău, Republic of Moldova
e-mail: lilia.tabara@gmail.com

Results on function approximation with Faber-Laurent polynomials in the topology of continuous functions spaces and in the Hölder's spaces are obtained. The reduction method, built on the Faber-Laurent polynomials for the approximate solving of singular integral equations and system of singular integral equations defined on a closed smooth contour, are proposed. The theoretical justification of this method in Hölder space is obtained. It is investigated the case when the contour is an ellipse.

Existence and approximation of solutions for generalized extended nonlinear variational inequalities

Balwant Singh Thakur

Pt. Ravishankar Shukla University, India
e-mail: balwantst@gmail.com

In this paper, we consider a new class of generalized extended nonlinear variational inequality problem involving set valued relaxed monotone operator and establish its equivalence with the fixed point problem. We also study criteria for existence of its solution. Iterative methods for approximate solution are also proposed and analyzed. Further, we study iterative method for finding common element of fixed point set of nonexpansive mapping, and solution set of the variational inequality problem.

Smooth variational principles and Dependent Choice

Mihai Turinici

Department of Mathematics, "Alexandru Ioan Cuza" University of Iaşi, Romania
e-mail: mturi@uaic.ro

The semi-bump variant of the Smooth Variational Principle in Deville, Godefroy and Zizler [J. Funct. Analysis, 111 (1993), 197-212] is equivalent with the Dependent Choice Principle; and as such, equivalent with Ekelandś Variational Principle [J. Math. Anal. Appl., 47 (1974), 324-353].

An integral operator on the classes $S^*(b)$ and $CVH(b)$

Nicoleta Ularu

University of Piteşti, Romania
e-mail: nicoletaularu@yahoo.com

The purpose of this paper is to study some properties related to convexity order and coefficients estimation for a general integral operator. We prove the convexity order for this operator using the analytical functions from the class of starlike functions of order and from the class CVH(b)and also we estimate the first two coeffcients for functions obtained by applying this operator on the class CVH(b).

Some geometrical aspects of the Γ correlated processes

Ilie Vălușescu

"Simion Stoilov" Institute of Matematics of Roamnain Academy, Bucharest, Romania
e-mail: Ilie.Valusescu@imar.ro

Some geometrical aspects of the Γ-correlated processes are analyzed, starting from the properties of a Γ-orthogonal projection, which is not a proper one. This "projection" is generalized to $\mathcal{H}^T$ - the cartesian product of T copies of $\mathcal{H}$, in order to be applied for periodically Γ-correlated processes with the period T. Geometrical results are generalized to Γ-correlated case, especially the problem of the angle between the past and the future of some Γ-correlated processes. Generalizing a result of Helson and Szegő, a Schauder basis can be obtained for Γ-correlated processes with a positive angle between the past and the future. The geometrical property of a process to have a stationary dilation (if any!) permits us to use some stationary techniques in the study of some nonstationary processes. This is the case at least for the processes very close to the stationary processes, such as period ically, harmonizable, or uniformly bounded linearly stationary processes. In the periodically Γ-correlated case it is proved that the positivity of the angle is preserved by its stationary dilation process. The generalized Friedrichs angle and other geometrical concepts are used in the study of some geometrical properties of periodically Γ-correlated processes, too.

On Solvents of Matrix Polynomials

Malika Yaici[1], Kamel Hariche[2]

[1] *University of Béjaïa, Algeria*

[2] *University of Boumerdes, Algeria*
e-mail: yaici_m@hotmail.com

Matrix polynomials play a central role in the mathematical description of the dynamics of multivariable systems. In this case MIMO systems are described by a matrix transfer function $G(s)$ expressed as a right (or left) matrix fraction description (RMFD or LMFD):

$$\begin{array}{c} G(s) = N_r(s)D_r^{-1}(s) \\ or \\ G(s) = D_l^{-1}(s)N_l(s), \end{array} \tag{1}$$

where N_r, D_r, N_l, D_l, are matrix polynomials.
In this paper, we will consider r-degree, m-th order monic matrix polynomials of the form

$$D(\lambda) = I_m\lambda^r + D_1\lambda^{r-1} + ... + D_{r-1}\lambda + D_r \tag{2}$$

where λ is a complex number and $D_i \in \mathbb{R}^{m\times m}$.
A latent root λ_i of $D(\lambda)$ is a complex number satisfying $\det D(\lambda_i) = 0$. A right latent vector $X_i \in \mathbb{R}^m$associated with λ_i satisfies $D(\lambda_i)X_i = \theta$ while a left latent vector Y_i is a row vector satisfying $Y_iD(\lambda_i) = \theta$ (θ being used here to indicate the zero vector).
A right solvent of $D(\lambda)$ is an $m \times m$ matrix R such that

$$R^r + D_1R^{r-1} + ... + D_{r-1}R + D_r = 0_m \tag{3}$$

while a left solvent is an $m \times m$ matrix L satisfying

$$L^r + L^{r-1}D_1 + ... + LD_{r-1} + D_r = 0_m. \tag{4}$$

The purpose of this paper is to show that right and left solvents can be constructed from latent roots and corresponding right and left latent vectors. It follows that each of the block roots contains a part of the latent structure of the matrix polynomial. A particular case of interest is the one where we can construct a set of block roots covering all the latent structure of $D(\lambda)$. This set is referred to as a complete set of block roots. The existence of right and left solvents is discussed in the paper as well as the existence of a complete set. Furthermore, the inverse $D^{-1}(\lambda)$ is given as a particular case of the block partial fraction expansion of a related rational matrix, this result being the main novelty of the paper.

2. PDEs with applications in Mechanics, Biology, etc.

Existence of waves for an atherosclerosis reaction-diffusion model

Narcisa Apreutesei

"Gheorghe Asachi" Technical University of Iasi, Romania
e-mail: napreut@gmail.com

We study a reaction-diffusion equation in an infinite two-dimensional strip with nonlinear boundary conditions. Such equation was suggested as a model of atherosclerosis development or other inflammatory diseases. In this context, nonlinear boundary conditions show the influx of white blood cells from blood flow into the tissue where the inflammation occurs. The existence of travelling waves for this equation is proved in the bistable case by the Leray-Schauder method. It is based on a topological degree for elliptic problems in unbounded domains and on a priori estimates of solutions. To this end, Fredholm property for associated linearized operators is proved and properness of the semilinear operators is studied in weighted Holder spaces.

Dirichlet problems with the mean curvature operator in Minkowski space

Cristian Bereanu

"Simion Stoilow" Institute of Mathematics of Romanian Academy, Bucharest, Romania
e-mail: Cristian.Bereanu@imar.ro

In this talk we present existence and multiplicity of classical positive radial solutions for Dirichlet problems with the mean curvature operator in Minkowski space. We use a combination of degree arguments, critical point theory for lower semicontinuous functionals and the upper and lower solutions method.

Hydromagnetic stagnation-point flow towards a moving plate

Bogac Bilgic[1], Serdar Baris[1], M. Salih Dokuz[2]

[1] *Faculty of Engineering, Department of Mechanical Engineering, Istanbul University, Turkey*
e-mail: bilgicbogac@gmail.com

[2] *Faculty of Mechanical Engineering, Istanbul Technical University, Turkey*

The paper aims to study the flow near the stagnation-point of an infinite plate moving in its own plane in the presence of a transverse uniform magnetic field. By using the appropriate transformations for the velocity components and temperature, the partial differential equations governing flow and heat transfer are reduced to a set of nonlinear ordinary differential equations. These equations are solved approximately using a numerical technique for the following two problems: (i) Two-dimensional stagnation-point flow on a mowing plate, (ii) Axisymmetric stagnation-point flow on a moving plate. The effects of the nondimensional parameters on the velocity components, wall shear stress, temperature and heat transfer are examined carefully.

Theoretical analysis on some simple flows of a binary mixture of incompressible Newtonian fluids

Bogac Bilgic[1], M. Salih Dokuz[2], Serdar Baris[1],

[1] *Faculty of Engineering, Department of Mechanical Engineering, Istanbul University, Turkey*
e-mail: bilgicbogac@gmail.com

[2] *Faculty of Mechanical Engineering, Istanbul Technical University, Turkey*

The problems involving some simple flows of a mixture of two incompressible Newtonian fluids are investigated theoretically. In the case of non-inertial flow, the partial differential equations governing the motion of the binary mixture are reduced to a system of coupled ordinary differential equations. The analytical solutions are obtained under very special conditions when all materials properties are assumed to be constants, and the only interaction force is drag resulting from relative velocity in a linear fashion for the following three problems: (i) the flow in a corner region with a moving wall, (ii) the flow between concentric cylinders, one of which is fixed and the other moving, (iii) the flow through a semi-annular channel.

Optical soliton perturbation by semi-inverse variational principle

Anjan Biswas[1,2], Luminița Moraru[3]

[1] *Department of Mathematical Sciences, Delaware State University, Dover, USA*

[2] *Department of Mathematics, Faculty of Science King Abdulaziz University, Jeddah, Saudi Arabia*

[3] *Physics Department, Faculty of Sciences and Environment, Dunărea de Jos University of Galaţi, Romania*
e-mail: biswas.anjan@gmail.com, Luminita.moraru@ugal.ro

This talk will be on perturbation of the nonlinear Schrödinger equation, with dual dispersion, that governs the propagation of solitons through nonlinear optical fibers. The semi-inverse variational principle is employed in order to obtain an analytical soliton solution in presence of the perturbation terms. There are three types of nonlinearity that will be studied. They are Kerr law, power law and the log law. The constraint conditions will naturally fall out in order for the soliton solutions to exist.

Thermal stresses in anisotropic porous cylinder

Emilian Bulgariu[1], Ghiba Ionel Dumitrel[2]

[1] *"Ion Ionescu de la Brad" University of Agricultural Sciences and Veterinary Medicine of Iaşi, Romania*

[2] *"Alexandru Ioan Cuza" University of Iaşi, Romania*
e-mail: bulgariu.emilian@gmail.com

We study the deformation of right porous cylinders subjected to a prescribed thermal field. We assume that the cylinder is filled by an inhomogeneous anisotropic porous material. We study two problems: the first one is the problem of extension-bending-torsion, when the thermal field is independent of the axial coordinate and the second one is the problem of extension-bending-torsion-flexure when the thermal field is considered linear in the axial coordinate. The considered problems are reduced to some generalized plane strain problems in the cross-section of the cylinder. Our analysis shows how the considered thermal fields influence the deformation of the porous cylinders.

The steady flow of an electroconducting fluid past a thin airfoil

Adrian Carabineanu

Faculty of Mathematics and Computer Sciences, University of Bucharest; "Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics, Bucharest, Romania
e-mail: acara@fmi.unibuc.ro

We use Fourier transforms in order to calculate the fundamental matrix of the MHD system consisting of linearized Euler and Maxwell equations. Then using a superposition of fundamental solutions we give integral representations for the presure,velocity and magnetic induction.The slip condition is imposed on the airfoil in order to obtain an integral equation for the pressure jump. Calculations are performed for the flat plate.

Perturbations of SG-hypoelliptic pseudo-differential operators on $L^p(\mathbb{R}^n)$ concerning essential spectra and semigroups of contractions

Viorel Catană

"POLITEHNICA" University of Bucharest, Romania
e-mail: catana_viorel@yahoo.co.uk

One of the main goal of this work is to feature some perturbation results regarding essential spectra and semigroups of contractions for a class of SG-hypoelliptic pseudo-differential operators ($SGH - \Psi DO$) introduced and studied by Camperi in [1]. Our investigation is done in the framework of L^p-Sobolev spaces $H^{s_1,s_2,p}(\mathbf{R}^n)$, $-\infty < s_1, s_2 < \infty$, $1 < p < \infty$ by means of minimal and maximal operators associated to ($SG - \Psi DO$) and is based on L^p-Sobolev estimates for ($SGH - \Psi DO$) which are the analogue of well-known Agmon-Douglis-Nirenberg estimates for elliptic partial differential operators. A variant for L^p-Sobolev spaces $H^{s,p}(\mathbf{R}^n)$, $-\infty < s < \infty$, $1 < p < \infty$ of a famous inequality namely Erhling's inequality is also used and a conjecture regarding this inequality is put forward. Finally a self-adjointness result is proved for such perturbation

of $(SGH - \Psi DO)$ on $L_2(\mathbf{R}^n)$ whose symbols are independent of spatial variables x in $\mathbf{R}^n$.
As an example of $(SGH - \Psi DO)$ such that we can apply the perturbation results from above we can take the operator $T_\sigma : \mathcal{S}(\mathbf{R}) \to \mathcal{S}(\mathbf{R})$ with the symbol $\sigma(x,\xi) = \langle\xi\rangle^\gamma x + i$ for (x,ξ) in $\mathbf{R}^2$ and $0 \leq \gamma < 1$, where $\langle\xi\rangle = (1 + |\xi|^2)^{1/2}$ defined by

$$T_\sigma u(x) = \sigma(x,D)u(x) = (2\pi)^{-1/2} \int_{\mathbf{R}} e^{ix\cdot\xi}(\langle\xi\rangle^\gamma x + i)\hat{u}(\xi)d\xi,$$

for $u \in \mathcal{S}(\mathbf{R})$, where $\mathcal{S}(\mathbf{R})$ is the Schwartz space.

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On the gravitation theory

Mircea Cazacu[1], Cabiria Andreian Cazacu[2]

1 "POLITEHNICA" *University of Bucharest, Faculty of Energetics, Bucharest, Romania*

2 *"Simion Stoilow" Institute of Mathematics of the Romanian Academy, Bucharest, Romania*
e-mail: Cabiria.Andreian@imar.ro

In this paper we try to formulate a gravitational theory, that have on its basis Isaac Newton's (1642-1727) idea [1][2], who interrogated how he explains himself the gravitational effect answered hypothesis non fingo this being the philosopher's occupation, but he as a science man, he deals only to discover the Mechanics laws. However interrogated in intimate manner, how can he to explain the gravitation, he supposes that this may be owed to an ether fluid more fine, that being attracted by the bodies is condensed on these, attracting only the other less bodies. In this aim we shall consider two plane sources of intensities Q_1, Q_2 to calculate the attraction force. One observes that the first term contained, as well as the third term, does not influence the result, because they have the zero values, while the central term with Q_1Q_2 has the following value, $F_{\text{gravit}} = Q_1Q_2/4\pi d^2$, [3], [4]. In conclusion we can observe, that the final expression is very similar with that of the gravitational atraction force between two bodies of masses M and m placed at a distance d between them.

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Variational inclusions for a class of nonlinear differential inclusions

Aurelian Cernea

Faculty of Mathematics and Computer Science, University of Bucharest, Bucharest, Romania
e-mail: acernea@fmi.unibuc.ro

We study nonlinear differential inclusions of the form

$$x' \in Ax + F(t,x), \quad x(0) = x_0,$$

where A is a m-dissipative operator on a Banach space X, $x_0 \in X$ and $F(.,.) : [0,T] \times X \to \mathcal{P}(X)$ is a set-valued map with nonconvex values that satisfies Fillipov type assumptions.
Using certain selection theorems, some existence results are obtained when X is separable and nonseparable.
We also establish several variational inclusions for solutions of the problem considered in separable Banach spaces.

Non-local evolution equations for lattice defects in finite elasto-plasticity

Sanda Cleja–Ţigoiu

Faculty of Mathematics and Computer Sciences, University of Bucharest, Romania
e-mail: stigoiu@yahoo.com

Within the second order finite elasto-plasticity we describe the lattice defects in terms of elements which characterize the plastic connection. The defects, like dislocations and disclinations, are characterized in terms of the Cartan torsion, while the micro cracks and micro voids are characterized in terms of the measure of the non-metricity of the plastic connections. We provide various type of non-local evolution equations for dislocations, disclinations and damage measure, which are compatible with the imbalance principle of the free energy. The provided evolution equations are strongly dependent on the expressions of the free energy density functions and involve the micro forces that have to be eliminated using the appropriate micro balance equations.

Interface cracks in fiber reinforced elastic composites

E. M. Crăciun[1], A. Carabineanu[2], D. Bigoni[3], M. Gei[3]

[1] *Ovidius University of Constanţa, Romania*

[2] *University of Bucharest, Romania*

[3] *University of Trento, Italy*
e-mail: emcraciun@yahoo.com; mcraciun@univ-ovidius.ro

A pre-stressed fiber reinforced elastic composite containing an interface crack in all three modes of classical fracture is studied. We consider the representation of the incremental fields for our initial deformed elastic composite, due to Guz and Soos (*Int.J.Engn.Sci.*1996). We formulate and solve the interface crack mathematical problem for all three modes of classical fracture, using a model of zero thickness linear interface similar to those used by Bigoni et al. (*Int. J. Solids Struct.* 1997, 1998, 2000). Using the boundary conditions of the interface crack in the pre-stressed elastic composite material, we solve the homogeneous and a nonhomogeneous Riemann - Hilbert problems. A nonhomogeneous linear complex differential equation having the unknown complex potential is obtained. We get the complex potentials, corresponding to each mode of the classical fracture. The incremental displacement and stress fields in the vicinity of the crack field are obtained using the complex potentials and the representation formulae.
The interaction of collinear interface cracks in the pre-stressed elastic composite is considered.

Study of some boundary value problems of micropolar thermoelasticity

Ion Crăciun

"Gheorghe Asachi" Technical University of Iaşi, Romania
e-mail: ion_craciun2008@yahoo.com

In this paper the linear theory of micropolar thermoelasticity of an isotropic and homogeneous solid in the case of stationary vibrations is considered. Then, by comparison, some interior and exterior boundary value problems of this theory are studied.

Wave propagation in the flat inhomogeneous waveguides

Irina Dmitrieva

Odessa National Academy of Telecommunications (ONAT), Ukraine
e-mail: irina.dm@mail.ru

We suggest explicit mathematical study of electromagnetic wave propagation in excited inhomogeneous linear isotropic media basing on the symmetrical differential Maxwell system [1]. The latter is investigated completely using inverse matrix operator construction [2] and is reduced to the general wave PDE (partial differential equation) regarding all scalar components of the unknown electromagnetic field vector intensities.
Criterion of equivalence between original system and wave equation is proved in the class of non generalized functions.
The relevant boundary problems of electromagnetic wave propagation in the flat inhomogeneous waveguides is proposed and solved analytically.

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Reduced-order approach for a pharmacokinetic model of subcutaneous injection of insuline analogues using POD-DEIM method

Gabriel Dimitriu

"Grigore T. Popa" University of Medicine and Pharmacy, Iaşi, Romania
e-mail: dimitriu.gabriel@gmail.com

Proper Orthogonal Decomposition (POD) is the mostly used and most successful model reduction technique. Unfortunately, for nonlinear PDEs, the efficiency in solving the reduced-order systems constructed from standard Galerkin projection with any reduced globally supported basis set, including the one from POD, is limited to the linear or bilinear part. A significant reduction in complexity is achieved by DEIM – a discrete variation of Empirical Interpolation Method (EIM), proposed by Barrault, Maday, Nguyen and Patera in: *An "empirical interpolation" method: Application to efficient reduced-basis discretization of partial differential equations*, C. R. Math. Acad. Sci. Paris, 339 (2004), 667-672.
In this work we study the application of DEIM combined with POD to provide the dimension reduction of a system of three nonlinear partial differential equations modeling the subcutaneous injection of insulin analogues. The equations describe the dynamics of long-acting insulin analogues in the hexameric form, followed by progessive dissociation into smaller units, dimers and monomers. Comparative numerical results are presented.

The two-scale convergence and the splitting operator method for convection-diffusion

Mihaela Dumitrache and Camelia Gheldiu

University of Piteşti
e-mail: mihaela.dumitrache@upit.ro

This paper presents the solution of a convection-diffusion problem which is considered on a composite material. In the first section of the paper we homogenize a heat transfer problem on a composite material with nonzero boundary conditions: the used technique is two-scale convergence. In the second part, for the homogenized convection-diffusion problem which was obtained in the first part, we apply a very good numerical method to approximate it: the splitting of the operator, more precisely, the Glowinski's scheme of the fractional step. A problem of this type is often seen to the components of some electronic devices.

A way to solve Goursat and Riemann problems for phase transforming thermo-elastic bars. Temperature effects in impact problems

Cristian Făciu

"Simion Stoilow" Institute of Mathematics of the Romanian Academy, Bucharest, Romania
e-mail: cristian.faciu@imar.ro

The longitudinal impact of two shape memory alloy (SMA) bars has been proposed in [1] as an effective mean for investigating the propagation of phase transformation fronts. A non-monotone stress-strain elastic relation has been used to describe a material capable of existing in three solid phases, the austenite phase (A) and two variants of martensite (M). In order to incorporate the important thermal effects due to the large amount of latent heat released or absorbed during a phase transformation we have considered in [2] a thermoelastic model for a SMA bar. This adiabatic thermodynamic framework, which permits the temperature to jump, leads to non-unique discontinuous solutions, even when the entropy inequality is satisfied. For such materials various *admissibility criteria* has been derived by introducing additional physical mechanisms. In [2], one has considered an augmented theory, which includes strain-rate and stress-rate effects, called Maxwellian rate-type effects, as dissipative mechanisms. We have established that the admissibility condition induced by the Maxwellian rate-type approach, coupled or not with Fourier heat conduction law is the *chord criterion with respect to the Hugoniot locus* in the strain-stress plane. By using a piecewise linear thermoelastic model for a three phase SMA bar and the chord criterion one determines unique solutions to the complete set of Riemann step-data problems. These consist of shock waves, wave fans and phase boundaries. Next, one uses systematically our Riemann solvers to construct solutions for the longitudinal impact of two SMA bars for a variety of impact conditions.

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New methods to provide exact solutions for some unidirectional motions of rate type fluids

Constantin Fetecău

Technical University Gheorghe Asachi of Iasi, Romania
e-mail: c_fetecau@yahoo.com

Based on three immediate consequences of governing equations corresponding to some unidirectional motions of rate type fluids, new motion problems are tackled for exact solutions. For generality, exact solutions are developed for shear stress boundary value problems of generalized Burgers

fluids. Such solutions, for which the shear stress instead of its differential expressions is given on the boundary, are lack in the literature for such fluids. Consequently, the first exact solutions for motions of rate type fluids induced by an infinite plate or a circular cylinder that applies a constant shear f or an oscillating shear to the fluid are here presented. In addition, all steady-state solutions can easily be reduced to known solutions for second grade and Newtonian fluids.

The homogenization of the controlled vibrations in a reinforced structure by thickness

Camelia Gheldiu, Raluca Mihaela Georgescu

University of Pitesti, Romania
e-mail: raluca.georgescu@upit.ro

The study of the exact intern controllability for the reinforced reticulated structures is presented. The convergence of the intern exact control for the case when δ tends to zero, where δ represents the thickness of the material in the periodicity cell is studied.

Laguerre collocation solutions to boundary layer type problems

Călin-Ioan Gheorghiu

"T. Popoviciu" Institute of Numerical Analysis, Cluj-Napoca
e-mail: ghcalin@ictp.acad.ro; ghcalin2003@yahoo.com

We introduce the Laguerre collocation method and show that it is a powerful tool in solving genuinely nonlinear, singularly perturbed, boundary value problems formulated on the half-line. First, we are concerned with solutions to classical Falkner-Skan (Blasius) problem. Second, a model equation for low Reynolds number flow, the so-called Cohen, Fokas, Lagerstrom model is considered. Third, the generalized eigenvalue problem corresponding to Ekman layer is analyzed. Eventually, we compute monotonously increasing solutions, the so-called bubble type solutions, to density profile equation. This equation is used in the description of the formation of microscopically bubbles in a non homogeneous fluid. We compare the results obtained in the last case with those carried out by shooting or polynomial collocation, both coupled with domain truncation. The collocation based on Laguerre functions avoid the domain truncation, exactly impose any kind of boundary conditions at infinity and resolve with high accuracy the sharpest interior or exterior layer of the solutions. The method is fairly easy implementable, reliable and more efficient in resolving the sharpest boundary layers than previously mentioned methods. Additionally, we solve with the same method the eigenvalue problem obtained by linearization around the constant solution of the fourth problem, which corresponds to the case of a homogeneous fluid (without bubbles), and observe that this solution is stable.

Computer algebra aided numerical solving nonlinear PDEs

Vladimir Gerdt, Yuri Blinkov

Joint Institute for Nuclear Research, Russian Federation
e-mail: gerdt@jinr.ru

First, we consider a five-parameter family of scalar nonlinear partial differential equations that contains the Korteveg-de Vries equation and the modified Korteveg-de Vries equation. To solve equations in this family numerically, we discretize them by using computer algebra assisted method based on combination of the finite volume method, numerical integration and difference elimination. To analyze quality of the obtained discretization we found a class of exact solutions and compared dynamics of numerical solutions and their exact counterparts. Second, we confront three finite difference approximations to the Navier–Stokes equations for the two-dimensional viscous incomressible fluid flows. Two of these approximations were generated by the same computer algebra assisted method as above. The third approximation was derived by the standard replacement of the temporal derivatives with the forward differences and the spatial derivatives with the central differences. We show that only one of these approximations is strongly consistent with the Navier–Stokes equations and present our numerical tests which show that this approximation has a better behavior than the other two.

Modified shape functions for the singular integration procedure when applying BEM for the 2D compressible fluid flow

Luminita Grecu

University of Craiova, Romania
e-mail: lumigrecu@hotmail.com

When applying Boundary Element Method (BEM) to solve boundary value problems of continuum mechanics, a very difficult challenge to overcome is to apply a good singular integration procedure, because almost all boundary integral formulations that arise have singularities. In this paper such a technique is applied in case of solving the Singular Boundary Integral Equation (SBIE) of the 2D compressible fluid flow around obstacles. The SBIE with sources distribution is considered in this paper and higher order boundary elements are used for its discretization. The singular integration procedure is based on modified shape functions, which are deduced by using series expansions for the basis functions choose for the local approximation models. So, this singular integration procedure is suitable for cases when higher order boundary elements are used for the SBIE discretization, as in our case. A computer code based on this method is made, and numerical solutions are found for some particular cases. The analytical checking shows very good agreements between the exact and the numerical solutions, fact that validates the proposed approach.

On the convergence of a class of approximating solutions of the Boltzmann equation in the whole space

Cecil P. Grünfeld[1,2] and Dorin Marinescu[2]

[1] *Institute of Space Science, Bucharest, Romania*

[2] *"Gh. Mihoc-C. Iacob" Institute of Mathematical Statistics and Applied Mathematics, Bucharest, Romania*
e-mail: grunfeld@spacescience.ro, marinescu.dorin@gmail.com

We consider a space-time discretized version of the Boltzmann equation, previously investigated in the context of the validation of the Nanbuś simulation method for the Boltzmann model in a bounded domain. Besides its usefulness in the validation of the Nanbuś method, the above considered space-discretized approximation may be of interest in developing new rigorous numerical methods for Boltzmann-like models. Our main result establishes sufficient conditions for the existence of solutions to the initial value problem for the Boltzmann equation in the whole space, which can be approximated convergently by the solutions of the aforementioned space-discretized Boltzmann equation.

Long-time behaviour for nonlocal problems

Liviu Ignat, Tatiana Ignat, Denisa Stancu-Dumitru

"Simion Stoilow" Institute of Mathematics of Romanian Academy, Bucharest, Romania
e-mail: liviu.ignat@gmail.com

In this talk we will present some nonlocal evolution problems that involve operators of the type:

$$\mathcal{L}u(x) = \int_{\mathbf{R}^d} J(x-y)(u(y)-u(x))\,dy$$

We analyze the asymptotic behaviour of the solutions of the following nonlocal convection-diffusion equation

$$u_t = J * u - u + G * u^2 - u^2.$$

The results are mainly obtained by scaling arguments and a new compactness argument that is adapted to nonlocal evolution problems.

Theorem. *Let $1 \le p < \infty$ and $\Omega \subset \mathbf{R}^d$ be an open set. Let $\rho : \mathbf{R}^d \to \mathbf{R}$ be a nonnegative smooth radial function with compact support, non identically zero, and $\rho_n(x) = n^d \rho(nx)$. Let $\{f_n\}_{n\ge 1}$ be a sequence of functions in $L^p((0,T)\times\Omega)$ such that*

$$\int_0^T \int_\Omega |f_n|^p \le M \tag{1}$$

and

$$n^p \int_0^T \int_\Omega \int_\Omega \rho_n(x-y)|f_n(t,x) - f_n(t,y)|^p dxdydt \le M. \tag{2}$$

1. If $\{f_n\}_{n\ge 1}$ is weakly convergent in $L^p((0,T)\times\Omega)$ to f then $f \in L^p((0,T), W^{1,p}(\Omega))$ for $p > 1$ and $f \in L^1((0,T), BV(\Omega))$ for $p = 1$.

2. Let $p > 1$. Assuming that Ω is a smooth bounded domain in $\mathbf{R}^d$, $\rho(x) \geq \rho(y)$ if $|x| \leq |y|$ and that

$$\|\partial_t f_n\|_{L^p((0,T),W^{-1,p}(\Omega))} \leq M \tag{3}$$

then $\{f_n\}_{n\geq 1}$ is relatively compact in $L^p((0,T)\times\Omega)$.

Computational analysis of the trajectories behavior for the mixing flow model

Adela Ionescu

University of Craiova, Department of Applied Mathematics
e-mail: adela0404@yahoo.com

This paper focuses on some new results in the computational analysis of the dynamic system associated to the mixing flow model. The mixing theory appears in an area with far from complete solving problems: the flow kinematics. Its methods and techniques developed the significant relation between turbulence and chaos. The turbulence is an important feature of dynamic systems with few freedom degrees, the so-called "far from equilibrium systems".
Studying a mixing for a flow implies the analysis of successive stretching and folding phenomena for its particles, the influence of parameters and initial conditions. In the previous works, the study of the 3D non-periodic models exhibited a quite complicated behavior. In agreement with experiments, they involved some significant events - the so-called "rare events". The variation of parameters had a great influence on the length and surface deformations.
In this paper there is continued the author's work started in recent years on turbulent mixing model. The 3D mixing flow phenomena had very few analytical and experimental tests, in order to prove that this model has a far from equilibrium behavior and it is very sensitive to the variation of its parameters and orientation versors. The comparisons of deformation (in length and surface) efficiency of 3D versus 2D case brought a very rich panel of random distributed events. An important partial conclusion was that the mixing, and especially the turbulent mixing, is introduced at irrational values of length and surface versors.
There is used a modern mathematical soft, MAPLE11, which has a lot of computational appliances. Crossing over from 2D to 3D case, it is easy to deduce the requirement of a special analysis of the influence of parameters on the behavior of this complex mixing flow. Since in the precedent work there were obtained numeric plots standing for the images of the deformation efficiency, in this paper the standpoint is a little changed. Using specific computational appliances, from "DEplot" package, there is started a computational comparison analysis of the flow behavior in 2D and 3D case. The basic analysis directions followed in the paper are focused on the 2D perturbed mixing model versus the 3D mixing flow model.
Thus, the panel of random distributed events would be completed with new special events and new data would be collected for the qualitative analysis of the mixing phenomena.

Regularized meshless solution for inverse boundary value problems in static thermoelasticity

Andreas Karageorghis[1], Daniel Lesnic[2], Liviu Marin[3,4]

[1] *Department of Mathematics and Statistics, University of Cyprus, Nicosia, Cyprus*

[2] *Department of Applied Mathematics, University of Leeds, Leeds, UK*

[3] *Institute of Solid Mechanics, Romanian Academy, Bucharest, Romania*

[4] *Centre for Continuum Mechanics, Faculty of Mathematics and Computer Science, University of Bucharest, Bucharest, Romania*
e-mail: marin.liviu@gmail.com

We propose the stable reconstruction of the boundary and internal thermoelastic fields for two- and three-dimensional solids from incomplete boundary data and additional noisy boundary or internal measurements. In order to achieve this, the method of fundamental solutions (MFS) is combined with the method of particular solutions (MPS) and the Tikhonov regularization method, while the optimal choice of the regularization parameter is made via a suitable criterion. The numerical stability, convergence, consistency and computational efficiency of the proposed method are investigated.

The Interval Lattice Boltzmann Method for transient heat transfer in a silicon thin film

Anna Korczak, Alicja Piasecka-Belkhayat

Institute of Computational Mechanics and Engineering, Poland
e-mail: anna.korczak@polsl.pl

In this paper the numerical modelling of heat transfer in a two-dimensional crystalline solid is considered. It is assumed that some parameters (the relaxation time and the boundary conditions) appearing in the mathematical model of the problem analyzed are given as interval numbers. The problem discussed has been solved using the interval form of the lattice Boltzmann method using the rules of the directed interval arithmetic.

An optimal design of the wind turbine blade geometry adapted to a specific site

Zine Labidine Mahri, Said Zid

University of Constantine, Algeria
e-mail: zlmahri@hotmail.com

The aerodynamic modeling of the wind turbine blades constitutes one of the most important processes in the design of the turbine. The rotor blades are the most important part of this turbine because of their aerodynamic shape and profiles that play the main role in extracting the wind energy.

The aerodynamic modeling is used in order to estimate the aerodynamic loads and the wind extracted power. This modeling must be done for a given wind speed and a given rotor blades.
The design of the blade geometry must provide the optimal shape of the rotor blade capable to produces the maximum extracted power.
In order to determine the optimal shape of the blades, one must compute the optimal parameters of the blade geometry such as the chord length distribution, the thikness and the twist angle distribution along the blade span.
In the aerodynamic modeling two aerodynamic theories are used, the first one is the axial momentum theory and the second is the blade element theory. In the first theory, the flow is considered to be completely axial, while in the second theory the effect of wake rotation is included, assuming that the flow downstream rotates. The momentum theory that employs simply the mass and momentum conservation principles cannot provide alone the necessary information for the rotor design. However, the blade element theory that uses the angular momentum conservation principal, gives complementary information about the blade geometry such as airfoil shape and twist distribution. When both theories are combined the aerodynamic loads and the produced power can be obtained. In order to compute the optimal parameters of the blade geometry that give the maximum power, an iterative algorithm is used until the maximum value of extracted power is reached.
This design must be done for a specific aerodynamic profile and a specific site, since we must use the characteristic wind speed data that gives the maximum available power in a given site. This characteristic wind speed is determined by statistical study of meteorological data. The design can be repeated for different sites and profiles. This design has a great impact on the turbine efficiency and consequently on its economical feasibility.

Two-scale model of cancellous bone structure

Przemyslaw Makowski

Institute of Computational Mechanics and Engineering, Silesian University of Technology, Gliwice, Poland
e-mail: przemyslaw.makowski@polsl.pl

This paper is devoted to multiscale model of cancellous (trabecular) bone structure. Finite element method (FEM) and numerical homogenization algorithm are used. The geometry of bone trabeculas was simplified so the heterogeneous macro-scale model of the tissue became a globally periodic structure. The size and shape of the numerical model is typical for bone specimens extracted from the head of human femur - cube with edge length of 10 mm.

A variational method for solving a class of boundary value problems arising from Contact Mechanics

Andaluzia Matei

University of Craiova, Romania
e-mail: andaluziamatei@inf.ucv.ro, andaluziamatei2000@yahoo.com

The present work is based on the recent article [1]. We focus on a variational method for solving a class of boundary value problems arising from Contact Mechanics. The variational support is an abstract mixed variational problem, the set of the Lagrange multipliers being dependent on the

solution. Firstly, we discuss the existence of the solution of the abstract problem. The discussion is based on a fixed point technique for weakly sequentially continuous maps. Next, we apply the abstract result to the weak solvability of a boundary value problem which models the antiplane frictional contact between a cylindrical deformable body and a rigid foundation. In addition, some 3D contact models leading to mixed variational problems with solution-dependent sets of Lagrange multipliers are indicated.

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On the numerical approximation of the phase-field system with non-homogeneous Cauchy-Neumann boundary conditions. Case 1D

Costică Moroșanu, Ana - Maria Moșneagu

Faculty of Mathematics, University "Al.I. Cuza", Iaşi
e-mail: c_moro@uaic.ro,anamaria.mosneagu@uaic.ro

A *scheme of fractional steps type*, associated to the nonlinear phase-field transition system in one dimension, is considered in this paper. To approximate the solution of the linear parabolic system introduced by such approximating scheme, we consider three finite differences schemes: **1-IMBDF** (first-order **IM**plicit **B**ackward **D**ifferentiation **F**ormula), **2-IMBDF** (second-order IMBDF) and **2-SBDF** (second-order **S**emi-implicit BDF). A study of stability and the numerical efficiency analysis of this new approach, are performed too.

Macroeconomic modeling of the economic activity, case of the Republic of Moldova

Elvira Naval

Institute of Mathematics and Computer Sciences of the MAS, Moldavia
e-mail: nvelvira@math.md

An approach to the macroeconomic models using in an economy in transition is presented in this article. An impressive variety of macroeconomic models previously studied by the author is structured as a model base. Database corresponding to these models is presented. A stochastic macroeconomic model is examined.

The theoretical approach of mechanical face seals hydrodynamic operation model

Constantin Onescu, Nicolae Popa

University of Piteşti, Romania
e-mail: constantin.onescu@upit.ro

The mechanical face seals are used in the mechanical systems whose proper operation depends, in a decisive manner, on the mechanical seal, even if the cost price is minimal regarding the whole system. This fact has determined the physicians, the mathematicians and the engineers to inquire more of these mechanical seals. The approximation of the thin films which were introduced by O. Reynolds in 1886 is the fundamental base of theoretical studies in this field. A well knowledge of the pressure field is therefore necessary in all the experimental studies, theoretical or numerical. Until now, the experimental study has been very difficult due to the measurement in the film that separates the two surfaces, when it exists, they are approximate. This fact is due to the complexity of the phenomena that occur and also due to their different nature (HD, THD, elastic, etc.). The typical configuration of oil cone (angle of inclination ŏ3b1) studied at the known hydrodynamic lubrication and under the name of skate with fix geometry, is typical for the axial housing. We meet this configuration at the mechanical face seal. Integrating the concluded equation for the particular case of the mechanical seal, we will determine the hydrodynamic pressure variation from the existing oil film between the main elements of the seal.

A reverse Saffman-Taylor instability

Gelu Paşa

"Simion Stoilow" Institute of Mathematics of Romanian Academy, Bucharest, Romania
e-mail: Gelu.Pasa@imar.ro

A new instability phenomenon was observed by Chan and Liang (1997) in the case of a particular vertical Hele-Shaw cell. When the inner part of the Hele-Shaw plates are coated with a thin surfactant layer and the cell is dipped into a pool with the same surfactant fluid, the interface pool-air becomes unstable, even the displaced fluid is less viscous (air). This is in contradiction with the Saffman - Taylor criterion obtained in the clean case. The new instability was partially confirmed by Krechetnikov and Homsy (2004), who pointed out that "is no dependence of the stability parameter on the displacing speed" (first lines of sect. 4.3) and that the variable surfactant concentration on the interface seems be the principal mechanism producing the new instability. We confirm these results by direct calculation. The thin fluid film is deposed on the plate by using the drag-out coating procedure, studied by Landau and Levich (1942). We analyse the stability of the free-surface of the film when the surface tension is *not constant*, due to the presence of a surfactant. We show that an instability appears in the horizontal direction (parallel with the liquid - pool surface) and obtain the expression of the growth constant in terms of the derivative of the surface tension as function of the surfactant concentration.

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The pulsatory liposome acting as a biomicroengine device

Dumitru Popescu

"Gh. Mihoc-C. Iacob" Insitute of Mathematical Statistics and Applied Mathematics, Bucharest, Romania
e-mail: popescu1947@yahoo.com

In this paper, we have considered the problem of the pulsatory lipid vesicle. In certain conditions, the vesicle swelling start again and it2019s the evolution of a lipid vesicle under positive osmotic stress may be a cyclic process. We have named it as a pulsatory lipid vesicle. The length time of each cycle increases with the rank of the cycle. Also, it is very important for biotechnological applications to know more about the solute amount delivered during each cycle. The pulsatory liposome may be regarded as a two stroke engine using the osmotic solute as a fuel. It is energized initial by establishment of transmembrane gradient of osmotic solute. The osmotic solute plays the role of fuel, which is consumed due to its deliver in external medium. But the consumed fuel is usefulness material for biotechnological applications.

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Mathematical Modelling of the Piezoelectric Effect of Human Long Bones

Dumitru Popescu[1], Iuliana Paşol[2]

[1] *"Gh. Mihoc-C. Iacob" Insitute of Mathematical Statistics and Applied Mathematics, Bucharest, Romania*

[2] *Department of Physiology and Biophysics, Faculty of Biology, University of Bucharest, Romania*
e-mail: popescu1947@yahoo.com, pasoliuliana@yahoo.com

In this work, we study the piezoelectric effect of human bones. The osseous tissue of bones have a solid phase organised as matrix composed from a network of colagen fibriles surounded by a layer of proteoglicans and composite material formed from minerals, as calcium phosphate (also

named calcium hydroxylapatite), arranged in microcrystals disolved in the colagen network. Both, the composite material and colegen network assures the brittle and elasticity of bones, respectively. Our attention is focused on the long bones which are supposed to the mechanical forces due to the external weights (inluding the human body) and internal action of muscles. We considered a long bone as big crystal composed from a lot of microcrystals. Under an applied mechanical stress (the bone is squeezed or stretched) a moving of inside electric charges take place and, as a consequence, an electric tension appears accros the bone. Taking into account by the mechanical structure and the ele ctrical properties of osseous material, in this paper we have made a mathematical modelling of the piezoelectric effect which appears in long human bones. Some biomedical applications are mentioned.

Some remarks on eigenvalue problems for the Laplace operator

Valentina Proytcheva[1], Snezhana Hristova[2]

[1] *Technical University of Sofia, branch Plovdiv, Bulgaria*

[2] *Plovdiv University, Plovdiv, Bulgaria*
e-mail: vproicheva@hotmail.com

This talk is concerned with the first nontrivial eigenvalue μ_1 of the free membrane problem

$$\Delta\nu_1 + \mu_1\nu_1 = 0 \text{ in } \Omega, \quad \text{and } \frac{\delta\nu_1}{\delta n} = 0 \text{ on } \Omega,$$

where $\frac{\delta\nu_1}{\delta n}$ is the outward normal derivative of nu_1 on the boundary of a bounded convex domain Ω in R^N, $n \geq 2$. A new proof of Paune-Weinberger inequality $\nu_1 \geq \left(\frac{\pi}{D}\right)^2$, where D is the diameter of Ω, will be presented.

Numerical Solution of Some Three-Dimensional Electrostatic Problems

Paţiuc Vladimir, Rîbacova Galina

Moldova State University, Chişinău, Republic of Moldova
e-mail: ribacus@yahoo.com

We consider some three-dimensional problems from the theory of electrostatics. The main focus is paid to the problem of determining the parameters of the electric field in the vicinity of the L-shaped support of a high voltage transmission line. The mathematical formulation of the problem is to determine the three-dimensional distribution of the potential and the intensity of the electrostatic field in a multiply-connected region, where the relative permittivity takes the piecewise constant values. In order to determine these parameters we solve the differential boundary-value problem numerically by means of finite volume method. The domain under consideration contains objects, whose geometric dimensions vary widely. Therefore, to achieve sufficient accuracy, we propose some optimization procedure for grid construction. The results of numerical simulation are represented and discussed.

Singular limits of solutions to the Cauchy problem for second order linear differential equations with positive powers of a positive defined operator

Galina Rusu

Moldova State University, Chişinău, Republic of Moldova
e-mail: rusugalinamoldova@gmail.com

Let H be a real Hilbert space endowed with the inner product $(\cdot,\cdot)$ and the norm $|\cdot|$. Let $A_i : D(A_i) \to H$, $i = 0, 1$, be two linear self-adjoint, positive defined operators. Consider the following Cauchy problem:

$$\begin{cases} \varepsilon\left(\ddot{u}_\varepsilon(t) + A_1 u_\varepsilon(t)\right) + \dot{u}_\varepsilon(t) + A_0 u_\varepsilon(t) = f_\varepsilon(t), \quad t \in (0,T), \\ u_\varepsilon(0) = u_{0\varepsilon}, \quad \dot{u}_\varepsilon(0) = u_{1\varepsilon}, \end{cases}$$

where $\varepsilon > 0$ is a small parameter($\varepsilon \ll 1$), $u_\varepsilon, f_\varepsilon : [0,T) \to H$.
We investigate the behavior of solutions $u_\varepsilon(t)$ to the perturbed system (1) when $\varepsilon \to 0$, $u_{0\varepsilon} \to u_0$ and $f_\varepsilon \to f$. We establish a relationship between solutions to the problem (1) and the corresponding solutions to the following unperturbed system:

$$\begin{cases} \dot{v}(t) + A_0 v(t) = f(t), \quad t \in (0,T), \\ v(0) = u_0. \end{cases}$$

Approximate schemes for fractional diffusion equation with two space variables

Ivan Secrieru

Moldova State University, Chişinău, Republic of Moldova
e-mail: secryah@yahooo.com

The process to determinate a border domain in plane, where the concentration of a dangerous substance, transported in atmosphere by some physical factors do not exceed the sanitary quota, is studied in [1]. Other problems of transport of a substance in some medium occur in many domains, including biology, hydrogeology, semiconductor and finance. The mathematical modeling of these problems depend of the domain where this process is studied. The main factors in modeling the transport of any substance in atmosphere are the diffusion process, absorbtion of substance and advection-convection process. The classical model of this problem with one space variable use the ordinary derivatives (cf.[2]). In recent years many authors use the fractional space derivative to modeling such process [3]. In this paper I consider following equation

$$\frac{\partial \varphi}{\partial t} - d_+(x)\frac{\partial^\alpha \varphi}{\partial_+ x^\alpha} - d_-(x)\frac{\partial^\alpha \varphi}{\partial_- x^\alpha} = f(x,t),$$

$$\varphi(x,0) = s(x), \qquad \varphi(L,t) = 0, \qquad \varphi(R,t) = b(x), \tag{1}$$

where $1 < \alpha \leq 2, L < x < R, 0 \leq t \leq T, d_+(x) \geq 0, d_-(x) \geq 0$. Left-hand (+) and right-hand (-)fractional derivatives of order α in (1) are defined by Riemann-Liouville formulas

$$\frac{\partial^\alpha \varphi}{\partial_+ x^\alpha} = \frac{1}{\Gamma(m-\alpha)})\frac{\partial^m}{\partial x^m}\int_L^x \frac{\varphi(\xi,t)d\xi}{(x-\xi)^{\alpha+1-m}},$$

$$\frac{\partial^\alpha \varphi}{\partial_- x^\alpha} = \frac{(-1)^m}{\Gamma(m-\alpha)})\frac{\partial^m}{\partial x^m}\int_x^R \frac{\varphi(\xi,t)d\xi}{(\xi-x)^{\alpha+1-m}}, \qquad (2)$$

where m is an integer such that $m-1 < \alpha \leq m$.
To construct an approximate scheme for (1) let τ be the time step, $t_n = n\tau$ and h is a space step of grid, $x_i = L + ih, \qquad i = 0, 1, 2, ..., M$. Let φ_i^n be the numerical approximate value of exact solution $\varphi(x_i, t_n)$.To discretize the fractional derivatives we will apply the Grunwald formula (cf.[3]): Using the weighted finite difference method with the parameter r at the points $t_n$and t_{n+1}.The approximate scheme of problem (1) can be written as a matrix form

$$[I - (1-r)A]\Phi^{n+1} = (I + rA)\Phi^n + \tau F^n, \qquad (3)$$

where $\Phi^n = [\varphi_1^n, \varphi_2^n, ..., \varphi_{M-1}^n]$.
Theorem 1. *The scheme (3) for problem (1) are unconditionally stable for* $0 < r \leq \frac{1}{2}$ *and stable for* $\frac{1}{2} < r \leq 1$ *with a supplementary restriction at choice of steps h and* τ.
Using the decomposition principle an analogous result hold in the case of equation with two space variables.

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Implicit parametrizations and applications in shape optimization

Dan Tiba

"Simion Stoilow" Institute of Mathematics of Romanian Academy, Bucharest, Romania
e-mail: Dan.Tiba@imar.ro

We discuss a differential equations treatment of the implicit functions problem. Our approach allows a precise and complete description of the solution, of continuity and differentiability properties. The critical case is also considered. The investigation is devoted to dimension two and three, but extensions to higher dimension are possible.

Modelling the electron movement in electric field - nonlinear aspects

Eugen Vasile

"POLITEHNICA" *University of Bucharest, Romania*
e-mail: evasevas@yahoo.com, evas@alpha.imag.pub.ro

When studying the oscillations of an electron forced by the electric field of an electromagnetic wave, the analytical integration of the equation of motion is possible only in some cases and is pretty difficult in the case of spatial non linearities of either the electric field or the environment (where the electron is only quasi free). An example of analytical calculi followed by numerical representations is done. In the general case of numerical integration, targeting the optical refractive index, a non-autonomous two-dimensional dynamical system is provided and solved with specific MathCAD subroutines. Exact analytical expression for the frequency dependence of the refractive index is obtained (useful when studying the plasmon behavior under laser radiation incidence) as well as numerical representations with phase portraits.

3. ODEs; Dynamical Systems

Global classification of configurations of singularities for quadratic differential systems with three real finite distinct singularities

J.C. Artés[1], J. Llibre[1], D. Schlomiuk[2], N. Vulpe[3]

[1] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, Spain*

[2] *Département de Mathématiques et de Statistiques, Université de Montréal, Canada*

[3] *Institute of Mathematics and Computer Science, Academy of Science of Moldova, Chişinău, Republic of Moldova*

e-mail: dasch@dms.umontreal.ca, nvulpe@gmail.com

In this work we consider the problem of classifying all configurations of singularities, both finite and infinite of quadratic differential systems, with respect to the *geometric equivalence relation* defined in [1]. This relation is finer than the *topological equivalence relation* which does not distinguish between a focus and a node or between a strong and a weak focus or between foci of different orders. Such distinctions are however important in the production of limit cycles close to the foci in perturbations of the systems. The notion of *geometric equivalence relation* of configurations of singularities allows us to incorporates all these important geometric features which can be expressed in purely algebraic terms. This equivalence relation is also finer than the *qualitative equivalence relation* introduced in [2]. The *geometric classification* of all configurations of singularities, finite and infinite, of quadratic systems was initiated in [3] where the classification was done for systems with total multiplicity m_f of finite singularities less than or equal to one. In this article we continue the work initiated in [3] and obtain the *geometric classification* of singularities, finite and infinite, for the subclass of quadratic differential systems possessing three distinct real finite singularities. We obtain 146 *geometrically distinct* configurations of singularities for this family. We also give here the global bifurcation diagram, of configurations of singularities, both finite and infinite, with respect to the *geometric equivalence relation*, for this class of systems. The bifurcation set of this diagram is algebraic. The bifurcation diagram is done in the 12-dimensional space of parameters and using the algebraic method of invariants of differential systems, developed by Sibirskii and his disciples (see for instance [4–8]), it is expressed in terms of polynomial invariants. The results can therefore be applied for any family of quadratic systems in this class, given in any normal form. Determining the geometric configurations of singularities for any such family, becomes thus a simple task using computer algebra calculations.

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Averaging Method in Multifrequency Systems with Delay

Yaroslav Bihun, Inessa Krasnokutska

Yuriy Fedkovych Chernivtsi National University, Ukraine
e-mail: yaroslav.bihun@gmail.com

Quite often mathematical models of oscillation processes are systems of differential equations, in which part of variables evolves with time slowly (slow or amplitude variables), while the others – fast (fast or phase variables). In many cases, by carrying in a small parameter $\varepsilon > 0$, this systems can be written as

$$\frac{da}{d\tau} = X(\tau, a, \varphi, \varepsilon), \quad \frac{d\varphi}{d\tau} = \frac{\omega(\tau, a)}{\varepsilon} + Y(\tau, a, \varphi, \varepsilon),$$

where $a \in D \subset R^n$, $\varphi \in R^m$; $m \geq 2$; ε is a small positive parameter, the vector functions X and Y are periodic in the variables φ with the period 2π. The main problem arising in the study of properties of solutions of the system is the problem of resonance relations between the components of the variable frequency vector $\omega(\tau, a)$ [1].
Noether boundary problems for ordinary differential equations and equations with lateness were researched in the work [2].
In this work we substantiate the method of averaging for the initial and boundary-value problem with linearly transformed argument and the frequency vector depending on slow variable. The obtained results are used for the investigation of the existence of the method of averaging for some boundary-value problems.

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Jacobi multipliers and first integrals for nonautonomous differential equations

Adriana Buică, Isaac Garcia

"Babes-Bolyai" University of Cluj-Napoca, Romania
e-mail: abuica@math.ubbcluj.ro

In this paper we consider nonautonomous differential systems of arbitrary dimension and first find expressions for their inverse Jacobi multipliers and first integrals in some nonautonomous invariant set in terms of the solutions of the differential system. Given an inverse Jacobi multiplier V, we find a relation between the Poincaré translation map Π at time T that extends to arbitrary dimensions the fundamental relation for scalar equations, $V(T,\Pi(x)) = V(0,x)\Pi(x)$, found in Trans. Amer. Math. Soc. 362 (2010), 3591-3612. We also give results that guarantee the existence of continua of T-periodic solutions for T-periodic systems.

An algebraic approach of homogeneous quadratic differential systems

Ilie Burdujan

"Ion Ionescu de la Brad" University of Agricultural Sciences and Veterinary Medicine of Iaşi, Romania
e-mail: ilieburdujan@yahoo.com

The study of Homogeneous Quadratic Differential Systems (shortly, HQDSs) can be approached along several ways in accordance with the tools used for it. The natural tool to use seems to be the classical invariant theory. Unfortunately, this way becomes very burdensome as soon as we pass to study of HQDSs on P3. Another way was proposed by L. Markus who remarked that a commutative algebra can be naturally assigned to any HQDS. In fact there exists a special interplay between HQDS-theory and commutative algebra-theory. Indeed, both commutative algebras and homogeneous quadratic dynamical systems are - each of them - naturally associated with covariant symmetric (1, 2)-tensors. Therefore, it is natural to put in correspondence the algebra and the HQDS that have the same structural (1,2)-tensor. This correspondence induces a 1-to-1 mapping between the classes of isomorphic commutative algebras and the classes of affine-equivalent HQDSs. It follows that the structural properties of the commutative algebras are intimately connected with the qualitative properties of the homogeneous quadratic differential systems. As a corollary, it results that the problem of classification, up to an affine equivalence, of quadratic differential systems is equivalent to the problem of classification, up to an isomorphism, of commutative algebras. In its turn, the study of any algebra, which is not simultaneously associative and commutative, is realized along two ways that depend on the existence or on the absence of any derivation. The existence of a derivation significantly facilitates the structural study of any algebra. By using algebraic tools, the classification up to an isomorphism of 3-dimensional commutative algebras, having at least a derivation, was performed. It is more difficult to achieve the study of the algebras without derivations. Such studies are based on the use of a complete system of the deviations from associativity and commutativity of the analyzed al gebras, which allow to organize their underlying vector spaces as homogeneous systems in Yamaguti's sense (i.e., they are

the tangent algebras to a homogeneous space). Notice that, in general, the homogeneous system associated to any algebra does not characterize the algebra, i.e. it does not allow the recovering of the structure tensor of the algebra. Indeed, any such an homogeneous system "ignores" (i.e. it is trivial on) the subalgebras that are both associative and commutative, if they exist. Yet, these homogeneous systems bring the most part of information about their "native" algebras. The obtained results are used for analyzing some of the usual mathematical models, formulated as quadratic differential systems, which arise in ecology, epidemiology, meteorology, physics, etc. The aim of these examples is to illustrate the power of the homogeneous systems as a useful tool for the structural studies of nonassociative algebras. In our study we do not use the classical theory of algebraic invariants which assumes to identify polynomial functions that have to be invariant under the action of an appropriate transformation group. We use some functions that have only a special property of invariance, the so called VNV-property, i.e. their property to be either zero or nonzero in any basis. Moreover, we are well helped by subalgebra lattices that are invariant objects of analyzed algebra. These lattices allow to identify partitions of the ground spaces of algebras as well as partitions of the set of solutions of the corresponding HQDSs. The obtained results concern the complete classification, up to an isomorphism, of algebras on R3 having at least a derivation. Several results are obtained for classes of algebras having no derivation.

Stiffness of the linear diffusion and wave-type partial differential equations

Elisabete Alberdi Celaya

Department of Applied Mathematics, EUIT de Minas y Obras Públicas, Universidad del País Vasco UPV/EHU, Spain
e-mail: elisabete.alberdi@ehu.es

After applying the Finite Element Method (FEM) to the one dimensional diffusion-type and wave-type linear Partial Differential Equations (PDEs) with boundary conditions and initial conditions, a first order Ordinary Differential Equation (ODE) system and a second order one are obtained respectively. This latter can be reduced to a first order system. We have studied the stiffness of the resulting systems.
We have numerically proved that the ODE system obtained from the diffusion equation presents more stiffness when the number of elements of the discretization or the thermal diffusivity are increased, or when the length of the rod is shortened. The relation between the eigenvalues of the ODE systems obtained from the diffusion and the wave equations has also been proved, being the eigenvalues of the second system the square root of the eigenvalues of the first one. Taking into account this result and the conclusions achieved for the diffusion equation, it is concluded that the ODE system that results from the wave-type PDE is more stiff as the number of elements or the speed of propagation are increased, or when the length of the string is shortened

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Using Numerical Differentiation Formulae in superfuture-point schemes

Elisabete Alberdi Celaya[1], Juan José Anza Aguirrezabala[2]

[1] *Department of Applied Mathematics, EUIT de Minas y Obras Públicas, Universidad del País Vasco UPV/EHU, Spain*

[2] *Department of Applied Mathematics, ETS de Ingeniería de Bilbao, Universidad del País Vasco UPV/EHU, Spain*
e-mail: elisabete.alberdi@ehu.es

One of the directions carried out in the search of higher order and more stable methods when solving Initial Value Problems (IVPs) has been the use of off-step points called superfuture points. Cash has introduced methods using superfuture points to solve stiff IVPs. These methods are known as extended BDF (EBDF) and modified extended BDF (MEBDF). They use two BDF predictors and one implicit multistep corrector. Both methods are A-stable up to order 4 and $A(\alpha)$-stable up to order 9, and the class MEBDF has better stability properties than the class EBDF.
We have modified the EBDFs and MEBDFs using the NDFs as predictors instead of the BDFs. We have done it in three different ways: changing both predictors of the EBDF and MEBDF schemes, changing only the first predictor and changing only the second one. All the new methods are A-stable up to order 4 and the new methods of the MEBDF scheme have better stability properties than the MEBDF has.

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Centers in cubic differential systems with invariant algebraic curves

Dumitru Cozma

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: dcozma@gmail.com

We consider cubic polynomial differential systems

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1}$$

where $P(x, y)$, $Q(x, y)$ are real and coprime polynomials. Let the origin $O(0, 0)$ be an isolated singularity of (1) with purely imaginary eigenvalues ($\lambda_{1,2} = \pm i$, $i^2 = -1$), i.e. a singular point of a focus or a center type.
The problem of distinguishing between a center and a focus for (1) is known as the center-focus problem. Although it dates from the end of the 19th century, it is completely solved only for quadratic systems, cubic symmetric systems and a few particular cases in families of polynomial differential systems of higher degree. In the last years, the relations established between invariant algebraic curves, Lyapunov quantities and integrability have provided new possibilities for approaching to the center-focus problem (see, for instance [1], [2]).
In this talk we discuss the difficulty of the center-focus problem and present the main mechanisms for proving the existence of a center in (1) with invariant algebraic curves: Darboux integrability and rational reversibility. Advances in computational algebra and computer technology made possible the application of these two mechanisms in solving of the center-focus problem for cubic differential systems with invariant straight lines and invariant conics, see, for example the recent monograph [3].

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A new characteristic of attractors of Iterated Function Systems

Valeriu Guţu

Moldova State University, Chişinău, Republic of Moldova
e-mail: vgutu@yahoo.com

Many authors have stated various conditions for a compact set to be an attractor, as well as some examples of compacta, which can not serve as the attractor of a hyperbolic Iterated Function System (IFS) (see, e.g., [1] and the bibliography therein). We study this problem from another

point of view, namely:
1. Which compacta can serve as attractors of hyperbolic IFSś?
2. Given a compact set, what is the minimal number of contractions of a hyperbolic IFS (provided it exists) needed to obtain this set as attractor?
For such compacta we put in discussion a new characteristic.
Let K be a nonempty compact set in a complete metric space. If K can be represented as the attractor of a hyperbolic IFS, consisting of a finite number of contractions, we denote by $\beta(K)$ the smallest number of such required contractions; otherwise $\beta(K) = +\infty$.
We study the characteristic β for plane compacts.
Theorem. *For any plane convex compact set K one has $\beta(K) \leq 3$.*
Theorem. *Let $K = K_1 \cup \cdots \cup K_m$ be a finite union of plane convex compact sets. Then K can be represented as the attractor of a finite hyperbolic IFS. Moreover, $\beta(K) \leq \beta(K_1) + \cdots + \beta(K_m)$.*
We show by examples that the last inequality can be an equality, as well as a strong inequality.
Remark. *In [2] it is shown that a disk can be represented as attractor of an IFS on the plane using nine contractions. By Theorem 1 a disc can be obtained using only three contractions. We show that this number of contractions cannot be decreased.*

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ψ- exponenatial stability for differential equations with "maxima"

Snezhana Hristova[1], Valentina Proytcheva[2]

[1] *Plovdiv University, Plovdiv,Bulgaria*

[2] *Technical University of Sofia, branch Plovdiv, Bulgaria*
e-mail: snehri@uni-plovdiv.bg, vproicheva@abv.bg

In the mathematical simulation in various important branches of control theory, pharmacokinetics, economics, etc., one has to analyze the influence of the maximal deviation of the regulated quantity. Such kind of problems could be adequately modeled by differential equations that contain the maxima operator and they are known as differential equations with "maxima" .
The purpose of this paper is to establish sufficient conditions for ψ-exponential stability for a class of nonlinear differential equations with "maxima". Modified Razhumikhin method and comparison results have been applied. A comparison scalar ordinary differential equation has been employed.

Sufficient conditions for Lurie problem in the context of Popov criterion applied to roll dynamics with delay

Ionel Cristinel Iorga[1], Adrian Toader[2]

[1] *University of Craiova, Romania*

[2] *"Elie Carafoli" National Institute, Romania*
e-mail: ioneliorga@yahoo.com, atoader@incas.ro

This paper is motivated by the study of absolute stability for systems with one non-linearity in the feedback path. Starting from the open-loop transfer function and using the Popov criterion sufficient conditions for absolute stability are set in the context of one delay variable. Also, a discussion is made in the case when the non-linearity is not active through the use of a theorem due to Jivotovski (in order to determine the stability for any delay). The dynamics considered is a generic roll one which can occur in aircraft motion, for example. **References**

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Fuzzy logic robust control of robot manipulators

Askin Mutlu, Recep Burkan

Faculty of Engineering, Department of Mechanical Engineering, Istanbul University, Turkey
e-mail: askin@istanbul.edu.tr

Robots are commonly used in various applications. Since robots are not able to act by themselves, they need to be programmed in order to be controlled. In this way, they act in a desired motion. Thus, robot control has been playing a vital role since robots existed, so a number of control methods have been developed. Since the robot parameters are varied due to the loads they carry, parameters are not calculated precisely. In the case of parametric uncertainty, adaptive or robust control methods are used. In this study, combination of adaptive robust control method were used, adaptive control gains were defined and stability of the uncertain system is guaranteed by using the Lyapunov stability theorem. The defined adaptive control gains are constant and they reduce tracking error. However, selection of the appropriate control parameters are difficult. A fuzzy logic compensator that would find this parameter, were designed and the effects of the parameters on the tracking error were investigated. Hence, a controller on which adapted robust controller and fuzzy logic are used together was designed in order to reduce tracking error to minimum level. This designed controller was used for controlling of robots. Improvement of the robust controller was aimed by using this controller. For computer simulations, a same trajectory and a same model are chosen. Known and proposed controllers are applied to same trajectory and same model under the same conditions and tracking performances are compared. Robust control is improved and tracking error level of the robot reduced to very small values by means of this fuzzy logic robust control algorithm.

Renormalization of a generalized Sierpinski gasket and Lindstrom's snowflake

Antonio Nuică

University of Piteşti, Romania
e-mail: antonio_74nm@yahoo.com

We expose an algorithmic method for determining the eigenoperators of the renormalization map for a given post critical finite self-similar fractal and apply it on the fractal called "generalized Sierpinski gasket"; the computation of the "highly-symmetric" irreducible eigenoperator for this fractal can be done by "triangle-star" transform, because its boundary has three points. For fractals with boundary having more than three points there is no chance of success without computer assistance. A Java program was developed to help in such cases and is used on Lindstrom's snowflake.

A new vision on solving the center-focus problem

Mihail Popa[1], Victor Pricop[1,2]

[1] *Institute of Mathematics and Computer Science, Academy of Sciences of Moldova, Moldavia*

[2] *"Ion Creangă" State Pedagogical University, Chişinău, Moldavia*
e-mail: popam@math.md, pricopv@mail.ru

Consider the nonlinear differential system

$$\frac{dx}{dt} = \sum_{i=0}^{\ell} P_{m_i}(x,y),\ \frac{dy}{dt} = \sum_{i=0}^{\ell} Q_{m_i}(x,y), \tag{1}$$

where P_{m_i}, Q_{m_i} are homogeneous polynomials of degree $m_i \geq 1$ in relation to x, y and $m_0 = 1$. The set $\{1, m_i\}_{i=1}^{\ell}$ consists of a finite number ($\ell < \infty$) of distinct integer numbers. The coefficients of the system (1) take values from the field of the real numbers $\mathbb{R}$.
It is known that if the roots of characteristic equation of the singular point $O(0,0)$ of the system (1) are imaginary, then the singular point O is a center (surrounded by closed trajectories) or a focus (surrounded by spirals) [1,2]. In this case the origin of coordinates is a singular point of the second type.
It was shown that the conditions for center are the vanishing of an infinite sequence of polynomials (Lyapunov's constants)

$$L_1,\ L_2,\ L_3,\ ..., L_k, ... \tag{2}$$

in coefficients of the right side of system (1). If at least one of the quantities (2) is not zero, then the origin of coordinates for the system (1) is a focus. These conditions are necessary and sufficient.
It is known [1] that there exists a finite number of polynomials (essential conditions of center) from (2)

$$L_{n_1},\ L_{n_2}, ..., L_{n_\omega}\ (n_i \in \{1,2,...,k,...\};\ i = \overline{1,\omega};\ \omega < \infty) \tag{3}$$

which, being equal to zero, vanish all other polynomials from (2), i. e. in the origin of coordinates we have a singular point of the type center, otherwise – focus.
In the case of system (1) from Hilbert's theorem on the finiteness of basis of polynomial ideals it

follows that in the mentioned sequence only a finite number of conditions for center *are essential*, the rest are consequences of them.
Then the center-focus problem for the system (1) takes the following formulation: *how many polynomials* (*essential conditions for center*) (3) *from* (2) *must be equal to zero in order that all other polynomials* (2) *would vanish*?
Using techniques developed in [3,4] we have established
Theorem 1. *The maximal number of algebraically independent* [3,4] *Lyapunov's constants from* (2)*, which participate in solving the center-focus problem for the system* (1)*, having at the origin of coordinates a singular point of the second type, do not exceed*

$$\varrho = 2\left(\sum_{i=1}^{\ell} m_i + \ell\right) + 3. \tag{4}$$

The main hypothesis. *The number of Lyapunov's constants which solve the center-focus problem for the system* (1) *does not exceed* (4). **Acknowledgement** *This research is partially supported by the grant 12.839.08.05F from SCSTD of ASM*

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Quartic differential systems with invariant straight lines along three directions

Vitalie Puţuntică

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: vitputuntica@mail.ru

Consider the quartic differential system of the form:

$$\begin{cases} \dot{x} = p_0 + p_1(x,y) + p_2(x,y) + p_3(x,y) + p_4(x,y) \equiv P(x,y), \\ \dot{y} = q_0 + q_1(x,y) + q_2(x,y) + q_3(x,y) + q_4(x,y) \equiv Q(x,y), \end{cases} \tag{1}$$

where real homogeneous polynomials p_i, $q_i \in \mathbb{R}$ $(i = \overline{0,4})$ of degree i in x, y.
A straight line $l(x,y) \equiv Ax + By + C = 0$ is said to be an invariant straight line of the system (1) if there exists a polynomial $K(x,y)$ such that

$$A \cdot P(x,y) + B \cdot Q(x,y) \equiv l(x,y) \cdot K(x,y).$$

In this paper is classified quartic differential systems having nine real invariant straight lines along three directions. The following result is obtained:

Theorem. Any quartic differential systems having nine real invariant straight lines along three directions via affine transformation of coordinates and time rescaling can by brought to one of the nine systems:

$$1)\begin{cases}\dot{x}=x(x+1)(x-a)(x-b),\\ \dot{y}=y(y+1)(y-a)(y-b),\\ a,b\in\mathbb{R}_+^*, a\neq b;\end{cases}\qquad 2)\begin{cases}\dot{x}=x(x+1)(x-1)(x-2),\\ \dot{y}=y(y+1)(y-1)(4x-3y-2);\end{cases}$$

$$3)\begin{cases}\dot{x}=x^2(1-x)(1-x+2y),\\ \dot{y}=y^2(y+1)(1-2x+y);\end{cases}\qquad 4)\begin{cases}\dot{x}=x(x+1)(x-1)(x-2y),\\ \dot{y}=y(y+1)(y-1)(y-2x);\end{cases}$$

$$5)\begin{cases}\dot{x}=x^2(x+1)(x-a),\\ \dot{y}=y^2(y+1)(y-a),\\ a\in\mathbb{R}_+^*;\end{cases}\qquad 6)\begin{cases}\dot{x}=x^3(x+1),\\ \dot{y}=y^3(y+1);\end{cases}$$

$$7)\begin{cases}\dot{x}=x^4,\\ \dot{y}=y^4;\end{cases}\qquad 9)\begin{cases}\dot{x}=x^3(x-2y),\\ \dot{y}=y^3(y-2x).\end{cases}$$

$$8)\begin{cases}\dot{x}=x^4,\\ \dot{y}=y^3(4x-3y);\end{cases}$$

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Continuous and discrete problems treated in a unified manner

Bianca Satco

"Stefan cel Mare" University of Suceava, Romania
e-mail: bisatco@eed.usv.ro

In this talk we present some methods to treat jointly continuous and discrete problems and, at the same time, impulsive problems. More precisely, we describe the measure differential equations approach and the time scales analysis and finally provide an existence result for measure driven problems.

Survey on quadratic differential systems

Dana Schlomiuk

Université de Montréal, Canada
e-mail: dasch@dms.umontreal.ca

Quadratic differential systems intervene in many areas of applied mathematics. They are useful tools in the process of creating mathematical models for very diverse problems. From the

theoretical point of view they are also interesting objects of study because some problems on these systems, formulated more than one hundred years ago are still unsolved today. During the past years we witnessed an increased, persistent and coherent activity in this area of research which led to a number of substantial results. In this lecture we shall survey some of the main achievements in the theory of quadratic differential systems and we shall point out which are the stumbling blocks on the road to the solution of these old problems.

Classification of cubic differential systems with degenerate infinity and five invariant straight lines

Alexandru Şubă [1], Vadim Repeşco [2]

[1] *Institute of Mathematics and Computer Science A.S.M., Chişinău, Republic of Moldova*

[2] *Tiraspol State University, Chişinău, Republic of Moldova*
e-mail: suba@math.md, repescov@gmail.com

We consider the real cubic differential system

$$\dot{x} = \sum_{j=0}^{3} P_j(x,y) \equiv P(x,y), \quad \dot{y} = \sum_{j=0}^{3} Q_j(x,y) \equiv Q(x,y), \tag{1}$$

where P_j, Q_j are homogeneous polynomial of degree j, $|P_3(x,y)|+|Q_j(x,y)| \not\equiv 0$ and $GCD\{P, Q\} = 1$. If $yP_3(x,y) - xQ_3(x,y) \equiv 0$, then the infinity for (1) is degenerate, i.e. consists only of singular points.

The straight line $l \equiv \alpha x + \beta y + \gamma = 0$, $\alpha, \beta, \gamma \in \mathbb{C}$, is said to be *invariant* for (1) if there exists a polynomial $K \in \mathbb{C}[x,y]$ such that the identity in x and y: $\alpha P + \beta Q \equiv (\alpha x + \beta y + \gamma)K$ holds. Moreover, if m is the greatest positive integer such that l^m divides $K(x,y)$, then we will say that the invariant straight line l has the parallel multiplicity equal to $m+1$. In the finite part of the phase plane the system (1) has at most nine singular points and no more than eight straight lines. Some properties of cubic systems with invariant straight lines are determined. The systems with the infinite line filled up with singularities (i.e. with the degenerated infinity) and straight lines of total multiplicity five are classified.

Theorem. *Any cubic system with degenerate infinity having invariant straight lines with total degree of invariance 5, via affine transformation and time rescaling can be written as one of the following* 24 *systems:*

1) $\begin{cases} \dot{x} = x\,(x+1)\,(x-a)\,, \\ \dot{y} = y\left(-a + c\,x - y + x^2\right), \\ a > 0,\ c \neq 2,\ a + c > 1; \end{cases}$

2) $\begin{cases} \dot{x} = x\,(x+1)\,(x-a)\,, \\ \dot{y} = y\left(b + (b-a)\,x - y + x^2\right), \\ a > 0,\ b > 0,\ b - a \neq 0; \end{cases}$

3) $\begin{cases} \dot{x} = x(x+1)(x-a), \\ \dot{y} = y(x+1)(x-a) + x^2 + y^2, \\ a > 0; \end{cases}$

4) $\begin{cases} \dot{x} = x(x+1)(x-a), \\ \dot{y} = (x+1)^2 + xy(x-a) + by^2, \\ a > 0,\ b > 0; \end{cases}$

5) $\begin{cases} \dot{x} = x^2(x+1),\ a > 0, \\ \dot{y} = y\left((a+1)x - y + x^2\right); \end{cases}$

6) $\begin{cases} \dot{x} = x^2(x+1),\ a \neq 0, \\ \dot{y} = y(a + ax - y + x^2); \end{cases}$

7) $\begin{cases} \dot{x} = (x-a)(x^2+1), \\ \dot{y} = y(1 - ac + cx - y + x^2), \\ a \in \mathbb{R},\ c \neq 0; \end{cases}$

8) $\begin{cases} \dot{x} = (x-a)(x^2+1), \\ \dot{y} = (x-a)^2 + y + \frac{1}{b}y^2 + x^2 y, \\ a \in \mathbb{R},\ b > 0; \end{cases}$

9) $\begin{cases} \dot{x} = x^2(x+1),\ a > 0, \\ \dot{y} = ax^2 + xy + ay^2 + x^2 y; \end{cases}$

10) $\begin{cases} \dot{x} = x^2(x+1),\ a \neq 0, \\ \dot{y} = a(x+1)^2 + ay^2 + x^2 y; \end{cases}$

11) $\begin{cases} \dot{x} = x^3,\ a > 0, \\ \dot{y} = y(ax - y + x^2); \end{cases}$ 12) $\begin{cases} \dot{x} = x^3,\ a > 0, \\ \dot{y} = ax^2 + ay^2 + x^2 y; \end{cases}$

13) $\begin{cases} \dot{x} = x(x-1)(y+a), \\ \dot{y} = y(y-1)(x+a), \\ a \notin \{-1; -1/2\},\ a < 0; \end{cases}$ 14) $\begin{cases} \dot{x} = x^2(y+a), \\ \dot{y} = y^2(x+b), \\ a > 0,\ b > 0,\ ab \neq 0; \end{cases}$

15) $\begin{cases} \dot{x} = (x^2+1)(y+a), \\ \dot{y} = (y^2+1)(x+a),\ a \neq 0; \end{cases}$ 16) $\begin{cases} \dot{x} = x(a - 2ay + x^2 + y^2), \\ \dot{y} = ay + (a-1)x^2 - \\ \quad -(a+1)y^2 + x^2 y + y^3, \\ a \notin \{0; 1/2; 1\}; \end{cases}$

17) $\begin{cases} \dot{x} = x + 2by + 2bx^2 - 2xy - 2by^2 + 2x^3 + 2xy^2, \\ \dot{y} = (2y-1)(2bx - y + x^2 + y^2),\ b \neq 0; \end{cases}$

18) $\begin{cases} \dot{x} = ax^2 + 2bxy - ay^2 + x^3 + xy^2, \\ \dot{y} = -bx^2 + 2axy + by^2 + x^2 y + y^3, \\ |a| + |b| \neq 0,\ a \geq 0; \end{cases}$

19) $\begin{cases} \dot{x} = x(x-1)(1 + ax + by),\ b(b+2) \neq 0, \\ \dot{y} = y(-1 + 2x + y + ax^2 + bxy), \\ \quad a < -1 < b \quad \text{or} \quad a < b < -1; \end{cases}$

20) $\begin{cases} \dot{x} = (1 + (x-a)^2)(x + by),\ b \neq 0, \\ \dot{y} = (a^2+1)(y - bx) + (ab-1)x^2 - 2axy - \\ \quad -(ab+1)y^2 + x^2 y + bxy^2; \end{cases}$

21) $\begin{cases} \dot{x} = x + cy + (2a+c)x^2 + 2(-1+ac)xy - cy^2 + (a^2 + b^2 - b + ac)x^3 \\ \quad +(-2a - c + a^2 c + b^2 c)x^2 y - (b - 1 + ac)xy^2, \\ \dot{y} = -cx + y + (b - ac)x^2 + 2(a+c)xy + (b - 2 + ac)y^2 + (a^2 + b^2 - \\ \quad -b + ac)x^2 y + (-2a - c + a^2 c + b^2 c)xy^2 - (b - 1 + ac)y^3, \\ bc(|a| + |b^2 - 1|) \neq 0; \end{cases}$

22) $\begin{cases} \dot{x} = x(1 + 2ax - 2y + (a^2 + b^2 - c)x^2 - 2axy - (c-1)y^2), \\ \dot{y} = y + cx^2 + 2axy + (c-2)y^2 + (a^2 + b^2 - c)x^2 y - 2axy^2 - (c-1)y^3, \\ bc(b^2 - c^2)(|a| + |b^2 - 1|) \neq 0; \end{cases}$

23) $\begin{cases} \dot{x} = x(1 + (a+b)x - 2y + (ab - c)x^2 - (a+b)xy + (1-c)y^2), \\ \dot{y} = y + cx^2 + (a+b)xy + (c-2)y^2 + (ab - c)x^2 y - (a+b)xy^2 + \\ \quad +(1-c)y^3, \\ c(b-a) \neq 0; \end{cases}$

24) $\begin{cases} \dot{x} = x(1 + (a+b)x - 2y + abx^2 - \alpha xy + cy^2), \\ \dot{y} = y(1 + \alpha x - (c+1)y + abx^2 - \alpha xy + cy^2), \\ \alpha = a + b + c - 1,\ ab(a-1)(b-1)(c-1) \neq 0,\ a > b. \end{cases}$

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Cubic differential systems with a straight line of maximal geometric multiplicity

Alexandru Şubă , Olga Văcăraş

Institute of Mathematics and Computer Science A.S.M., Republic of Moldova
e-mail: suba@math.md; vacaras_olga@mail.md

We consider the real polynomial system of differential equations

$$\dot{x} = P(x,y), \qquad \dot{y} = Q(x,y), \quad GCD(P,Q) = 1 \tag{1}$$

and the vector field $\mathbb{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}$ associated to system (1).

Denote $n = \max\{\deg(P), \deg(Q)\}$. If $n = 3$ then system (1) is called cubic.
A straight line $\alpha x+\beta y+\gamma = 0,\ \alpha,\beta,\gamma \in \mathbb{C},\ |\alpha|+|\beta| \neq 0$ is called invariant for (1) if there exists a polynomial $K(x,y)$ such that the identity $\alpha P(x,y)+\beta Q(x,y) \equiv (\alpha x+\beta y+\gamma)K(x,y),\ (x,y) \in \mathbb{R}^2$ holds. In the work [1] there are introduced the following definitions of the multiplicity of an invariant algebraic curve: algebraic multiplicity, geometric multiplicity, integrable multiplicity, infinitesimal multiplicity, holonomic multiplicity and the relations between these definitions are established. In this paper all cubic systems which have a straight lines of maximal geometric multiplicity are determined.

Definition 1. *An invariant straight line $L = 0$ for a cubic vector field $\mathbb{X}$ has* geometric multiplicity *m if there exists a sequence of cubic vector fields $\mathbb{X}_k$ converging to $\mathbb{X}$, such that each $\mathbb{X}_k$ has m distinct invariant straight lines $L_{1,k} = 0, ..., L_{m,k} = 0$, converging to $L = 0$ as $k \to \infty$, and this does not occur for $m+1$.*

Theorem. *For cubic systems the geometric multiplicity of an invariant straight line is at most seven.*

Any cubic system having an invariant straight line of the geometric multiplicity seven via affine transformations and time rescaling can be brought to the form

$$\dot{x} = x^3, \quad \dot{y} = 1 + ax^2 + bx^3 + 3x^2y. \tag{2}$$

For (2) only the straight line $L = x$ is invariant. The following cubic system

$$\begin{aligned}
\dot{x} = \;& x(x - 3\epsilon + 2bx\epsilon^3 - 4b\epsilon^4)(x - 3\epsilon + 2a\epsilon^3 + 2bx\epsilon^3 + 4b\epsilon^4),\\
\dot{y} = \;& 1 + ax^2 + bx^3 + 3x^2y - 4ax\epsilon - 12xy\epsilon + 2a\epsilon^2 - 12bx\epsilon^2\\
& +9y\epsilon^2 + 8b\epsilon^3 - 4axy\epsilon^3 - 12xy^2\epsilon^3 + 10ay\epsilon^4 - 24bxy\epsilon^4\\
& +24y^2\epsilon^4 + 48by\epsilon^5 + 8ay^2\epsilon^6 - 12bxy^2\epsilon^6 + 16y^3\epsilon^6 + 8aby\epsilon^7\\
& +72by^2\epsilon^7 + 32b^2y\epsilon^8 + 8ay^2\epsilon^9 + 32by^3\epsilon^9 + 48b^2y^2\epsilon^{10}\\
& +16b^2y^3\epsilon^{12}
\end{aligned} \tag{3}$$

has seven distinct invariant straight lines:

$$\begin{aligned}
&L_1 = x;\ L_2 = x - \epsilon - 2a\epsilon^3 - 4y\epsilon^3 - 8b\epsilon^4 - 4by\epsilon^6;\\
&L_3 = x - 2\epsilon - 2y\epsilon^3\ L_4 = x - 4\epsilon - 4y\epsilon^3 - 4b\epsilon^4 - 4by\epsilon^6;\\
&L_5 = x - 3\epsilon + 2bx\epsilon^3 - 4b\epsilon^4; L_6 = x - \epsilon - 4y\epsilon^3 - 4by\epsilon^6.\\
&L_7 = x - 3\epsilon + 2a\epsilon^3 + 2bx\epsilon^3 + 4b\epsilon^4.
\end{aligned}$$

If $\epsilon \to 0$ then (3) tends to (2) and L_i, $i = 1, ..., 7$ converge to the invariant straight line $L:\ x = 0$.

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4. Probability Theory, Mathematical Statistics, Operational Research

Modeling and calibrating the federal funds market- micro level

Elena Cristina Cânepă

Carnegie Mellon University, PIttsburgh, USA
e-mail: elencristina2@gmail.com

In this paper we model and calibrate the deposit processes of the banks in the federal funds market. We also study the relationship between the asset sizes and the corresponding estimated parameters of the deposit processes. We model and calibrate the deposit processes as a Brownian motion, as a geometric Brownian motion, as an Ornstein-Uhlenbeck process and as a geometric Ornstein-Uhlenbeck process. The goodness-of-fit tests show that the best model among the proposed ones is the geometric Ornstein-Uhlenbeck process, followed by the Ornstein-Uhlenbeck process, the geometric Brownian motion and the Brownian motion with drift, respectively.

On the existence of Berge equilibrium with pseudocontinuous payoffs

Messaoud Deghdak

Département de Mathématiques, Faculté des Sciences Exactes, Université de Constantine 1, Algeria.
e-mail: deghdak.messaoud@gmail.com

The concept of Berge equilibrium goes back to the book of Berge (1957) and was later formalized by Zhukovskii (1994) for differential games. For a non cooperative game with finite number of persons, this equilibrium means that if each person plays his strategy at a Berge equilibrium, then he obtains the maximum payoff if all the remaining players play their strategy in the Berge equilibrium. It is worthnoticing that the Berge equilibrium is totally different from the Nash equilibrium since the Nash equilibrium is stable with respect the deviation of any unique player. For the concept of Nash equilibrium, we refer the reader to the paper of Nash (1951). The existence of Berge equilibrium has been studied by Nessah et al (2007), Larbani and Nessah (2008), Abalo and Kostreva (2004, 2005) and Colman et al (2011). More recently, Musy et al (2012) have established the existence of Berge equilibrium without using Nash equilibrium. Previously mentioned works, the authors have assumed that payoffs of persons are continuous. However, many games as the oligopolies of Bertrand (1883) and Hotelling (1929) have discontinuous payoffs. Several authors have studied the existence of Nash equilibrium where payoffs are not necessarily continuous. Let us quote for example, Morgan and Scalso (2007). In their paper, Morgan and Scalso (2007) have proved the existence of Nash equilibrium with "pseudocontinuous" payoffs. In this work we prove the existence of Berge equilibrium with "pseudocontinuous" payoffs.

Parallel algorithms for solving information extended games

Boris Hâncu

Moldova State University, Chişinău, Republic of Moldova
e-mail: boris.hancu@gmail.com

Let $\Gamma = \langle I; X_p, p \in I; H_p : X \to \mathbb{R} \rangle$ be the strategic form of the static noncooperative games with complete and partial perfect information. All players know exactly the payoff functions and

they know the sets of strategies. Players $r \in I, r \neq i, j$ do not know what kind of the strategy will be chosen by the players, but players i and j know what kind of the strategy will be chosen by each others. These condition stipulate that we can use the set of the informational extended strategies of the player i (respectively j) which is the set of the functions $\Theta_i = \{\theta_i : X_j \to X_i\}$ (respectively $\Theta_j = \{\theta_j : X_i \to X_j\}$) such that $\forall x_j \in X_j$, $\theta_i(x_j) \in X_i$ (respectively $\forall x_i \in X_i$, $\theta_j(x_i) \in X_j$). The payoff functions of the players will be $\mathcal{H}_p : \Theta_i \times \Theta_j \times X_{[-ij]} \to R$ for all $p \in I$. Players i, j do not know what kind of the informational extended strategy will be chosen by the players. The game described above will be denoted by $Game\left(i \overset{\inf}{\leftrightarrows} j\right)$ and will be treated as the Bayesian game because all informational extended strategies, for example θ_i of the player i, generate the uncertainty of the players $p \in I \backslash \{i\}$ about the complete structure of the payoff function $H_p\left(\theta_i(x_j), \theta_j(x_i), x_{-ij}\right)$ in the game with non informational extended strategies. So players does not know some parameter of the game they are playing, notably the payoff functions in case if they use the information non-extended strategies strategies $x_p \in X_p$. To solve the $Game\left(i \overset{\inf}{\leftrightarrows} j\right)$ we convert the game with informational extended strategies to Bayesian game and following Harsanyi's 1967-68 trilogy we can reduce the analysis of a game with incomplete information to the analysis of a game with complete (yet imperfect) information, which is fully accessible to the usual analytical tools of game theory.

For two persons game $Game\left(1 \overset{\inf}{\leftrightarrows} 2\right)$ we construct the following Bayesian game $\Gamma = \left\langle I = \{1, 2\}, S_1(\Delta_1), S_2(\Delta_2), \Delta_1, \Delta_2, p, q, \widetilde{H}_1, \widetilde{H}_2 \right\rangle$, where the set of types of the player 1 and 2 are $\Delta_1 = \{\delta_1^j, j = 1, ..., m_1\}$ correspondingly $\Delta_2 = \{\delta_2^k, k = 1, ..., m_2\}$ and mean the following: the player 1 (player 2) is of the type δ_1^j (type δ_2^k) if he choose the informational extended strategy $\theta_1^j \in \Theta_1$ (strategy $\theta_2^k \in \Theta_2$). The sets of strategies are $S_1(\Delta_1) \equiv \Theta_1$ and $S_2(\Delta_2) \equiv \Theta_2$. For this game we construct the following Harsanyi-Selten game $\Gamma^* = \left\langle J, \{R_j\}_{j \in J}, \{U_j\}_{j \in J} \right\rangle$ where $R_j = \begin{cases} \Theta_1 & j \in J_1, \\ \Theta_2 & j \in J_2. \end{cases}$, and for all $j \in J_1,\ k \in J_2$, $U_j\left(\theta_1^j, \{\theta_2^k)\}_{k \in J_2}\right) = \sum\limits_{k \in J_2} p(\theta_2^k | \theta_1^j) H_1\left(\theta_1^j(y), \theta_2^k(x)\right)$, $U_k\left(\left\{\theta_1^j\right\}_{j \in J_1}, \theta_2^k\right) = \sum\limits_{j \in J_1} q(\theta_1^j | \theta_2^k) H_2\left(\theta_1^j(y), \theta_2^k(x)\right)$. The strategies profile $\left(\left\{\widetilde{\theta}_1^j\right\}_{j \in J_1}, \left\{\widetilde{\theta}_2^k\right\}_{k \in J_2}\right)$ in game Γ^* are Nash equilibrium if for all $j \in J_1$, $k \in J_2$, $\theta_1^j \in \Theta_1$, $\theta_2^j \in \Theta_2$ the following conditions are satisfied

$$\sum_{k \in J_2} p(\delta_2^k | \delta_1^j) H_1\left(\widetilde{\theta}_1^j(y), \widetilde{\theta}_2^k(x)\right) \geqq \sum_{k \in J_2} p(\delta_2^k | \delta_1^j) H_1\left(\theta_1^j(y), \widetilde{\theta}_2^k(x)\right) \forall y \in Y,$$

$$\sum_{j \in J_1} q(\delta_2^k | \delta_1^j) H_2\left(\widetilde{\theta}_1^j(y), \widetilde{\theta}_2^k(x)\right) \geqq \sum_{j \in J_1} q(\delta_1^j | \delta_2^k) H_2\left(\widetilde{\theta}_1^j(y), \theta_2^k(x)\right) \forall x \in X.$$

It is easy to proof that the Nash equilibrium profiles in the game Γ^* are the Bayes-Nash equilibrium profiles in the informational extended game $Game\left(1 \overset{\inf}{\leftrightarrows} 2\right)$.

To find the Bayes-Nash equilibrium profiles in the bimatricial informational extended game we also construct and analyze a parallel algorithms using the parallel clusters and standardized functions of MPI and OpenMP for parallel applications development on computer systems with common and distributed memory.

A geometric program to optimize the evolution of stochastic systems with final sequence states

Alexandru Lazari

Moldova State University, Republic of Moldova
e-mail: lazarialexandru@mail.md

In this paper the stochastic systems with final sequence states X, independent states and unit transition time are analyzed. The evolution time $T(p)$ of these systems is studied, where p represents the distribution of the states of the system. The problem of minimization the expectation $E(T(p))$ of evolution time $T(p)$ is considered.
The main idea of the elaborated method for solving this problem is to study the derivative of generating function in the point 1 as a function of discrete distribution p of the states. The value of this function for arbitrary distribution p^* of the states represents the expectation of the evolution time of the corresponding stochastic system. It is theoretically grounded that the expression $E(T(p)) + 1$ represents a posynomial for every distribution p of the states. So, is obtained the geometric program: minimize $E(T(p)) + 1$ subject to $\sum_{k \in \overline{X}} p_k \leq 1$, that can be solved efficiently using geometric programming, where $\overline{X}$ represents the set of the states that participate in the final sequence states X. For some particular cases the explicit solution is obtained.

Modeling with Set-Valued and Fuzzy Stochastic Differential Equations

Marek T. Malinowski

Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Gora, Poland
e-mail: M.Malinowski@wmie.uz.zgora.pl

A great number of real phenomena in control theory, physics, biology, economics can be modelled by stochastic differential equations. A typical feature of such phenomena is uncertainty. This term is mostly understood as stochastic uncertainty and methods of probability theory are utilized in its analysis. However, uncertainty can result from the second source: vagueness (sometimes called ambiguity, fuzziness, softness). It appears, for instance, when the data of considered systems are imprecise due to lack of precision of measuring instruments. This type of uncertainty is not stochastic. With the notions of set-valued stochastic differential equations and fuzzy stochastic differential equations we offer the new mathematical tools that can be appropriate in modeling systems subjected to two combined types of uncertainties, i.e. randomness and vagueness (fuzziness), simultaneously. We will present the existence and uniquenes theorems for solutions to set-valued and fuzzy stochastic differential equations. We will consider the solutions with increasing and decreasing diameter of their values. Some applications of set-valued and fuzzy stochastic differential equations will be shown.

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Optimality and duality in multiobjective programming with fuzzy-valued objective functions under generalized type-I invexity

Toni Mihalcea

Faculty of Mathematics and Computer Sciences, University of Bucharest, Romania
e-mail: toni_mihalcea@yahoo.com

In this paper we present optimality conditions and duality for multiobjective programming problems with fuzzy-valued objective functions under generalized type-I invexity. Weak, strong and converse duality theorems are proved in the generalized V-invexity type I setting for fuzzy optimization. We develop the pseudo, quasi, quasi-pseudo and pseudo-quasi type-I invexity for fuzzy multiobjective optimization problems.

On Some Mathematical Models for the Management of Pension Funds

Iulian Mircea, Mihaela Covrig

'Academia de Studii Economice din Bucuresti
e-mail: mirceaiulian91@yahoo.com, mihaela_covrig@yahoo.com

One of the largest sources of risk faced by life insurance companies and pension funds is the longevity risk: the risk that members of some reference population might live longer on average than anticipated, affecting their pricing and reserving calculations. In recent years, due to increased life expectancy and decrease in the number of employee taxpayers, major difficulties have occurred on the annuity market and pension funds. Hence, it has become more important for insurance companies and pension funds to find a suitable and efficient way to cross-hedge or to transfer part of the longevity risk to reinsurers or to financial markets. The markets for longevity derivatives are starting to develop, as the insurance industry faces some specific challenges related to longevity risk. In this study, we prove statistical properties of the Asymmetric Power ARCH (A-PARCH) process and we develop some models of mortality rates and of pricing the longevity risk. We make some remarks r egarding forecasting mortality rates using various models. For the A-PARCH process we exhibit an existence condition for certain fractional moment of interest in these models and we

show that there exists a value of the heteroskedasticity parameter such that it minimizes the first-order autocorrelation of squares of the power-transformed series. We propose some mathematical models in forecasting mortality rates for the Romanian population, and also, we present some models for the securitization of longevity bonds or loans. We provide a numerical illustration of the above models.

Kendall and Pollaczek-Khintchin equations for queueing models with semi-Markov switching

Mishkoy Gheorghe, Bejenari Diana, Mitev Lilia

Academy of Sciences of Moldova, Free International University of Moldova, Chişinău, Republic of Moldova
e-mail: gmiscoi@ulim.md

Queueing models with semi-Markov switching, from applicative point of view, present a big interest in the scientific world. In particular case, polling models find a wide application in the different fields, such as computer systems, transportation, telecommunications, manufacturing and others [1, 2]. In the paper will be presented and discussed some results that can be viewed as generalizations of well-known classical Kendall and Pollaczek-Khintchin equations [3, 4]. Numerical algorithms for solving generalized Kendall and Pollaczek-Khintchin equations for exhaustive polling models and priority models with semi-Markov switching will be proposed. The particulary examples obtained from mentioned algorithms for some probabilistic characteristics of studied models will be discussed. For a large class of generalized priorities models the analogical characteristics were obtained in [5]. This work is supported partially by STCU.F/5854 grant and by A.S.M. grant 13.819.18.05A

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A modelling system for seaport activities

Gheorghe Mishkoy, Alina Costea, Ionela Rodica Ticu

Academy of Science of Moldova, Chişinău, Republic of Moldova
e-mail: alina.costea2009@yahoo.com

In this paper we shall approach a modelling system for seaport activities based on the average waiting time and average queue length of ships in the seaport. We shall propose some suggestions for deepening and expanding this modelling system.

Solving of the Currency Exchange Problem using a dynamic model

Ludmila Novac

Moldova State University, Moldavia
e-mail: novac-ludmila@yandex.com

The Problem of Currency Exchange is a simple situation well known for everybody, however the mathematical model for a such simple operation of currency exchange could be very complicated if we will analyse all parameters which can have important influence for certain of the participants of this operation.
Actually, for the most part of the people this kind of transaction represents a simple necessity, but in the same time a number of such consecutive operations could be the start of a successful business transaction for some people (as a legal currency exchange point). Certainly, there exists the risk for a big loss if the transaction is not adequate to the situation. This can happen in the case when somebody don't make the complete investigation before making the transaction.
In this work we analyse the problem when a person has to do some operation of currency exchange and choose some exchange point, which is the most adequate for his interests. If we will consider a more complicated situation for some exchange operations of the same participants, during a period of some days or weeks, we can construct a dynamical model with complete (or incomplete) information.
Therefore, we construct the dynamical model with two players, one of them is the "seller" and the other is the "buyer". As we know, in such situation each of them want to have some profit (or at least a minimum loss). Thus, the important decision is, "when" and "which operation" is better to do ("to buy" or "to sell") for the buyer; and for the seller is important to establish the most adequate price for each type of currency.

A New Multiplicative Fuzzy Regression Function Approach

Nimet Yapici Pehlivan, Ismail Burhan Türkşen

Selçuk University, TOBB–Economics and Technology University, Turkey
e-mail: nimet@selcuk.edu.tr

"Fuzzy Regression Functions (FRF)" which was first proposed by Türkşen (2008) are presented to be determined by the least squares estimation (LSE) technique for the development of fuzzy system models. FRF is also called as "Additive Fuzzy Regression Functions (AFRF)". In the

AFRF approach, various transformations of the membership values are used as new variables in addition to original input variables. At first, fuzzy clustering algorithms such as Fuzzy c-means clustering (FCM) or Improved Fuzzy Clustering (IFC) are used for obtaining membership values of input variables. Secondly, membership values and input variables are used by the LSE technique. In this study, we propose a new "Multiplicative Fuzzy Regression Functions (MFRF)" approach. In the proposed approach, various transformations of the membership values are considered as a multiplier of original input variables.
Finally, effectiveness of the proposed approach is compared to several well-known existing methods. Computational results is verified with respect to RMSE and R-square measures for both training and testing data sets.

Equilibrium of Bayesian fuzzy economies and quasi-variational inequalities with random fuzzy mappings

Monica Patriche

Faculty of Mathematics and Computer Sciences, University of Bucharest, Romania
e-mail: monica.patriche@yahoo.com

In this paper, we introduce a Bayesian abstract fuzzy economy model and we prove the Bayesian fuzzy equilibrium existence. As applications, we prove the existence of the solutions for two types of random quasi-variational inequalities with random fuzzy mappings and we also obtain random fixed point theorems.

Efficient frontiers and absolute portfolios

Marius Radulescu[1], Sorin Radulescu[1],Gheorghita Zbaganu[1], Constanta Zoie Radulescu[2]

[1] *"Gheorghe Mihoc-Caius Iacob" Institute of Mathematical Statistics and Applied Mathematics, Bucharest, Romania*

[2] *National Institute for Research and Development in Informatics, Bucharest*
e-mail: mradulescu.csmro@yahoo.com, radulescucz@yahoo.com

In the paper are defined the notions of efficient frontier set and efficient frontier function of a parametric optimization problem. We formulate several portfolio selection problems which are nonlinear programming problems. Two of them are minimum variance type problems and the other two are maximum expected return type problems. Taking into account various hypotheses on the covariance matrix and on the vector of means the duality between minimum variance type problems and maximum expected return type problems is investigated. We are interested when the efficient frontier sets of the minimum variance type problems and of the maximum expected return type problems are equal. Generalization of the problems studied to the case of mean-risk models is suggested. Let $X = (X_1, \cdots, X_n)$ be a financial market. A portfolio is a vector $\mathbf{t} = (t_1, \cdots, t_n)$ of non-negative numbers which add up to 1. The return of the portfolio $\mathbf{t}$ is the number $\zeta = \mathbf{t}'\mathbf{X}$. Suppose that $\mathbf{X}$ is a random vector. The problem we investigate is the following: how should be the distribution of the random vector $\mathbf{X}$ in order that all the risk averse decision makers agree that some fixed portfolio $\mathbf{t}^0$ is the best? Precisely, our problem is: in what conditions $\mathbf{t}'\mathbf{X}$ is dominated by $\mathbf{t}^{0\prime}\mathbf{X}$ for all portfolios $\mathbf{t}$ in the stochastic order given by risk - averse utilities? We answer completely this problem if all the components of $\mathbf{t}^0$ are positive or if all but one components are equally to 0.

Generalized equilibrium problems related to Ky Fan inequalities

Ionel Roventa

Department of Mathematics, University of Craiova, Romania
e-mail: ionelroventa@yahoo.com

In this paper we study a generalized equilibrium problem by using a non symmetric extension of Ky Fan inequality. Note that many classes of problems belong the class of equilibrium problems, such as saddle point problems, fixed point problems and variational problems. As an application, we present a fixed point type algorithm.
More precisely, by using a non symmetric extension of Ky Fan inequality we solve a special class of equilibrium problems, which can be fitted into the field of minimax inequalities. By using such type of generalized equilibrium problems we consider a fixed point type algorithm based on two iterative sequences. The main ingredient is given by the classical Knaster-Kuratowski-Mazurkievicz theorem.
Acknowledgement. This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, project number PN-II-RU-TE-2011-3-0223.

On some generalization of univariate distributions based on truncated moments of order statistics

Alin Rusu

Faculty of Mathematics and Computer Science, University of Bucharest, Romania
e-mail: rusualinmarian@yahoo.com

In this paper we present some generalizations of well known univariate continuous distributions based on a truncated moment of the i-th order statistics. Thus some of the known results are special cases of the results presented here.

Statistical testing of random number generators

Emil Simion

"POLITEHNICA" *University of Bucharest, Bucharest, Romania*
e-mail: esimion@fmi.unibuc.ro

In this paper we propose an improvement of the decision regarding the randomness, proposed by National Institute of Standards and Technologies (NIST) in the guideline Statistical Test Suite (STS) Special Publication (SP) 800-22, on computing the second order error (the probability of acceptance a false hypothesis).

Beta-coalescents and their lengths

Iulia Stanciu, Goetz Kersting, Anton Wakolbinger

Goethe University, Frankfurt, Germany
e-mail: stanciu@math.uni-frankfurt.de

Coalescent processes are theoretical population genetics models for the genealogies of (haploid) populations. If the population size is n, the corresponding coalescent can be graphically represented as a rooted tree started with n leaves. The (total, external and internal) lengths of a coalescent tree - the sums of the lengths of its branches - play a central role in the statistical analysis of the number of mutations that affect the individuals in the population. We review recent results on the fluctuations of the a) total branch length and b) total external branch length in a Beta$(2-\alpha, \alpha)$-coalescent as the population size n tends to infinity. This is based on joint work with Götz Kersting and Anton Wakolbinger (University of Frankfurt).

Sufficient efficiency criteria in multiobjective fractional programming with generalized n-set functions

Andreea Mădălina Stancu, Ioan M.Stancu-Minasian

"Gh. Mihoc-C. Iacob" Institute of Mathematical Statistics and Applied Mathematics, 050711 Bucharest, Romania
e-mail: andreea_madalina_s@yahoo.com; stancu_minasian@yahoo.com

We consider some types of generalized convexity and discuss new global semiparametric sufficient efficiency conditions for a multiobjective fractional programming problem involving n-set functions.

Fuzzy Cost Multiobjective Transportation Problem with Time Restriction

Alexandra Tkacenko

State University of Moldova, Moldavia
e-mail: alexandratkacenko@gmail.com

The multicriteria optimization problems with fuzzy coefficients are the most important, because of its especially practical applications in the various managerial decision processes. In this paper is presented an interactive solving approach of the multiobjective transportation problem with fuzzy cost coefficients and the time restriction. The mathematical model of the proposed problem is the follow:

$$\min z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^1 x_{ij} \quad \min z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^2 x_{ij}$$
$$\dots$$
$$\min z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r x_{ij} \tag{1}$$

$$\min z_{r+1} = \max_{ij}\{t_{ij} \,|x_{ij} > 0\} \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \forall i = \overline{1,m} \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \forall j = \overline{1,n} \quad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

where: c_{ij}^k, $k = 1,\ldots,r$, $i = 1,\ldots,m$, $j = 1,\ldots,n$ unit costs, which correspond to the certain interpretation of the respective criteria, being of fuzzy type, $t_{ij}-$ unit transportation time from source i to destination j; a_i- availability of source i, b_j- requirement of destination j, $x_{ij}-$ transported amounts from source i to destination j, which are only positive.
The proposed solving approach is based on the interval presentation of each cost criterion coefficients. Finding the probabilistic belong parameter for each cost value to its interval for every criterion, we can find iterative the set of efficient solutions for the multiple criteria transportation model for every parameter value. The proposed algorithm was tested successfully on some examples.

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Mathematical theory of Pareto-Nash-Stackelberg game-control processes

Valeriu Ungureanu

Moldova State University, Chişinău, Republic of Moldova
e-mail: V.A.Ungureanu@gmail.com

Games and Control Processes of Pareto-Nash-Stackelberg (PNS) type are defined. Mathematical models are presented and solving principles are identified. Methods for computing sets of equilibria in PNS games are exposed. A direct-straightforward method for solving linear discrete-time optimal control problem is applied to solve control problem of a linear discrete-time system as a mixture of multi-criteria Stackelberg and Nash games. For simplicity, the exposure starts with the simplest case of linear discrete-time optimal control problem and, by sequential considering of more general cases, investigation finalizes with the highlighted single and set valued PNS control problems. Solution principles are compared and their equivalence is proved.

Parameters estimation for the bivariate Sarmanov distribution with normal marginals

Raluca Vernic, Elena Pelican

"Ovidius" University of Constanţa, Romania
e-mail: rvernic@univ-ovidius.ro

Recently, Sarmanov's family of bivariate distributions gained interest due to its flexible structure that easily joins given marginals. In this paper, we focus on the parameters estimation for the bivariate Sarmanov distribution with normal-type marginals. The maximum likelihood method is discussed and illustrated on both real and simulated data.

Stochastic comparisons and orders

Gheorghiță Zbăganu

Faculty of Mathematics and Computer Science, University of Bucharest, Romania

"Gh. Mihoc-C. Iacob" Institute of Mathematical Statistics and Applied Mathematics, Bucharest, Romania
e-mail: gheorghitazbaganu@yahoo.com

Somebody has to choose between two queues. He knows the waiting times and the serving times. What queue is better to choose?
A device has the life time T_1 and another one, with the same performances has the life time T_2. Which of them has a longer life time?
A financial market has n assets "1","2",...,"n" with the rates of returns $X_1, ..., X_n$. A portfolio of sum S is a vector $x = (x_1, ..., x_n)$ such that $x_j \geq 0$ and $x_1 + x_2 + ... + x_n = S$. The vector $\xi = x_1X_1 + x_2X_2 + ... + x_nX_n$ is the return of the portfolio $\boldsymbol{x}$. Which portfolio is the best?
How can we compare two financial markets $\boldsymbol{X} = (X_1, ..., X_n)$ and $\boldsymbol{Y} = (Y_1, ..., Y_n)$?
An ensurer has a homogeneous portfolio of n ensured clients which have the claims $X_1, ..., X_n$. What is the best insurance premium if the accepted default probability is ϵ?
All these questions have something in common: how can we compare two random vectors? Or, simpler, two random variables? The first is called *the multivariate problem* and the last one is *the univariate* one?
The principle of expected utility gives a possible answer.
A utility is a continuous increasing function $u : D \to \mathbb{R}$ where $D \subset \mathbb{R}$ is a set with some special properties. The principle says that $\boldsymbol{X} \prec_u \boldsymbol{Y}$ ($\boldsymbol{Y}$ is preferable to $\boldsymbol{X}$) if $\mathrm{E}u(\boldsymbol{X}) \leq \mathrm{E}u(\boldsymbol{Y})$. The set $\boldsymbol{C} = \{u \text{ utility} : \boldsymbol{X} \prec_u \boldsymbol{Y}\}$ is a closed cone. Thus thye study of various stochastic orders involves the study of the generators of various cones of continuous increasing functions.

Modeling of adsorption kinetics of fluorine onto modified trepel

Veaceslav Zelentsov, Tatiana Datsko

The Institute of Applied Physics of the Academy of Sciences of Moldova, Chişinău, Republic of Moldova
e-mail: vzelen@yandex.ru

The kinetic of adsorption of fluorine onto modified trepel has been modeled using pseudo-first-order model (Lagergren), pseudo-second-order model and intraparticle diffusion model. The kinetic process parameters constant K, τ and a_m at solution temperatures 293, 303 and 313K. Applicability of the models was evaluated by statistical parameters – the correlation coefficient R^2, Chi-Square value χ^2 and also normal standard deviation, Δq. The kinetic sorption curves showed better fit with pseudo-second-order model. The mechanism of adsorption of fluoride onto trepel includes two stages – fast, which is controlled by intraparticle diffusion of fluoride into the pores of the sorbent and slow, which is responsible for the chemical reaction of the adsorbate with the adsorbent. The value of activation energy of the system (35.4 kJ mol-1) indicates the significance of diffusion in the sorption process.

5. Algebra, Logic, Geometry (with applications)

Applications of Finsler-type structures in image registration

Vladimir Balan,[1] Jelena Stojanov[2]

[1] *"POLITEHNICA" University of Bucharest*

[2] *University of Novi Sad, Technical Faculty M̈ihajlo Pupin; Serbia*
e-mail: vladimir.balan@upb.ro, jelena@tfzr.uns.ac.rs

The talk describes several Finslerian extensions of the Riemannian geodesic active field (GAF) methodology used in image registration, which generalizes the Casseles-Kimmel-Sapiro weighted length problem. The proposed extension relies on the mean curvature flow - whose Finslerian extension is explicitly determined, where the integration kernel of the weighted Polyakov energy functional is regarded as an application-dependent weight function. The customization of the Finsler-GAF technique for image enhancement is discussed as well.

On the structure of grid

Natalia Bobeica, Liubomir Chiriac

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: nbobeica1978@gmail.com, llchiriac@gmail.com

A groupoid is distributive if $(x \cdot y) \cdot z = (x \cdot z) \cdot (y \cdot z)$ and $x \cdot (y \cdot z) = (x \cdot y) \cdot (x \cdot z)$ hold for all x, y, z in the groupoid. An element e in a groupoid is called idempotent if $e \cdot e = e$ and injective if $x = y$ whenever $x \cdot e = y \cdot e$ or $e \cdot x = e \cdot y$. Let $(G, \cdot)$ be a topological distributive groupoid on a closed real interval $[0, e]$ such that 0 is a zero for G, e is an injective idempotent, and $xe \cdot ey = ye \cdot ex$ for all $x, y \in G$. Then $(G, \cdot)$ will be a grid. In this paper we determine the structure of grid. The results established are related to the research papers [1, 2].

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Distributive topological groupoids and paramedialitivity

Natalia Bobeica, Liubomir Chiriac

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: nbobeica1978@gmail.com, llchiriac@gmail.com

A topological groupoid is a Hausdorff space on which there is defined a jointly continuous binary operation. A topological groupoid is distributive if $(x \cdot y) \cdot z = (x \cdot z) \cdot (y \cdot z)$ and $x \cdot (y \cdot z) = (x \cdot y) \cdot (x \cdot z)$ hold for all x, y, z in the groupoid. A paramedial topological groupoid is one in which $(x \cdot y) \cdot (u \cdot v) = (v \cdot y) \cdot (u \cdot x)$ holds for all x, y, u, v. An element e in a groupoid is called idempotent

if $e \cdot e = e$, bijective if the maps $x \to x \cdot e$ and $x \to e \cdot x$ are homeomorphisms and injective if $x = y$ whenever $x \cdot e = y \cdot e$ or $e \cdot x = e \cdot y$. The results established are related to the research papers [1, 2, 3].

Theorem 1. *If a paramedial topological groupoid $(S, \cdot)$ contains a bijective idempotent e and if " $\circ$ " is defined on S by $xy = ey \circ xe$ then $(S, \circ)$ is a commutative semigroup having e as identity.*

Theorem 2. *Suppose $(M, \cdot)$ is a distributive groupoid with an injective idempotent e and let a, b be in M. If $ae \cdot eb = be \cdot ea$ then $(\{e, a, b\}^e, \cdot)$ is a paramedial subgroupoi of M. If e is a bijective idempotent, then $(\{e, a, b\}^e, \circ)$, where " $\circ$ " is given by $ex \circ ye = yx$, is a commutative semigroup with identity e.*

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The cocartesian product and pairs of conjugate subcategories

Botnaru Dumitru[1], Băes Elena[2]

[1] *Tiraspol State University, Chişinău, Republic of Moldova*

[2] *Technical University of Moldova, Chişinău, Republic of Moldova*
e-mail: dumitru.botnaru@gmail.com, baeselena@yahoo.it

We examine a construction that allows new reflective subcategories to be obtained. For the notions and notation see [1].

Let $\mathcal{K}$ be a coreflective subcategory, and $\mathcal{R}$ a reflective subcategory in the category $\mathcal{C}_2\mathcal{V}$. For any arbitrar object X of the category $\mathcal{C}_2\mathcal{V}$, examine the following construction: $k^X : kX \to X$ is $\mathcal{K}$-coreplica and $r^{kX} : kX \to rkX$ is $\mathcal{R}$-replica of respective objects. Cocartesian square construct: $\overline{v}^X \cdot k^X = u^X \cdot r^{kX}$.

Definition. Full subcategory of all objects isomorphic to objects of type $\overline{v}X$ is called the cocartesian right product of subcategories $\mathcal{K}$ and $\mathcal{R}$ and is denoted $\overline{\mathcal{V}} = \mathcal{K} *_{dc} \mathcal{R}$.

Lemma 1. $\mathcal{R} \subset \mathcal{K} *_{dc} \mathcal{R}$.

Theorem 1. *Let $\mathcal{K}$ be a coreflective subcategory, and $\mathcal{R}$ a reflective subcategory in the category $\mathcal{C}_2\mathcal{V}$. If $\mathcal{K} \subset \widetilde{\mathcal{M}}$, and $\mathcal{S} \subset \mathcal{R}$, then cocartesian product is a reflective subcategory.*

Example 1. *For any coreflective subcategory $\mathcal{K}$ we have $\mathcal{K} *_{dc} \Pi = \Pi$.*

Example 2. *For any coreflective subcategory $\mathcal{K}$ we have $\mathcal{K} *_{dc} \mathcal{S} = \mathcal{S}$.*

Theorem 2. *Let $(\mathcal{K}, \mathcal{L})$ a pair of conjugate subcategories, and $\mathcal{K}$- a reflective subcategory of category $\mathcal{C}_2\mathcal{V}$. Then:*

1. $\mathcal{K} *_{dc} \mathcal{R} = Q_{\varepsilon\mathcal{L}}(\mathcal{R})$, $Q_{\varepsilon\mathcal{L}}(\mathcal{R})$ is a full subcategory of all $\varepsilon\mathcal{L}$-factorobjects of the objects of subcategory $\mathcal{R}$.

2. $\mathcal{K} *_{dc} \mathcal{R}$ is a reflective subcategory of category $\mathcal{C}_2\mathcal{V}$.

3. Subcategory $\mathcal{K} *_{dc} \mathcal{R}$ is closed with respect to $(\varepsilon\mathcal{L})$-factorobjects.

4. $\overline{v} \cdot k = r \cdot k$

5. If $r(\mathcal{K}) \subset \mathcal{K}$, then coreflector functor $k : \mathcal{C}_2\mathcal{V} \to \mathcal{K}$ and reflector functor $\overline{v} : \mathcal{C}_2\mathcal{V} \to \mathcal{K} *_{dc} \mathcal{R}$ switch: $k \cdot \overline{v} = \overline{v} \cdot k$.

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The lattice of semireflexive sucategories

Botnaru Dumitru[1], Cerbu Olga[2]

[1] *Tiraspol State University, Chişinău, Republic of Moldova*

[2] *Moldova State University, Chişinău, Republic of Moldova*
e-mail: dumitru.botnaru@gmail.com, olga.cerbu@gmail.com

It is examined the lattice $\mathbb{R}_{sr}$ of semireflexive subcategories in the category of locally convex topological vector Hausdorff spaces - $\mathcal{C}_2\mathcal{V}$. Concerning to the terminology see [1-3].

1. Theorem. *$\mathbb{R}_{sr}$ is a complete lattice with the minimal element Π - the subcategory of the complete spaces with week topology and the maximal element $\mathcal{C}_2\mathcal{V}$.*

The right product of the subcategory $\widetilde{\mathcal{M}}$ of the spaces with Mackey topology possess the following properties:

2. Theorem. *The application $\widetilde{\mathcal{M}}\times_d\colon \mathbb{R} \longrightarrow \mathbb{R}$ has the following properties:*

1. Let be $\mathcal{R} \in \mathbb{R}(\mathcal{E}_u)$. Then $\widetilde{\mathcal{M}} \times_d \mathcal{R} = \mathcal{C}_2\mathcal{V}$.

2. Let be $\mathcal{R} \in \mathbb{R}(\mathcal{M}_p)$. Then $\widetilde{\mathcal{M}} \times_d \mathcal{R} = \mathcal{R}$.

3. Let be $\mathcal{R} \in \mathbb{R}(\mathcal{E}_u, \mathcal{M}_p)$. $\mathcal{R} \in \mathbb{R}_{sr}$ iff $\widetilde{\mathcal{M}} \times_d \mathcal{R} = \mathcal{R}$.

4. Let be $\mathcal{R} \in \mathbb{R}$ şi $\widetilde{\mathcal{M}} \times_d \mathcal{R} = \mathbb{R}(\mathcal{E}_u)$. Then $\widetilde{\mathcal{M}} \times_d \mathcal{R} = \mathcal{C}_2\mathcal{V}$.

3. Example. *Let $q\Gamma_0$ be the subcategories of quasicomplete spaces, but $s\mathcal{R}$ the subcategory of semireflexive spaces. Then*

$$\mathcal{S} \times_{sr} (q\Gamma_0) = \widetilde{\mathcal{M}} \times_d (\mathcal{S} \cap q\Gamma_0) = s\mathcal{R}.$$

4. Example. *For any reflective subcategory $\mathcal{R}$ with the properties $\mathcal{S} \subset \mathcal{R} \subset \mathcal{N}$ we have*

$$\mathcal{R} \times_{sr} q\Gamma = s\mathcal{R}.$$

In particulary,

$$\mathcal{S} \times_{sr} q\Gamma = s\mathcal{N} \times_{sr} q\Gamma = \mathcal{N} \times_{sr} q\Gamma = s\mathcal{R}$$

(see [3] Theorem 3.15).

5. Example. *Let Γ be a reflective subcategory so that $q\Gamma_0 \subset \Gamma \subset p\Gamma_0$. Then*

$$s\mathcal{R} = \mathcal{S} \times_{sr} \Gamma.$$

6. Example. *For any reflective subcategory Γ with the properties $\Gamma_0 \subset \Gamma \subset q\Gamma_0$ the subcategory $\mathcal{S} \times_{sr} \Gamma$ is a semireflexive.*

7. Example. *For any c-reflective subcategory $\mathcal{R}$, so that $\mathcal{R} \subset \mathcal{S}ch$, it follows that the subcategory $\mathcal{R} \times_{sr} \Gamma$ is semireflexive.*

8. Example. *Let $\mathcal{R}$ be the semireflexive subcategory, but $\Gamma \in \mathbb{R}(\mathcal{M}_p)$. Then $\mathcal{R} \cap \Gamma$ is a semireflexive subcategory.*

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The left and right product in the description of the pairs of conjugate subcategories

Botnaru Dumitru[1], Ţurcanu Alina[2]

[1] *State University from Tiraspol, Chişinău, Republic of Moldova*

[2] *Technical University of Moldova, Chişinău, Republic of Moldova*
e-mail: dumitru.botnaru@gmail.com, alina.turcanu@gmail.com

In this paper the pairs of conjugate subcategories are described with the help of the left and right product of two subcategories. Therefore the properties of the pairs of conjugate subcategories may be studied with the help of the left and right product. Referring to the terminology see [1-2].
1. Definition [1]. *Let $\mathcal{R}$ be a reflective nonull subcategory of the category of locally convex topological vector Hausdorff spaces - $\mathcal{C}_2\mathcal{V}$, and $\mathcal{K}$- a coreflective nonull subcategory, with the respective functors $r : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{R}$ and $k : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K}$. We state that $(\mathcal{K}, \mathcal{R})$ forms a pair of conjugate subcategory if the functors k and r verify the following relations:*
a) $k \cdot r = k$;
b) $r \cdot k = r$.
2. Theorem. *Let $\mathcal{K}$ be a coreflective subcategory, but $\mathcal{R}$ - a reflective subcategory of the category $\mathcal{C}_2\mathcal{V}$, the subcategory $\mathcal{K}$ contains the spaces with Mackey topology $(\widetilde{\mathcal{M}} \subset \mathcal{K})$, but $\mathcal{R}$ contains the spaces with weak topology $(\mathcal{S} \subset \mathcal{R})$.*
1. The following affirmations are equivalent:
a) $k \cdot r = k$;
b) $\mathcal{K} *_d \mathcal{R} = \mathcal{C}_2\mathcal{V}$.
2. The following affirmations are equivalent:
a) $r \cdot k = r$;
b) $\mathcal{K} *_s \mathcal{R} = \mathcal{C}_2\mathcal{V}$.
3. The following affirmations are equivalent:
a) $k \cdot r = k$ *and* $r \cdot k = r$ *meaning* $(\mathcal{K}, \mathcal{R})$ *is a pair of the conjugate subcategory;*
b) $\mathcal{K} *_d \mathcal{R} = \mathcal{C}_2\mathcal{V}$ *and* $\mathcal{K} *_s \mathcal{R} = \mathcal{C}_2\mathcal{V}$.
3. Remark. *So we can affirm that the pairs of conjugate subcategories can be defined via right and left products.*

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Location problem for complex of abstract n-polytops

Sergiu Cataranciuc

Moldova State University, Chişinău, Republic of Moldova
e-mail: s.cataranciuc@gmail.com

A complex of abstract n-polytops with an even number of facets is studied. For this complex, the median problem that is known as the location problem is solved. We search the median for 1-dimensional skeleton of the complex with weighted vertices and edges. The algoritm to calculate the median without using any metric is described. The proposed algorithm generalizes the known algorithm for solving this problem in the case of complexes of abstract n-dimensional cubes. Such complexes have been studied by the author in previous works. The problem becomes very difficult for complex of abstract n-polytops with an odd number of edges.

Variational principles in vector-optimization

Mitrofan Choban

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: mmchoban@gmail.com

Let E be a Banach reticulate lattice. Then, for each lower semi-continuous mapping $f : X \to E$, the notions of: minimization problem (f, X, E), minimal solutions, Pareto (effective) solutions, infimal elements, minimization sequences and well-posedness, are defined.
We denote by $C(X, E)$ the Banach lattice of all bounded continuous mappings $g : X \to E$ with sup-norm.
For any continuous bounded from below function $f : X \to E$ it is proved that:
- the set $\{g \in C(X, E) : \textit{minimization problem (f+g, X, E) has a Pareto solution}\}$ is dense in the space $C(X, E)$;
- if X is a first-countable space, then the set $\{g \in C(X, E) : \textit{minimization problem (f+g, X, E) has a well-posed Pareto solution}\}$ is dense in the space $C(X, E)$.
Some results about Pareto solutions are obtained for semi-continuous functions. In particular, the analogous of the Bishop-Phelps theorem and of Ekeland variational principle are proved.

On extensions with first-countable remainders

Mitrofan M.Choban, Laurenţiu I.Calmuţchi

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: mmchoban@gmail.com

Any space is considered to be a non-empty completely regular T_1-space. If X is a dense subspace of a space Y, then Y is called an extension of X and the subspace $Y \setminus X$ is called remainder of X. An extension Y of a space X is called a p-extension if $Y \setminus X \neq \emptyset$ and there exist a metric space Z and a perfect mapping $g : Y \longrightarrow Z$ such that $g(X) \cap g(Y \setminus X) = \emptyset$.
A space X is a paracompact p-space if and only if there there exist a metric space Z and a perfect mapping $g : X \longrightarrow Z$ (see [2]).

An extension Y of a space X is called: an fc-extension if $Y \setminus X \neq \emptyset$ and the space Y has a countable base at any point $p \in Y \setminus X$; a one-point fc-extension if $Y \setminus X$ is a singleton subset of Y; a compact p-extension if Y is a p-extension and and $Y \setminus X$ is a compact subset of Y; a metric p-extension if Y is a p-extension and and $Y \setminus X$ is a metric subspace of Y.
Denote by $E_p(X)$ the poset of all p-extensions of the space X, by $M_p(X)$ the poset of all metric p-extensions of the space X, by $Com_p(X)$ the poset of all compact p-extensions of the space X, by $S_p(X)$ the poset of all one-point p-extensions of the space X, by $E_{fc}(X)$ the poset of all fc-extensions of the space X, by $S_p(X)$ the poset of all one-point fc-extensions of the space X.
The present research was motivated initially by the Bel'nov's study of the poset $M(X)$ of metric extensions of a locally compact metric space X [3] and by the M.Henriksen, L.Janos and R. G. Woods study of the poset $S(X)$ of one-point metric extensions of a locally compact metric space X [4].
Using the notion of the extension traces from [1, 4], are constructed all one-point fc-extensions of the spaces. Let X be a non-empty space. The following assertions are equivalent:
X is not a pseudocompact space;
$E_{fc}(X) \neq \emptyset$;
$S_{fc}(X) \neq \emptyset$.
We also prove that for the Lindelöf spaces X and Y the posets $S_{fc}(X)$ and $S_{fc}(Y)$ are order-isomorphic if and only if the spaces $\beta X \setminus X$ and $\beta Y \setminus Y$ are homeomorphic.

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The Topological Groupoids with Multiple Identities

Liubomir Chiriac

Tiraspol State University, Moldavia
e-mail: llchiriac@gmail.com

We study the (n,m)-homogeneous isotopies of topological goupoids with multiple identities and relation between distributivity, paramediality and asociativity. The concept of multiple identities and homogeneous isotopies facilitates the study of topological goupoids with (n,m)-identities and homogeneous quasigroups. We prove some properties of a class of a (n,m)-homogeneous

Tools from combinatorial commutative algebra in discrete tomography

Mihai Cipu

"Simion Stoilow" Institute of Mathematics of the Romanian Academy. Bucharest,Romania

e-mail: Mihai.Cipu@imar.ro

Roughly speaking, tomography is a collection of techniques allowing one to construct an image from the scattering data associated with cross-sectional scans of an object. The appropriate mathematical problem is to determine a real function with bounded support knowing only its integrals along low-dimensional linear manifolds. This question has been considered almost one century ago by J. Radon. More recently has been a lot of interest in the discrete version of the problem, when the function is defined on a finite set of lattice points, takes only finitely many values, and the only available information is the sums of the function along all lines in a finite number of directions.
Such problems are ill-posed in the sense that for most instances solutions are not guaranteed to exist and even when they exist, solutions lack unicity and stability. Quantitative results about the sought-for functions are obtained with the help of algorithms inspired by ideas in linear algebra, multivariate calculus, probability theory, polynomial algebra and incorporating techniques ranging from iterative solutions of linear systems to Monte Carlo techniques to generating functions.
The aim of our talk is to add to the available arsenal results and techniques developed in combinatorial commutative algebra. The talk provides an outlook on the use of toric ideals, Markov bases, Gröbner bases in discrete tomography.

Hilbert - Mumford - Nagata Theorem on invariants and some topological aspects of the finite generation of subalgebras.I

Adrian Constantinescu

"Simion Stoilow" Institute of Mathematics of the Romanian Academy, Bucharest, Romania
e-mail: adrian.constantinescu@imar.ro

Let us recall the well known central Theorem of the Invariant Theory :
Theorem. (Hilbert - Mumford - Nagata) *Let k be an algebraically closed field, A a k-algebra of finite type and G a reductive (in particular linearly reductive) algebraic Lie group over k acting rationally on A. Then the subalgebra of invariants $A^G = \{a \in A | ga = a \text{ for each } g \in G\}$ is finitely generated over k.*
It is well known that the famous original proof of this Theorem is a highly technical and hermetic one.
In this talk we intend to present some more conceptual variants of some parts of this proof, by using some topological conditions for finite generation of subalgebras of algebras of finite type over a field k.
More precisely, we focus mainly on the following
Theorem. *Let $f : X \longrightarrow Y$ be a universally open surjective morphism of reduced schemes over a field k, with X an algebraic k-variety (i.e. X is of finite type over k). Then Y is also an algebraic k-variety.*
If k is the complex number field C, based on *Gelfand* topologies on the maximal spectra of C-subalgebras, we propose also another alternative topological proof for the finite generation of A^G.

Criterion for completeness relative to implicit reducibility in the chain super-intutionistic logics

Ion Cucu[1], Mefodie Raţiu[2]

[1] *Moldova State University, Chişinău, Republic of Moldova*

e-mail: cucuion2012@gmail.com

[2] *Academy of Sciences of the Republic of Moldova*

One examine chain logics C_2, C_3, ..., intermediary between classical and intuitionistic logics. They are also the logics of pseudo-Boolean algebras of type $< E_m, \&, \vee, \supset, \neg >$, where E_m is the the chain $0 < \tau_1 < \tau_2 < \ldots < \tau_{m-2} < 1$ $(m = 2, 3, \ldots)$. A formula F is called implicit expressible in the logic L by the system Σ of formulas, if the relation

$$L \vdash (F \sim q) \sim ((G_1 \sim H_1)\& \ldots \&(G_k \sim H_k)),$$

is true, where q do not appear in F, and formulas G_i and H_i, for $i = 1, \ldots, k$ are explicit expressible in L via Σ. The formula F is said to be implicit reducible in logic L to formulas of Σ if there exist a finite sequence of formulas C_1, C_2, ..., C_l where C_l coincide with F and for $j = 1, \ldots, l$ formula C_j is implicit expressible in L by $\Sigma \cup \{G_1, \ldots, G_{j-1}\}$. The system Σ is called complete relative to implicit reducibility in logic L if any formula of logical L is implicit reducible in L by Σ.
Present paper contains the criterion for recognition of completeness with respect to implicit reducibility in the logic C_m, for any $m = 2, 3, \ldots$. The criterion is based on 13 closed pre-complete classes of formulas.
E. Post obtained the criterion for functional completeness in classical logic which gives us an algorithm, permitting, for each finite system of Boolean function, given by formulas or tables, to recognise if it is possible to obtaine by superpositions any Boolean function. Analogous criteria of completeness have been obtained by I. Rosenberg (1965) in the general k - valued logic for $k > 2$, as well as in the propositional intuitionistic logic [M. Ratsa (1981)], etc. Each of these criteria is based on the finite number of closed (relative to expressibility in corresponding logic) classes of functions or formulas that are pre-complete (i.e. maximal between those not-complete).
Theorem. Relative to completeness with respect to implicit reducibility any chain logic is equal to one and only one of the next 4 logics: the absolute contradictory logic, the classical logic C_3 or C_4 logic.

Points stabiliser on the elements of simmetry in manifolds

Florin Damian

Moldova State University, Chişinău, Republic of Moldova
e-mail: fl_damian@yahoo.com

In the lecture we will discuss the geometry of hyperbolic manifolds in a term of the stabilizer of a point. On invariant submanifolds, with respect to a subgroup of the isometry group, points of general position have isomorphic stabilizers. A number of theorems of mathematical crystallography will be examined simultaneously on the universal covering space and on the factor spaces (manifolds). We will also discuss the role of the Dirichlet-Voronoi polyhedron in finding the point stabilizer, the principle of symmetry of symmetry elements [1] and its connection with geometric criterion of the normal subgroup.

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On the completion of Gieseking manifold

Ion S.Gutsul

Institute of mathematics and Computer Science of theAcademy of Sciences of Moldova, Moldavia
e-mail: igutsul@mail.ru

In the work [1] W.Thurston developed the theory of the completion of incomplete orientable hyperbolic manifold. But he did not consider the completion of incomplete non-orientable hyperbolic manifolds. It is possible to develop the theory of the completion of such non-orientable manifolds, too, but the method of Thurston cannot be used in this case. For this case the hyperbolic space should be considered from the point of view of synthetic geometry, i.e. Poincare models or some other models of the hyperbolic space cannot be used.
The present communication is devoted to the study of the geometry of the completion of so called Gieseking manifold. In the work [2] Gieseking constructed a complete non-orientable hyperbolic manifold with finite volume by giving a scheme of the identification of faces of the regular hyperbolic simplex with all the vertices being on the absolute. If we begin to deform the simplex keeping the scheme of the identification of its faces, we will obtain an incomplete manifold provided dihedral angles of the simplex satisfy the equations: $\alpha+\beta+\gamma=\pi, sin\alpha \times sin\beta = sin^2\gamma$. The last equation is obtained when considering the corresponding picture on a horosphere. Note that the horosphere plays a key role in our method. When completing this manifold we obtain a countable series of non-orientable orbifolds O_k if the dihedral angles of the simplex with all the vertices being on the absolute are as follows: $\alpha=\pi/2-\pi/(2k)-\gamma/2, \quad \beta=\pi/2+\pi/(2k)-\gamma/2$ and γ satisfies the equation $sin\gamma/2=\sqrt{5-\sqrt{25-16cos^2(\pi/2k)}}/2\sqrt{2}, k=2,3,...$
The volumes of the orbifolds O_k are bounded and form a convergent sequence, the limit of the sequence is the volume of the regular hyperbolic simplex with all the vertices being on the absolute.

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New generalizations of BCI and BCK algebras and of Hilbert algebras

Afrodita Iorgulescu

Academy of Economic Studies, Bucharest, Romania
e-mail: afrodita.iorgulescu@ase.ro

Hilbert algebras were introduced in a dual form in 1950, by Henkin. *BCK algebras* and *BCI algebras* were introduced by K. Iséki in 1966, as algebraic models of BCK-logic and of BCI-logic, respectively. Most of the commutative algebras of logic (such as residuated lattices, Boolean algebras, MV algebras, Wajsberg algebras, BL algebras, Gödel algebras, product algebras, Hilbert algebras, Heyting algebras, NM algebras, MTL algebras, IMTL algebras, R_0 algebras, weak-R_0 algebras etc.) can be expressed as particular cases of BCK algebras. The commutative groups are

particular cases of BCI algebras.
BCH algebras were introduced by Hu and Li in 1983.
BCC-algebras, also called *BIK*$^+$*-algebras*, were introduced by Y. Komori in 1984.
BZ algebras, also called *weak-BCC algebras*, were introduced in 1995, by X.H. Zhang and R. Ye.
BE algebras were introduced by H.S. Kim and Y.H. Kim in 2006.
Pre-BCK algebras were introduced in 2010 by D. Buşneag and S. Rudeanu.
In this paper, we proved in what conditions BCH algebras coincide with BCI algebras. While connecting the old algebras BCK, BCI, BCC, BZ, BCH, BE and pre-BCK algebras, we have obtained nine new algebras, which are generalizations of BCI and BCK algebras. We have also obtained generalizations of Hilbert algebras. Finally, we have obtained examples of all these new algebras.

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Group transversals in the infinite sharply 2-transitive permutation group

Eugene Kuznetsov

Institute of Mathematics and Computer Science, Academy of Sciences of Moldova, Chişinău, Republic of Moldova
e-mail: kuznet1964@mail.ru

Definition. *Let G be a group and H be its subgroup. Let $\{H_i\}_{i\in E}$ be the set of all left (right) cosets in G to H, and we assume $H_1 = H$. A set $T = \{t_i\}_{i\in E}$ of representativities of the left (right) cosets (by one from each coset H_i and $t_1 = e \in H$) is called a* ***left (right) transversal*** *in G to H.*
On any left transversal T in a group G to its subgroup H it is possible to define the following operation (*transversal operation*):

$$x \overset{(T)}{\cdot} y = z \quad \overset{def}{\Longleftrightarrow} \quad t_x t_y = t_z h,\ h \in H,$$

Definition. *If a system* $< E, \overset{(T)}{\cdot}, 1 >$ *is a loop (group), then such left transversal* $T = \{t_x\}_{x \in E}$ *is called a* ***loop (group) transversal****.*
An infinite sharply 2-transitive permutation group G is studied. As it was proved in [1], the set T of all elements of order 2 from G form a loop (stable) transversal in the group G to its subgroup $H = St_1(G)$. Moreover, this set T is a normal subset in the group G.
For an arbitrary (fixed) element $t_i \in T$ let us consider its centralisator $C_i = C_G(t_i)$:

$$C_i = C_G(t_i) = \{g \in G | gt_i = t_i g\}.$$

It is proved the following
Theorem. *The following statements are true:*

1. *For any* $i \in E - \{1\}$ *the set* C_i *is a left transversal in* G *to* H*,*
2. *For any* $i \in E - \{1\}$ *the left transversal* C_i *is a group transversal in* G *to* H*,*
3. *For any* $i, j \in E - \{1\}$ *transversal operations* $< C_i, \cdot >$ *and* $< C_j, \cdot >$ *are isomorphic.*

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The minor groups of $\overline{4}$-symmetry by the generating discrete groups of category G_{320}

Alexandru Lungu, Marina Roşca

Moldova State University, Chişinău, Republic of Moldova
e-mail: lungu.al@gmail.com

One of recent generalizations of classical symmetry is $\overline{P}-$symmetry [1]. The groups $G^{(P)}$ of $\overline{P}-$symmetry with the generating group G of classical symmetry, the defining group P (transitive group substitutions on the set N = {1,2, ..., m}), the kernel H of accompanying homomorphism $\varphi : G \to AutP$ and $Im\varphi = \Phi$ are subgroups, which verify certain conditions, of the right semi-direct product of P with G, in other words: $G^{(P)} \leq P \overrightarrow{\lambda}_{\varphi H(\Phi)} G$.
The minor groups of $\overline{P}-$symmetry are deriving from the groups P and G, when the kernel H of accompanying homomorphism φ is known, by the following steps: 1) to find in G all proper subgroup H' with the index equal to the order of P and for which there is the isomorphism χ of factor-group H/H'' and P' ($\chi : H/H'' \to P'$ by the rule $\chi(hH'') = p$), where $e \leq P' \leq P$ and $H'' = H' \cap H$; 2) to construct a right quasi-homomorphism ψ of the group G onto the P by the rule $\psi(gH') = p$ and which preserves the correspondence between the elements of H/H'' and P' received as the result of isomorphism χ; 3) to combine pairwise each g' of gH' with $p = \psi(g')$; 4) to introduce into the set of all these pairs the operation $p_i g_i * p_j g_j = p_k g_k$, where $g_k = g_i g_j$, $p_k = p_i \overrightarrow{\varphi_{g_i}}(p_j)$, $\overrightarrow{\varphi_{g_i}} = \varphi(g_i)$ and $\overrightarrow{\varphi_{g_i}}(p_j) = g_i p_j g_i^{-1}$.
From the 31 generating groups G of the category G_{320} (crystallographic tablet groups) are obtained 76 minor groups of $\overline{4}$-symmetry ($P \cong C_4$). The structure of groups inferred is described and theirs polynomial symbols are elaborated. Namely: $G|H'[(P, P_i)|P'; H/H'''/H'']$, where $H/H'' \cong P'$, $H'''/H'' \cong P_i^{'} (= P_i \cap P')$ and P_i is the stationary subgroup of the group P.

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An intrinsic characterisation of minimal surfaces

Sergiu Moroianu

"Simion Stoilow" Institute of Mathematics of the Romanian Academy, Bucharest, Romania
e-mail: sergiu_moroianu@yahoo.com

A Ricci surface is a Riemannian 2-manifold $(M; g)$ whose Gaussian curvature K satisfies the nonlinear degenerate elliptic equation

$$K\Delta(K) + g(dK; dK) + 4K^3 = 0$$

, where Δ is the Laplace operator. Every minimal surface isometrically embedded in $\mathbb{R}^3$ is a Ricci surface of non-positive curvature. At the end of the 19th century Ricci- Curbastro has proved that conversely, every point x of a Ricci surface has a neighborhood which embeds isometrically in $\mathbb{R}^3$ as a minimal surface, provided $K(x) < 0$. Together with Andrei Moroianu, we prove this result in full generality by showing that Ricci surfaces can be locally isometrically embedded either minimally in $\mathbb{R}^3$ or maximally in $\mathbb{R}^{2;1}$, including near points of vanishing curvature.

Neutrosophic information in the framework of multi-valued representation

Vasile Pătrașcu

Tarom Information Technology, Otopeni,Romania
e-mail: patrascu.v@gmail.com

The paper presents some steps for multi-valued representation of neutrosophic information. These steps are provided in the framework of multi-valued logics using the following logical value: true, false, contradictory, neutral, unknown and saturated. Also, this approach provides some calculus formulae for the following neutrosophic features: truth, falsity, contradiction, neutrali-ty, ignorance, entropy, definedness, and saturation. In addition, it was defined net truth, definedness and neu-trosophic score.

On some classes of functions

Dorin Pavel

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: dorinp@mail.md

Let $\mathbb{R}$ be the space of reals, $\mathbb{R}^\infty = \mathbb{R} \cup \{+\infty\}$. We assume that $x \leq +\infty$ and $x + (+\infty) = (+\infty) + x = +\infty$ for each $x \in \mathbb{R}^\infty$. On the space $\mathbb{R}^\infty$ we consider the topology with the open base $\{V_t = \{x \in \mathbb{R}^\infty : t < x\} : t \in \mathbb{R}\}$. Then $\mathbb{R}^\infty$ is a T_0-space.
A pseudo-quasimetric on a set X is a function $\rho : X \times X \to \mathbb{R}^\infty$ with the properties $\rho(x, x) = 0$

and $\rho(x,z) \leq \rho(x,y) + \rho(y,z)$ for all $x,y,z \in X$. The pseudo-quasimetric ρ is a quasimetric if $\rho(x,y) + \rho(y,x) = 0$ if and only if $x = y$. If ρ is a pseudo-quasimetric and $\rho(y,x) = \rho(x,y)$ for all $x,y \in X$, then ρ is called a pseudometric. If ρ is a pseudometric and a quasimetric, then ρ is called a metric.
On the space $F(X \times X)$ of all functions on a space $X \times X$ consider the topology generated by distance of uniform convergence $d(f,g) = sup\{|f(x,y) - g(x,y)| : x,y \in X\}$.
In the space $F(X \times X)$ we investigate the distinct closed subspaces. For instance, the subspaces:
- $F_s(X \times X) = \{g \in F(X \times X) : g(x,y) = g(y,x)$ *for all* $x,y \in X\}$ of all symmetrical functions;
- $F_d(X \times X) = \{g \in F(X \times X) : g(x,y) + g(y,x) = 0$ *imply* $x = y$, $g(x,y) \geq 0$ *for all* $x,y \in X\}$ of all distantial functions;
- $F_t(X \times X) = \{g \in F(X \times X) : g(x,z) \leq g(x,y) + g(y,z)$ *for all* $x,y,z \in X\}$ of all triangular functions.
We study some dense subsets of these subspaces.

Middle Bol loops

Syrbu Parascovia

Moldova State University, Chişinău, Republic of Moldova
e-mail: syrbuviv@yahoo.com

A groupoid $(Q,\cdot)$ is said to be a loop if the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions in Q, for every $a,b \in Q$, and there exists an element $e \in Q$, such that $e \cdot x = x \cdot e = x$, for every $x \in Q$. A loop $(Q,\cdot)$ is called a right (left) Bol loop if it satisfies the identity $(xy \cdot z)y = x(yz \cdot y)$ (respectively, $x(y \cdot xz) = (x \cdot yx)z$) [1,2]. The theory of left (right) Bol loops can be rewritten by a mirror reflection, using the theory of right (left) Bol loops [3]. If a loop is both a left and a right Bol loop then it is a Moufang loop.
A loop $(Q,\cdot)$ is said to be a middle Bol loop if the identity $(x \cdot y)^{-1} = y^{-1} \cdot x^{-1}$ (called the anti-automorphic inverse property) is universal in $(Q,\cdot)$, i.e. if every loop isotope of $(Q,\cdot)$ satisfies the anti-automorphic inverse property. Middle Bol loops have been defined by V. Belousov in [1] and are considered, for example, in [1,4-7]. It is shown in [1] that a loop $(Q,\cdot)$ is middle Bol if and only if the corresponding primitive loop $(Q,\cdot,/,\backslash)$ satisfies the identity

$$x(yz\backslash x) = (x/z)(y\backslash x), \tag{1}$$

and that (1) is universal.
Middle Bol loops are isostrophes of right (left) Bol loops [7]. More precisely, if $(Q,\cdot)$ is a right (left) Bol loop and $(Q,\circ)$ is the corresponding middle Bol loop then $x \circ y = y^{-1}\backslash x$ and $x \cdot y = y//x^{-1}$ (resp., $x \circ y = x/y^{-1}$ and $x \cdot y = x//y^{-1}$), where "/" ("\") is the left (right) division in $(Q,\cdot)$ and "//" is the left division in $(Q,\circ)$) [4,5]. Invariants under this isostrophy are studied in the present work. It is known [4,5] that two middle Bol loops are isotopic (isomorphic) if and only if the corresponding right (left) Bol loops are isotopic (isomorphic). Hence, the isotopy-isomorphism property [1,2] is invariant under the considered isostrophy. The "behavior" of the autotopisms, automorphisms, pseudo-automorphisms, congruences and normal congruences under the considered isostrophy was partially studied in [4,5]. New connections between the structure of middle Bol loops and those of the corresponding right (left) Bol loops are described and some related groups (the multiplication groups, the groups of regular substitutions, the groups of pseudo-automorphisms (left, right, middle [10])) are considered. In particular, we announced in [6] that the commutant of a middle Bol loop $(Q,\circ)$ (i.e. the set of all elements of Q which commute with all elements of Q) is a subloop in $(Q,\circ)$. It is known that the commutant of a Moufang loop is a subloop but there exist (left, right) Bol loops which commutants are not subloops [8,9].
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Infinitely many precomplete relative to parametric expressibility classes of formulas in the maximal non-tabular extension of the provability logic

Andrei Rusu

Information Society Development Institute, A.S.M, Chişinău, Republic of Moldova
e-mail: andrei.rusu@idsi.md

Artificial Intelligence (AI) systems simulating human behavior are often called intelligent agents. These intelligent agents exhibit somehow human-like intelligence. Intelligent agents typically represent human cognitive states using underlying beliefs and knowledge modeled in a knowledge representation language, specifically in the context of decision making [1]. In the present paper we investigate some functional properties of the underlying knowledge representation language of intelligent agents which are based on the provability logic G [2].

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Implication filters in BL-algebras

Borumand Saeid, Neda Mohtashamnia

Shahid Bahonar University of Kerman, Iran
e-mail: a_b_saeid@yahoo.com

We introduce the notion of implication filters in BL-algebras and then state and prove some theorems which determine the relationships of these filters and other filters in BL-algebras.

Some results in t-convex vague sets

Borumand Saeid, Marjan Kuchaki Rafsanjani

Shahid Bahonar University of Kerman, Iran
e-mail: a_b_saeid@yahoo.com

In this paper, we introduce the notion of t-convex vague sets and study their properties in details.

On the lattice of quasivarieties of quasigroups

Vasile Ursu

"Simion Stoilow" Institute of Mathematics of the Romanian Academy, Bucharest, Romania
e-mail: Vasile.Ursu@imar.ro

It is proved that if a lattice of subquasivarieties of a quasivariety of quasigroups is modular, then it is distributive. On the other hand, the lattice of subvarieties of any variety of quasigroups is modular. More examples of quasivarieties of quasigroups are constructed, the subquasivarieties of which make non-modular lattices.

A matrix approach for divisibility properties of the generalized Fibonacci numbers

Aynur Yalciner

Selcuk University, Turkey
e-mail: aynuryalciner@gmail.com

In this talk we give divisibility properties of the generalized Fibonacci numbers by matrix methods. We also present new recursive identities for the generalized Fibonacci numbers.

6. Computer Science

Algebraic structure of the ordered tables

Lucia Bitcovschi

Moldova State University, Chişinău, Republic of Moldova
e-mail: bitco@mail.md

In this communication is described in detail one of the most important and used data structures - ordered table. Formal definitions have been introduced for registration table, key, table, ordered table. The properties of ordered tables and operations on them in terms of algebraic structures is investigated. It was proposed in C++ the implementation of a classes generic parameterized with abstract class to represent tabular entries that allow modelling work with tables sorted in terms of algebraic structures.
Let $E = \{e_1, e_2, ..., e_n\}$ be a set of all tabulated records by the same structure with highlighted key. For key is established order relation which allows us ordering tabular records by key.
Definition. *By the ordered table on the set E, marked* $T(e_{i_1}, e_{i_2}, ..., e_{i_k})$, *we understand the ordered set* $\{e_{i_1}, e_{i_2}, ..., e_{i_k}\}$, *where* $e_{i_j} \in E, j = 1, 2, ..., \kappa$, *as* k *– is the number of elements in the table* $T(e_{i_1}, e_{i_2}, ..., e_{i_k})$, *and the element* e_{i_1} *– is called the first element of the table T, and* e_{i_k} *– the last element of the table T.*
Rremark. *The ordered table may be empty, if it has no record. We will denote a empty ordered table, with* $\emptyset$*", so T() =*$\emptyset$*. Ordered table* $T(e)$ *is composed of a single element* $e \in E$.
Denote by $\Re = \{\mathrm{T}_1, \mathrm{T}_2, ..., \mathrm{T}_m, ...\}$ the set of all possible ordered tables defined on the set E. It was introduced a set of operations for ordered tables similar to those described in [1,2].
It is demonstrated that the ordered tables have the neutral element and is resoluble.
Theorem. *The groupoid* $(\Re, +)$ *is semigroup.* This demonstrate that *semigroup* $(\Re, +)$ is a commutative monoid.

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On the impact of explicit or semi-implicit integration methods over the stability of real-time numerical simulations

Teodor Cioacă, Horea Cărămizaru

"POLITEHNICA" *University of Bucharest, Romania*
e-mail: t.cioaca@gmail.com

Physics-based animation of soft or rigid bodies for real-time applications often suffers from numerical instabilities. We analyse one of the most common sources of unwanted behaviour: the numerical integration strategy. To assess the impact of popular integration methods, we consider a scenario where soft and hard constraints are added to a custom designed deformable linear object.

Since the goal for this class of simulation methods is to attain interactive frame-rates, we present the drawbacks of using explicit integration methods over inherently stable, implicit integrators. To help numerical solver designers better understand the impact of an integrator on a certain simulated world, we have conceived a method of benchmarking the efficiency of an integrator with respect to its speed, stability and symplecticity.

Towards a hybrid wavelet-based multiresolution representation of terrain data

Teodor Cioacă, Bogdan Dumitrescu, Mihai-Sorin Stupariu

"POLITEHNICA" *University of Bucharest, Bucharest, Romania*
e-mail: t.cioaca@gmail.com

Handling, processing and carrying out numerical simulations on terrain surfaces is an intricate task, concerning a wide and interdisciplinary audience. We propose a method capable of filtering data stored at the vertices and edges of a triangular irregular network. The data, obtained through either the sampling of a heightmap or gathered from LiDAR scans, is subjected to a feature detection preprocessing stage. A feature based heuristic mesh decimation (via estimating the discrete thin plate energy at each vertex) is employed to help preserve important mesh vertices, while producing a multiresolution hierarchy. This data representation can be subsequently treated using a graph-based wavelet lifting scheme, taking advantage of the mesh connectivity and edge weights. Due to the nature of our measurements, a large palette of numerical information can be synthesized using this approach.

Comparative Study of Some Algorithms for Face Recognition

Lăcrămioara Liță, Elena Pelican

"Ovidius" University of Constanţa, Constanţa, Romania
e-mail: epelican@univ-ovidius.ro

Face recognition is one of the most prevalent problems of pattern recognition and a current issue in the context of nowadays technology progress. This real-life problem needs real-time answers, so a variety of algorithms have been developed to address this issue. In this paper we present a comparative study for some standard algorithms and an own algorithm for face recognition. The algorithms are tested on benchmark datasets and on an own dataset. Experiments are also conducted on a bigger face dataset that includes different datasets.

Greedoids and unique perfect matchings

Vadim E. Levit[1], Eugen Mândrescu[2]

[1] *Ariel University Center of Samaria, Israel*
e-mail: levitv@ariel.ac.il

[2] *Holon Institute of Technology, Israel*
e-mail: eugen_m@hit.ac.il

Many problems in combinatorial optimization can be described using algebraic structures called matroids, which permit greedy algorithms to find optimal solutions. Introduced as generalizations of matroids, greedoids capture more problems whose solutions can be found by greedy methods. Further, it has been revealed that for many optimization problems, in order to run the greedy algorithm it was enough to have a structure satisfying a weaker than the hereditary property, namely, the accessibility property.
A set S is a local maximum stable set of graph G if S is a maximum stable set of the subgraph induced by its closed neighborhood. Let $\Psi(G)$ denote the family of all local maximum stable sets. Nemhauser and Trotter Jr. (Mathematical Programming, 1975) proved that any $S \in \Psi(G)$ is included in a maximum stable set of G, and interpreted this assertion as a sufficient local optimality condition for a binary integer programming formulation of the weighted maximum stable set problem.
It is known that $\Psi(G)$ forms a greedoid for every forest G. Bipartite graphs, triangle-free graphs, unicyclic graphs G whose $\Psi(G)$ is a greedoid were also characterized (by exhibiting a close relationship between some special matchings and local maximum stable sets of a graph) (V. E. Levit and E. Mandrescu, Discrete Applied Mathematics, 2002, 2004, 2007). Recently, it was also shown that $\Psi(G)$ forms a greedoid (in fact, an interval greedoid), whenever it satisfies the accessibility property (V. E. Levit and E. Mandrescu, Discrete Mathematics, 2012).
In this work we combine topics of well-covered graphs and greedoids, by showing that if $G = (V, E)$ is a very well-covered graph (i.e., all its maximal stable sets are of the size $|V|/2$), then $\Psi(G)$ forms a greedoid if and only if G has a unique perfect matching (V. E. Levit and E. Mandrescu, Discrete Applied Mathematics, 2012).

A new Intrusion Detection System based on MLP Neural Network and Parallelism Technique

Marjan Kuchaki Rafsanjani, Mohammad Hassan Nataj, Arsham Borumand Saeid

Shahid Bahonar University of Kerman, Iran
e-mail: marjankuchaki@yahoo.com

Nowadays, with the expansion of the computer networks, Network Security more than ever, has become an issue of concern to the researchers. That is why Intrusion Detection is known as one of the key factors in securing computer networks. Intrusion Detection and Prevention in computer networks is one of the main issues of security conferences of advanced societies. Hence, in order to act against the attacks, many different methods have been implemented in the form of Intrusion Detection Systems (IDS) which are applied in computer networks. The main goal of an Intrusion Detection System is to control the network traffic and analyze the users' behaviors. Therefore, the activities of a system are fallen into 2 categories by intrusion detection systems:

"normal activities" and "intrusive (suspected) activities". Intrusion Detection Systems usually determine the type of the attacks or classify them into specific groups. In this research, we applied Neural Network with Supervised Learning in order to implement the intrusion detection system. Moreover, in this project, we used the method of Parallelization with real time application of the system processors to detect the system's intrusions. Using this method enhanced the speed of the intrusion detection. In order to train and test the neural network, NSLKDD database was used. Creating some IDS layers each of which is considered as a single Agent, we start to detect the network intrusions precisely. In this proposed design, we have classified the attacks in 4 groups and detect each group by using an Agent equipped with IDS. These Agents act independently and report the intrusion or non-intrusion in the system; the results achieved by the Agents will be studied in the Final Analyst and at last the analyst reports that whether there has been an intrusion in the system or not.

A survey on replication methods in data grid

Marjan Kuchaki Rafsanjani[2], Zahra Miroei[1]

[1] *Islamic Azad University, Kerman, Iran*

[2] *Shahid Bahonar University of Kerman, Iran*

e-mail: marjankuchaki@yahoo.com

Data grid is a set of computational sources and distributed storage that is not restricted to a certain storage place. One of important tasks of data grid is to share data, increase data availability and reliability. Today, considering scientific applications that produce numerous data, sharing and availability of data by users and researchers throughout the world, grid systems have become very important and considerable for researchers. One of methods that increase data availability and reliability is data replication in different sites of the system. Many researchers have investigated on this issue and offered several algorithms. In present paper, researches done in this context will be studied and compared.

Workflow Petri nets in modeling of parallel processes

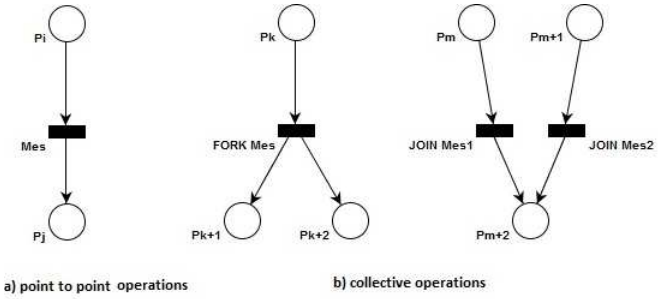
Inga Titchiev

'Institute of Mathematics and Computer Science, Moldavia

e-mail: inga.titchiev@gmail.com

Parallel computing is a form of computation in which many calculations are carred out on a parallel machine simultaneously, operating on the principle that large problems can often be divided into smaller ones, which are then solved concurrently (in parallel)[1]. Efficient processing of parallel algorithms in time and space can bring significant benefits in various fields such as medicine, biology, astronomy, forensics.

One of the standards that ensure a maximum portability of applications is OpenMPI. To represent existing operations in OpenMPI, will be used the formalism of workflow Petri nets[2]. They allow to model this thing in a very intuitive graphical manner. In MPI two types of operations exist: *point to point operations* and *collective operations.*



Open MPI operations by means of workflow Petri nets

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Specifying complex systems: the Petri nets formalism

Cristian Vidrașcu

"Alexandru Ioan Cuza" University of Iaşi, Iaşi, Romania

e-mail: vidrascu@info.uaic.ro

An important area of research in computer science is that of models for parallel, concurrent and/or distributed systems. Finding suitable mathematical models is important in order to cope with the increasing complexity of real systems and for studying their properties. There is a wide range of studies dedicated to this topic and among the most frequent approaches we can enumerate: Petri nets, process calculi (CSP, CCS, ACP, pi-calculus etc.), abstract state machines and temporal logic (LTL, CTL, TLA etc.). Petri nets were introduced by C. A. Petri in the early 1960s as a graphical and mathematical tool for modelling such systems, and they proved to be a proper formalism for specifying and analyzing information processing systems that are characterized as being parallel, concurrent, distributed, asynchronous, nondeterministic, and/or stochastic. As a graphical tool, Petri nets can be used as a visual-communication aid similar to flow charts, block diagrams, and networks. As a mathematical tool, it is possible to set up state equations, algebraic equations, and other mathematical models governing the behaviour of systems. Petri nets are a powerful language for system modelling and validation. They are now in widespread use for a very wide variety of applications because of their generality and adaptability. They have been successfully used for concurrent and parallel systems modelling and analysis, communication protocols, performance evaluation and fault-tolerant systems. Because of their numerous applications in areas like engineering, economics, medicine, education and science, Petri nets became a very prolific research field soon after their introduction, such that today the Petri nets research is materialized in a large number of publications in prestigious journals and conference proceedings. Among the most important research centers where Petri nets are studied one can mention: University of Hamburg, Technical University of Munich, Humboldt University of Berlin, University of Aarhus, Eindhoven University of Technology, etc.

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7. Education

Formarea gândirii probabiliste la elevi şi modelarea unor experimente aleatoare

Olga Benderschi, Andrei Poştaru

Unviersitatea de Stat a Moldovei, Chişinău, Republica Moldova
e-mail: obenderschi@yahoo.com

Teoria probabilităţilor este o structură matematică, care verifică un anumit sistem de axiome. Acest concept stă la baza oricărui curs de Teoria probabilităţilor în şcoala superioară. Accentul se pune pe însuşirea aparatului matematic, necesar la examinarea modelelor probabiliste. În liceu o astfel de abordare nu pare raţională.
În liceu predarea elementelor de teoria probabilităţilor are următoarele scopuri: în primul rând, elevul trebuie să se familiarizeze cu unele legităţi care apar în procesele în care factorul aleator intervine în mod esenţial; în al doilea rând, elevul trebuie să înveţe să creeze modele matematice probabiliste pentru unele situaţii (fenomene) reale; în al treilea rând, elevul trebuie să poată să utilizeze aceste modele matematice pentru studierea situaţiilor reale şi pentru a prognoza rezultatele experimentelor respective. Aceste competenţe trebuie să asigure formarea la elevi a unei gândiri probabiliste.
Scopurile menţionate urmează să fie atinse cu ajutorul unui sistem complex de exerciţii (rezolvate sau propuse spre rezolvare). În principiu, aici este vorba de exerciţii care se referă la experimente aleatoare devenite ëlasice; legate de aruncarea unei monede (sau a câtorva monede), aruncarea zarului (o dată sau de câteva ori), alegerea la întâmplare a unui punct (sau a câtorva puncte) dintr-o mulţime oarecare de puncte, extragerea unei bile (sau a câtorva bile) dintr-o urnă etc. Modelarea unor astfel de experimente este, credem noi, un suport bun, dar şi un instrument eficient pentru crearea la elevi a unei gândiri probabiliste. Modelarea se poate efectua utilizând tabele de numere aleatoare sau elaborând programe în diverse limbaje de programare.

Role of matematics in high school physics exam preparation

Viorica Cornelia Bronţ

Liceul Tehnologic "IOAN BOCOCI" Oradea, Romania
e-mail: vigyorika@yahoo.com

Our paper focuses on baccalaureate exam preparation by insisting on the physical side of mathematics, on problem solving. The remembering of math concepts they learned, and of applied physics. Concentration in classes IX and X on fundamental areas of classical physics is justified as follows:There are areas whose applications in technical and everyday life are most commonly encountered;Are accessible areas both on the students understanding of scientific ideas on fundamental and experimental approach;There are areas known to students of previous classes and thus allow the cover any gaps in their previous learning and curricular standards and overcome achieving outstanding academic performance. Framework objectives: Identifying data and mathematical and physical relations and their correlation depending on the context in which they were defined.

Descompunerea in factori ireductibili a polinoamelor omogene f(a,b,c) de grad 3

Bogdan Cânepâ
e-mail: bogdan_canepa@yahoo.com

In aceasta lucrare prezint cateva aplicatii ale geometriei algebrice in obtinerea unor identitati in 3 variabile. Aceste identitati de tip A=B, odata obtinute, pot fi verificate prin calcul direct si de catre un elev de liceu dar problema reciproca nu e la fel de simpla: daca ni se da termenul din stanga A, cum putem gasi termenul din dreapta B? Geometria algebrica ne furnizeaza o metoda generala prin care putem da raspuns la aceasta intrebare. Aceste aplicatii sunt interesante pentru studentii care au urmarit un curs de baza de geometrie algebrica. In plus, pun in evidenta capacitatea acesteia de a rezolva probleme de algebra pe o cale pur geometrica, utilizand teoreme si rezultate referitoare la curbe, singularitati si multiplicitati.

The perspective of projecting a schedule based on the integrated approach of mathematics curriculum in secondary education system

Cerasela Liliana Cozoș

"Ion Bănescu" High School
e-mail: ceracozos@yahoo.com

At the European and international levels, the educational policies sustain the reorganization of matters promoted by schools according to the relation theory-practice. At present, Romanian education faces the following dilemma: students are not sufficiently prepared in order to transfer the scientific information achieved during their school classes, to the everyday context they meet. A number of students are not able to find or to keep a job, because they are unable to apply concretely in practice what they have learned at school. The present study proposes a new approach of mathematics curriculum corresponding to secondary education. It focuses on a module concept and on its organization method from the perspective of projecting a schedule based on the integrated approach of this curriculum. Within this paper, I propose some methodological guidelines which could be used for developing general and "transversal" skills expected at the level of each thematic module and of each year of study for the upper cycle of secondary education (classes XI-XII).

Aspecte metodice privind elaborarea unui demers didactic pentru formarea competenelor specifice la matematic n nvmntul gimnazial

Neculae Dinuță

Universitatea din Piteşti, România
e-mail: neculae.dinuta@yahoo.com

Articolul prezintă un aspect important al proiectării didactice, acela al elaborării unui demers didactic pentru formarea competenţelor specifice la matematică înînvăţământul gimnazial. În prima parte, sunt prezentate o serie de aspecte teoretice privind modul de formare al competenţelor, prin acţiuni de iniţiere în metodologia predării-învăţ ării-evaluării matematicii, pentru a putea

conştientiza conţinutul şi algoritmul structurării adecvate a acestor acţiunii, ca proces legic formator de performanţe. Partea a doua prezintă câteva aspecte metodice privind modul concret de elaborare a unui demers didactic pentru formarea competenţelor specifice, la o secvenţă didactică a programei şcolare pentru clasa a VII-a. În acest sens, pentru fiecare competenţă specifică s-au formulat obiectivele operaţionale, s-au selectat conţinuturile matematice, s-a adoptat strategia didactică şi s-au elaborat instrumentele de evaluare. Articolul se încheie cu câteva concluzii despre importanţa cunoaşterii traseului legic de proiectare a competenţelor specifice şi de finalizare prin stabilirea unor obiective de evaluare, ce evidenţază, nu numai comportamentul elevului, ci şi modul de manifestare al său.

Teacher training programs focused on innovative blended learning practices in Math Education

Silvia Făt

Dept. of Teacher Training, University of Bucharest, Romania
e-mail: silvia.fat@g.unibuc.ro

This article presents a new project implemented recently in Romanian educational system (POSDRU/87/1.3/S/62882) and focusing on the use of ICT in Math teacher training. This is one of the various innovative initiatives of the national teacher training programs based on the newest tendencies in curriculum design, on diversity of the structure of the training, and flexibility of the transferable credits according to the national training standards. Interactive strategies especially created with modern digital resources (e-learning practices) are an important development aspectof the current teacher career. This program is developed in the blended learning system, one of the most modern concepts of training pedagogy, even if it has been little known in what concerns the professional development of the teachers. Providing an efficient offer for development of teachers creates obvious benefits to the concrete practices of teaching and learning in Romanian schools.

Dreapta în spaţiul euclidian tridimensional prezentaťa cu ajutorul calculatorului

Raluca Mihaela Georgescu

'University of Pitesti, Romania
e-mail: raluca.georgescu@upit.ro

Se prezinta o metodă de abordare cu ajutorul mediului Maple a unor probleme de geometrie analitică referitoare la dreapta in spatiul euclidian tridimensional. Problemele sunt rezolvare mai nti clasic, la tablă, urmărind aplicarea formulelor prezentate la curs, apoi, aceleaşi probleme se rezolvă cu ajutorul mediului Maple, urmărindu-se şi reprezentarea grafică a soluţiei.

Examinarea unui model didactic de predare-învăţare a compartimentului "Modelare şi metode de calcul" în cursul liceal

Liliana Mihalache, Chiriac Liubomir

Universitatea de Stat Tiraspol, Chişinău, Republica Moldova
e-mail: lilia.mihalache@gmail.com

În lucrare este examinat un model didactic de predare-învăţare a compartimentului "Modelare şi metode de calcul", treapta liceală

The growth of efficacy of network technologies with introduction of the integrated model of estimation

Margarita Nizhegorodova

Tiraspol State Univesity, Chişinău, Republic of Moldova
e-mail: m.nizhegorodowa@gmail.com

The network technologies training are an important part of the training of specialists in the field of computer science. The labor market has a growing need for specialists who are competent and skilled in the usage of modern network technologies, and are also able to work effectively in their professional sphere according to the world standards and are ready for a permanent professional growth. Effectiveness of the learning process of the unit Computer Network Technology disciplines depends on the used methods and training tools, and techniques for objective assessment of learning outcomes.

Integral estimation of the training level of university students of Direction " Computer Science and Engineering"

Svitlana Pomian

Tiraspol State University, Chişinău, Republic of Moldova
e-mail: svpomian@mail.ru

The current labor market tends to produce more and more stringent requirements for the qualification of the graduates. In order to meet the demands of the labor market, it is necessary to have a clear system of governance of the educational process of higher education institution, which should be sufficiently tight with one hand and flexible on the other. To describe and compare the models the key concepts of qualification are used: products, product quality, quality levels, quality's measurement and assessment, methods for evaluating the quality. Evaluation of the quality of education is an attempt to translate qualitative characteristics of the object, in this case the educational process in its broadest sense, diverse, multifaceted, multidimensional into quantitative. The object of study is a level of professional competence (training), of the graduate students, focused on a narrow area in the future professional activity. The subject of the study is the formation of an integrated assessment of the students2019 training level, the choice of one of the possible areas of professional activity that matches best the level of training of graduate students and a quantitative assessment of the training level.

Invarianţi proiectivi în geometria constructivă

Sergiu Port

UPS "Ion Creangă" Chişinău, Republic of Moldova
e-mail: serghport@yahoo.com

Din cursul de geometrie proiectivă este cunoscută noţiunea de corespondenţă polară în raport cu cercul.
Deoarece figura F ce posedă proprietăţii ale proiectării centrale poate fi construită cu rigla unilaterală, în problemele propuse construcţiile sunt executate numai cu rigla unilaterală.

Problema 1.
Este dată polara d, construiţi polul în raport cu cercul ω numai cu rigla unilaterală.
Construcţie (fig. 1)

1. $\{C, D\} \notin \omega(A, AB) : \{C, D\} \in d$
2. $\{E, H\} \in \omega$
3. $(CE) \cap \omega = G,\ (CH) \cap \omega = F$
4. $(CF) \cap (EH) = I$
5. $[EF] \cap [GH] = J$
6. $\{K, L\} \in \omega$
7. $(DK) \cap \omega = N,\ (DL) \cap \omega = M$
8. $(NM) \cap (KL) = O$
9. $[MK] \cap [NL] = P$
10. $(OP) \cap (IJ) = Q, Q$ polul construit

Demonstraţia se bazează pe definiţia corespondenţei polare în raport cu un cerc şi proprietăţii: dacă punctul C se află pe polara punctului I, atunci şi I se află pe polara punctului C.

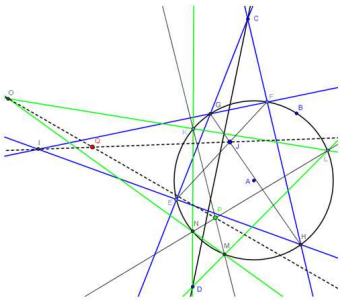

Fig. 1

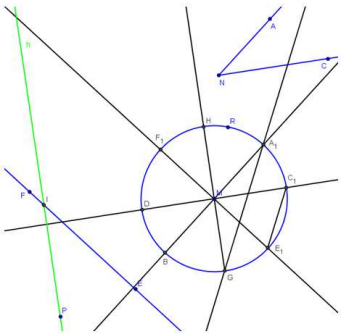

Fig. 2

În cadrul cercetării vom accentua, că construcţia dată este valabilă pentru orice dreaptă, considerată polară, ce intersectează în două puncte cercul ω. Iar în cazul când polara este tangentă, atunci însuşi punctul de tangenţă este pol. Pentru cazul când polara nu intersectează cercul ω, polul se află în interiorul cercului ω, iar construcţia va fi asemănătoare celei date.

Problema 2.
Se dă cercul $\omega(M, MR)$. Să se construiască, doar cu rigla unilaterală, prin punctul dat P o dreaptă ce va forma cu dreapta dată FE, un unghi egal cu unghiul dat ANC.
Construcţie (fig. 2)

1. $(C_1D) \| (NC) :\ \{C_1, D\} \in \omega,\ M \in [C_1D]$
2. $(A_1B) \| (AN) :\ \{A_1, B\} \in \omega,\ M \in [A_1, B]$
3. $(F_1E_1) \| (FE) :\ \{F_1, E_1\} \in \omega,\ M \in [F_1E_1]$
4. $(A_1, G) \| (C_1E_1) : G \in \omega$
5. $[GM) \cap \omega = H$
6. construim o dreaptă h prin $P \| (HG)$
7. $h \cap FE = I$

(PI)dreapta căutată

Demonstraţie: Încât $(A_1G)\|(C_1E_1) \Rightarrow [A_1C_1] = [GE_1] \Rightarrow \angle GME_1 = \angle A_1MC_1 = \angle ANC \Rightarrow \angle PIE = \angle ANC$.
Construcţia dată este valabilă pentru orice poziţie a datelor. Iar în cazul când centrul cercului coincide cu vârful unghiului construcţia devine mai simplă deoarece se omit unii paşi.

Metasystems Learning Design in Digital Textbook Use and Development

Elena Răilean

Academy of Sciences of Moldova, Moldavia
e-mail: elenarailean32@gmail.com

Metasystems Learnning Design approach drove scientists to explore new ways of solving issues related to interface and content design. The modern digital textbooks are assumed to be both the pedagogical resource and the learning tool. However, the approach of metasystem thinking goes beyond linear, system, systemic or critical thinking. This means that the procedure of transformation from one state to other makes it something more than a set of individual procedures. As was noted by Flood (2010, p. 269-270) the world is systemic, which means that phenomena are understood to be an emergent property of an interrelated world. Emergence and interrelatedness are the fundamental ideas of systems thinking. An emergent property of a whole arise where a phenomenon cannot be comprehended in terms only of properties of constituent parts. Instead of this, the whole is greater than the sum of its parts. This is well-known synergistic effect. But, how to obtain the synergistic effect in learning environment with real students? How to engage all students in learning? How to keep intrinsic motivation? The aim of this article is to analyse the similarities and differences between linear, systems and metasystems thinking. Our hypotheses is that metasystems thinking involves modelling of learning processes on the base on inter-dependencies between cognitive systems, personal learning environment and theirs transformations during learning. We used the classic understanding of "metasystems", proposed by Klir (1990, p. 325): meta X is the name of things or systems, which are bigger than X in sense that it is more organized, have higher logical type or it is analysed in more general sense (where X-system). In our understanding the metasystems thinking can be developed, if learning will be design on the base on metasystems cross-principles. Theirs norms of application require that students will personalize the content and will automate the core concepts at the level of application (according to Bloom taxonomy) using self-assessment tests, concept mapping tool and writing conclusions. Mathematically, the concept of metasystems thinking can be modelling through knowledge graph and probabilistic causation models, using Bayes' rule. The core idea is to integrate the set of variables V and parameters N with processes P, which could be represented in a optimized knowledge graph structure.

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Problems of teacher training for activities with children gifted in mathematics

Larisa Sali

Universitatea de Stat Tiraspol, Chişinău, Republica Moldova
e-mail: salilarisa@yahoo.com

The purpose of the research is the establishment of epistemological, theoretical and practical benchmarks for the development of a pedagogical model for future mathematics teachers' training in organizing and conducting educational activities with gifted and talented children. In this context, the following are elaborated: a conceptual model that describes the interconnection of the components of the process of professional formation of mathematics teachers at diverse levels; a set of recommendations concerning the creation of optimal conditions for the formation of competencies in organizing and deploying extracurricular activities, activities with children gifted in mathematics and the monitoring of these processes.

Formation of competencies in Geometry from a constructive perspective in the 5^{th} and 6^{th} Grades

Larisa Sali

Universitatea de Stat Tiraspol, Chişinău, Republica Moldova
e-mail: salilarisa@yahoo.com

According to Piaget, the principles of learning are an active process and should be whole, authentic and real. Human cognitive development is a continually adaptive process of assimilation, accommodation, and correction. The cognitive constructivist theoretician S. Papert characterizes behavioral approaches as "clean" teaching, whereas constructivist approaches are "dirty" teaching. The contrast emphasizes the differences between approaches that isolate and break down knowledge to be learned (clean) and approaches that are holistic and authentic (dirty). A bit more detail describing the contrast of the two approaches to instruction - didactic (behavioral) and constructivist was offered by J.V.Copley. Students must be given opportunities to construct knowledge through their own experiences and through more emphasis on learning in a meaningful context. Technology provides essential tools with which to accomplish the goals of a constructivist classroom. Constructivist approaches to technology in the classroom are not yet commonplace. In this presentation, we describe several types of cognitive constructivist learning environments designed to shape geometrical abilities in the 5^{th} and 6^{th} grades based on Robert O. McClintock's and John B. Black's model of learning environments. The recommendations are elaborated taking into account the psycho-pedagogical particularities of 11-13 years old children. We offer a description of an instructional approach using some aspects of the GEOGEBRA software that supports constructivist learning environments.

Matematica şi ştiinţele economice î n societatea cunoaşterii

Liliana-Camelia Săndulescu, Mariana Vladu

Universitatea de Stat Tiraspol, Chişinău, Republica Moldova
e-mail: lucsandra@yahoo.com

Documentele Uniunii Europene pun în evidenţă faptul că, societatea bazată pe cunoaştere face posibil accesul la informaţie şi noi tehnologii, prin noi forme de interacţiune socială şi de expresie culturală, oferind oportunităţi de a participa şi a influenţa dezvoltarea societăţii în care trăim.
Consiliul European din martie 2000 de la Lisabona a stabilit ca obiectiv strategic, pentru următorul deceniu, ca Europa să aibă economia cea mai competitivă şi dinamică din lume, bazată pe cunoaştere, capabilă de o creştere durabilă. Toate acestea sunt posibile printr-o transformare fundamentală a educaţiei din întreaga Europă, pentru a putea dezvolta noi competenţe, atitudini şi abilităţi necesare pentru competitivitate pe piaţa muncii. În acest context, au fost adaptate sistemele educaţionale la realităţile mediului socio-economic actual, prin revizuirea politicilor educaţionale, ce a dat naştere unui nou concept educaţional, sintetizat prin sintagma învăţarea centrată pe competenţe. La nivel european s-au parcurs mai multe etape:
- promovarea importanţei competenţelor transversale pentru facilitarea transferului cunoştinţelor,
- adoptarea transdisciplinarităţii ca principiu de organizare a conţinuturilor. Centrarea educaţiei pe formarea competenţelor răspunde cerinţelor crescânde şi diversificate ale societăţii faţă de şcoală. Competenţele oferă avantajul transferabilităţii şi o mai uşoară integrare socio-economică a absolvenţilor.
Printre competenţele cheie, recomandate de Comisia Europeană, se află competenţele matematice, în ştiinţe şi tehnologii, precum şi asumarea iniţiativei şi antreprenoriat sau sensibilizare la cultură şi exprimare culturală.
Articolul abordează problematica transdisciplinarităţii matematicii cu ştiinţele economice, prin exemple concrete de calcul matematic în economia şi gestiunea firmei, investiţii, etc. Într-o lume în continuă schimbare, aplicarea unui proiect în viaţa socială, economică sau educaţională este supus unor riscuri din ce în ce mai mari, iar eşecul poate afecta societatea pe perioade mari de timp. În vederea minimizării riscului, se fac calcule matematice ce pot reda, cu precizie foarte mare, eficienţa unui proiect pe termen mediu şi lung.

Asupra arhitecturii cursului de "Teoria probabilităţilor şi statistică matematică" pe platforma on-lineMOODLE

Vladislav Seiciuc, Victor Seiciuc, Eleonora Seiciuc

Universitatea Cooperatist Comercială din Modova, Universitatea de Stat din Modova, Chişinău, Republica Moldova
e-mail: seiciuc@mail.ru, runsmar@yahoo.com, eseiciuc@mail.ru

În lucrare este prezentat cursul universitar "Teoria probabilităţilor şi statistica matematică", creat on-line pe platforma electronică MOODLE (Modular Object-Oriented mediu dinamic de învăţare) pentru instruirea la distanţă. Acest curs electronic, numit în continuare modul, ideile şi direcţiile principale de elaborare a arhitecturii sale le are formulate, ajustate şi realizate în lucrarea [1]. Modulul are la bază:
1) Cursul universitar tradiţional [2] de Teorie a Probabilităţilor şi Statistica Matematică, predat

studenţilor la Universitatea Cooperatist - Comercială din Moldova pe parcursul ultimilor 15 ani, fiind recent editat în a doua ediţie.
2) Experienţa acumulată în rezultatul participării la stagiile de formare în domeniul e-learning-ului, activităţi în proiectul TEMPUS 516597 – Tempus 1- 2011-1-FR "Crearea unei reţele universitare tematice in ştiinţele aplicate şi economice în Moldova" (ghidat de Jean BARLOY, AgroCampus Ouest, or. Rennes, Franţa), ce s-au petrecut în ultimii doi ani în cadrul:
-instituţiilor superioare de învăţământ UTM, ASEM, UCCM din Republica Moldova;
- la Universitatea Tehnică "Gh. Asachi" din Iaşi, România;
- în Programul stagiului de totalizare intermediară la AgroCampus-ul Ouest, din or. Rennes, Franc ta.

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Proprietatile simedianei la rezolvarea problemelor de geometrie competitiva

Marcel Teleucă

Universitatea de Stat Tiraspol, Chişinău, Republica Moldova
e-mail: mteleuca@gmail.com

Proprietaţile simedianei deseori pot fi folosite la rezolvarea problemelor de concurs. Sunt utilizate în soluţii următoarele proprietaţii de bază ale simedianei:

1. simediana AD a triunghiului ABC şi mediana AM sunt simetrice faţă de bisectoarea unghiului BAC.
2. cele trei simediane ale tringhiului sunt concurente în punctual lui Lemoine.
3. simediana AD împarte latura BC în raportul $\frac{AB^2}{AC^2}$ (rezultă din teorema lui Steiner).

Cu ajutorul acestor proprietăţi sunt rezolvate mai multe probleme cu grad diferit de dificultate, spre exemplu:

1. Fie ABC un triunghi dreptunghic în A înscris în cercul ω. Tangentele în punctele A şi B la ω se intersectează în P. Să se arate că dreapta PC trece prin mijlocul înălţimii din vîrful A. *Gazeta Matematică.*
2. Trei puncte A, B, C se află, în această ordine, pe o dreaptă. Fie Γ un cerc care trece prin A şi C cu centrul în exterior dreptei AC, punctul P este intersecţia tangentelor la cerc, trasate prin A şi C, iar cercul Γ intersectează segmentul PB în Q. Să se demonstreze că intersec tia bisectoarei unghiului AQC cu dreapta AC nu depinde de alegerea cercului Γ. *IMO Shortlist 2003.*

3. Două cercuri se intersectează în punctele A şi B, iar una din tangentele commune intersectează cercurile în punctele P şi Q. Tangentele în punctele P şi Q la cercul circumscris triunghiului APQ se intersectează în S, iar H este simetricul punctului B faţă de dreapta PQ. Demonstraţi că punctele A, S şi H sunt coliniare. *Baraj Vietnam 2001.*

4. Fie ABC un triunghi scalen ascuţitunghic şi fie M, N şi P mijloacele laturilor BC, CA şi CB respective. Mediatoarele laturilor AB şi AC intersectează dreapta AM în punctele D şi E, iar dreptele BD şi CE se taie în F. Să se arate că punctele A, N, F şi P se află pe un cerc. *USAMO 2008.*

5. Fie ω cercul circumscris triunghiului ascuţitunghic ABC. Tangentele la ω în B şi C se intersectează în P, iar AP intersectează latura BC în punctul D. Punctele E şi F sunt situate pe laturile AC şi AB astfel încît $DE \parallel BA$ şi $DF \parallel CA$.

   (a) Să se arate că punctele F, B, C şi E sun conciclice.

   (b) Fie A_1 centrul cercului circumscris patrulaterului FBCE. Analog definim punctele B_1 şi C_1. Demonstraţi că dreptele AA_1, BB_1 şi CC_1 sunt concurente.
   Baraj China 2005.

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Elemente din teoria congruenţilor pentru liceeni

Boris Ţarălungă

UPS "Ion Creangă" Chişinău, Republica Moldova
e-mail: Borisstar@mail.ru

În lucrare se abordează următoarele tipuri de probleme: 1) Să se determine restul de la divizarea unui număr la un alt număr; 2) Să se determine ultimile 2 (respectiv, 3) cifre a unui număr reprezentat ca puterea căruiva număr; 3) Să se determine, dacă un număr dat se divide la un careva număr. Aceste probleme, cât şi altele necesită un efort sporit din partea liceenilor. Ele pot fi rezolvate cu mult mai uşor, dacă se aplică proprietăţile congruenţilor în mulţimea numerelor întregi. Din aceste motive, considerăm că este binevenită studierea la orele facultative în liceu a elementelor din teoria congruenţilor în mulţimea numerelor întregi.

Xcolony: a modular paper game used in learning geometry

Consuela Luiza Voica

Herăstrău Middle School, Bucharest, Romania
e-mail: consuelavoica@yahoo.com

The present paper investigates the use of a modular paper game in learning geometry. We present the game and the conclusions we obtained in using this game during two summer camps, in 2012 and 2013. The experiment involved students in grades 5 to 9, engaged in a guided play workshop. We report the conclusions on the use of this paper game regarding teaching – learning geometry.

Romanian students' misconceptions in learning Math – an analysis of the TIMSS 2007 results

Cristian Voica, Ariana - Stanca Văcăreţu

Dept. of Mathematics, University of Bucharest, Romania
e-mail: voica@gta.math.unibuc.ro

Trends in International Mathematics and Science Study) 2007 has been conducted in 59 countries around the world and its results provide information on both, fourth grade and eighth grade students' performance in mathematics and science. This study is widely recognized as one of the most influential studies, as it provides broad information about students' achievement in mathematics. This paper presents the most relevant eighth grade Romanian students' misconceptions in learning mathematics. The misconceptions were identified through both, a deep analysis of the TIMSS 2007 public items and the analysis of the eighth graders answers corroborated with the Romanian math curriculum, as part of a project (POSDRU/55/1.1/S/35279). We targeted in our analysis the Number, Algebra, and Geometry topics, even though some of the TIMSS 2007 items referred to the Data and Chance topic. The reason of our approach is based on the fact that in the Romanian math curriculum from the 5th to the 8th grade the Data and Chance topic has a very low representation. The aim of our paper is to raise math teachers' awareness towards students' misconceptions in learning math and to give them food for thinking on how they may manage these misconceptions.

Supporting mathematics students in their first years' studies: the tutorial system applied in FMI

Cristian Voica

Dept. of Mathematics, University of Bucharest, Romania
e-mail: voica@gta.math.unibuc.ro

The transition from high school to university is a challenge for many students. Mathematics students especially need support in at least two directions: to adapt their learning strategies; and to experiment new ways of thinking and diversified approaches. In this paper, we present a tutorial system developed in the Faculty of Mathematics and Computer Sciences from Bucharest University. Based on the results obtained during the last year, we can assume that our tutorial system has positive effects on some relevant learning strategies.

Referitor la interacţiunile matematicii cu teoria informaţiei în aspect istoric

Ion Valuţă, Evelina Valuţă

Universitatea Tehnică a Moldovei, Chişinău, Republica Moldova
e-mail: valutse@mail.md

Interacţiunile între ştiinţe se manifestă în mod diferit în fiecare perioadă de dezvoltare a lor. Perioadele de dezvoltare a matematicii au fost studiate din diferite puncte de vedere în [1],[2]. Cele ale informaticii, probleme mult mai dificile, în [2],[3]. Dar cum să înţelegem, ce este informaţia în cel mai general sens al acestei noţiuni? O caracterizare în sensul universal pe care dorim să îl promovăm se găseşte la Norbert Wiener: "Informaţia este informaţie, dar nu materie şi nu energie" [4]. Pentru a elimina negaţia, dar şi tautologia, ne permitem să reformulăm sentinţa lui Wiener în felul următor: "Universul constă din materie, energie şi informaţie". Informaţia fiind, o formă de manifestare, exprimare a orice (inclusiv a materiei şi energiei), Cu milioane de ani în urmă vietăţile încep să recepţioneze informaţia prin organele de simţ. Urmează codarea informaţiei, cea mai importantă şi eficientă fiind cea sonoră, ce duce la apariţia limbilor naturale, deci şi a noţiunilor matematice. Un punct crucial a fost momentul înţelegerii că bătrânii, deşi nu pot, dar ştiu, cunosc, începându-se cultul învăţătorului, înţeleptului. Matematica devine ştiinţă teoretică. Pe parcurs apar dispozitive mecanice de prelucrare a informaţiei: abacul, aritmometrele, inclusiv cele electrice. Crucial devine mijlocul secolului XX când, pe de o parte se introduce unitatea de măsură a informaţiei, generându-se Teoria matematică a informaţiei, pe de altă parte, confecţionându-se tehnica electronică de calcul, cu consecinţele ce urmează. Astfel, datorită interacţiunii în decurs de secole a trei ramuri de activitate intelectuală: a) Teoria informaţiei (informaţia, în cel mai larg sens al cuvântului, informatica), cu modalităţile de codare, stocare şi memorizare a informaţiei, cunoştinţelor ca atare; b) matematica, cu modalităţile sale de prelucrare a informaţiei; c) dispozitivele, maşinile de calcul, calculatoarele, ultimele, în aspectul lor electronic s-au constituit tehnologiile informaţionale contemporane.

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